ASIAN JOURNAL OF MATHEMATICS AND APPLICATIONS
Volume 2014, Article ID ama0181, 11 pages
ISSN 2307-7743
http://scienceasia.asia

A DECOMPOSITION OF CONTINUITY IN IDEAL BY USING SEMI-LOCAL FUNCTIONS

R. SANTHI AND M. RAMESHKUMAR

ABSTRACT. In this paper we introduce and investigate the notion of α-$\mathcal{I}_s$-open, semi-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-open sets via idealization by using semi-local functions and we obtained new decomposition of continuity.

1. Introduction

Ideal in topological space have been considered since 1930 by Kuratowski [9] and Vaidyanathaswamy[14]. After several decades, in 1990, Jankovic and Hamlett[6] investigated the topological ideals which is the generalization of general topology. Where as in 2010, Khan and Noiri[7] introduced and studied the concept of semi-local functions. Tong [13] and Hatir et al.[4] introduced B-sets in 1989 and C-sets in 1996 respectively to obtain a decomposition of continuity in topological space. Finally in 2002 Hatir et al. [5] introduced $B_{\mathcal{I}}$-sets, $C_{\mathcal{I}}$-sets, α-$\mathcal{I}$-sets, semi-$\mathcal{I}$-sets to obtain a decomposition of continuity in ideal topological spaces.

In this paper we introduce $B_{\mathcal{I}s}$-sets, $C_{\mathcal{I}s}$-sets, $S_{\mathcal{I}s}$-sets, α-$\mathcal{I}_s$-sets, semi-$\mathcal{I}_s$-sets and pre-$\mathcal{I}_s$-sets to obtain a decomposition of continuity in ideal topological spaces by using semi-local functions.

2. Preliminaries

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), $cl(A)$ and $int(A)$ denote the closure and interior of A in (X, τ) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$ (2)$A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

2010 *Mathematics Subject Classification.* 54A05, 54C10.
Key words and phrases. α-$\mathcal{I}_s$-open, semi-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-open sets.

If $(X, \tau)$is a topological space and $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal topological space or an ideal space.

Let $P(X)$ be the power set of X. Then the operator $()^* : P(X) \to P(X)$ called a local function [9] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X : U \cap A \notin \mathcal{I}$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(\mathcal{I}, \tau)$ in case there is no confusion. For every ideal topological space $(X, \tau, \mathcal{I})$ there exists topology τ^* finer than τ, generated by $\beta(\mathcal{I}, \tau) = \{U \setminus J : U \in \tau$ and $J \in \mathcal{I}\}$ but in general $\beta(\mathcal{I}, \tau)$ is not always a topology. Additionally $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ. Throughout this paper X denotes the ideal topological space $(X, \tau, \mathcal{I})$ and also $int^*(A)$ denotes the interior of A with respect to τ^*.

DEFINITION 2.1. Let (X, τ) be a topological space. A subset A of X is said to be semi-open [10] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by SO(X) (resp. SC(X)). The semi-closure of A in (X, τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by $scl(A)$.

DEFINITION 2.2. For $A \subseteq X, A_*(\mathcal{I}, \tau) = \{\ x \in X\ /\ U \cap A \notin \mathcal{I}$for every $U \in SO(X)\}$ is called the semi-local function [7] of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{U \in SO(X) : x \in U\}$. We simply write A_* instead of $A_*(\mathcal{I}, \tau)$ in this case there is no ambiguity.

It is given in [2] that $\tau^{*s}(\mathcal{I})$ is a topology on X, generated by the sub basis $\{\ U - E : U \in$ SO(X) and E$\in$ I $\}$ or equivalently $\tau^{*s}\mathcal{I} = \{U \subseteq X : cl^{*s}(X - U) = X - U\}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathcal{I})$ is defined as follows: for A $\subseteq X, cl^{*s}(A) = A \cup A_*$ and int^{*s} denotes the interior of the set A in $(X, \tau^{*s}, \mathcal{I})$. It is known that $\tau \subseteq \tau^*(\mathcal{I}) \subseteq \tau^{*s}(\mathcal{I})$. A subset A of $(X, \tau, \mathcal{I})$ is called semi-$*$-perfect [8] if $A = A_*$. $A \subseteq (X, \tau, \mathcal{I})$ is called $*$-semi dense in-itself [8](resp. semi-$*$-closed [8]) if $A \subset A_*$ (resp. $A_* \subseteq A$).

DEFINITION 2.3. A subset A of a topological space X is said to be

(1) α-open [12]if $A \subseteq int(cl(int(A)))$,
(2) semi-open [10] if $A \subseteq cl(int(A))$,
(3) pre-open [11] if $A \subseteq int(cl(A))$,
(4) β-open if [1] $A \subseteq cl(int(cl(A)))$,
(5) α^*-set [4] if $int(A) = int(cl(int(A)))$,
(6) C-set [4] if $A = U \cap V$, where U is an open set and V is an α^*-set,

(7) t-set [13] if $int(A) = int(cl(A))$,
(8) B-set [13] if $A = U \cap V$, where U is an open set and V is a t-set.

DEFINITION 2.4. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

(1) α-$\mathcal{I}$-open [5]if $A \subseteq int(cl^*(int(A)))$,
(2) pre-$\mathcal{I}$-open [3] if $A \subseteq int(cl^*(A))$,
(3) semi-$\mathcal{I}$-open [5] if $A \subseteq cl^*(int(A))$.

LEMMA 2.5. *For a subset A of a topological space (X, τ), the following properties hold:*

(1) $scl(A) = A \cup int(cl(A))$,
(2) $scl(A) = int(cl(A))$, *if A is open.*

DEFINITION 2.6. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

(1) t-$\mathcal{I}$-set [5] if $int(cl^*(A)) = int(A)$,
(2) α^*-$\mathcal{I}$-set[5] if $int(cl^*int(A))) = int(A)$,
(3) s-$\mathcal{I}$-set [5] if $cl^*(int(A)) = int(A)$.

LEMMA 2.7. [7] *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B be subsets of X. Then for the semi-local function the following properties hold:*

(1) *If $A \subseteq B$ then $A_* \subseteq B_*$.*
(2) *If $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$*
(3) *$A_* = scl(A_*) \subseteq scl(A)$ and A_* is semi-closed in X.*
(4) *$(A_*)_* \subseteq A_*$.*
(5) *$(A \cup B)_* = A_* \cup B_*$.*
(6) *If $\mathcal{I} = \{\phi\}$, then $A_* = scl(A)$.*

3. α-$\mathcal{I}_s$-open, semi-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-open sets

In this section we introduced α-$\mathcal{I}_s$-open, semi-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-open sets and studied some of their properties.

DEFINITION 3.1. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

(1) α-$\mathcal{I}_s$-open if $A \subseteq int(cl^{*s}(int(A)))$,
(2) semi-$\mathcal{I}_s$-open set if $A \subseteq cl^{*s}(intA))$,
(3) pre-$\mathcal{I}_s$-open set if $A \subseteq int(cl^{*s}(A))$.

PROPOSITION 3.2. *For a subset of an ideal topological space the following hold:*

1. *Every α-$\mathcal{I}_s$-open set is α-open.*
2. *Every semi-$\mathcal{I}_s$-open set is semi-open.*
3. *Every pre-$\mathcal{I}_s$-open set is pre-open.*

Proof.

1. Let A be a α-$\mathcal{I}_s$-open set. Then we have $A \subseteq int(cl^{*s}(int(A))) = int((int(A))_* \cup int(A) \subseteq int(scl(int(A)) \cup int(A)) \subseteq int(cl(int(A)) \cup int(A)) \subseteq int(cl(int(A)))$. This shows that A is an α-open.
2. Let A be a semi-$\mathcal{I}_s$-open set. Then we have $A \subseteq cl^{*s}(int(A)) = (int(A))_* \cup int(A) \subseteq scl(int(A)) \cup int(A) \subseteq cl(int(A)) \cup int(A)) \subseteq cl(int(A))$. This shows that A is semi-open.
3. Let A be a pre-$\mathcal{I}_s$-open set. Then we have $A \subseteq int(cl^{*s}(A)) = int(A_* \cup A) \subseteq int(scl(A) \cup A) \subseteq int(cl(A) \cup A) \subseteq int(cl(A))$. This shows that A is pre-open.

REMARK 3.3. *Converse of the Proposition 3.2 need not be true as seen from the following example.*

EXAMPLE 3.4. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Set $A = \{b, c\}, B = \{a, b, c\}$. Then A is semi-open but not semi-$\mathcal{I}_s$-open, B is an α-open but it is not an α-$\mathcal{I}_s$-open set.*

EXAMPLE 3.5. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Set $A = \{a, b, c\}$. Then A is pre-open but not pre -$\mathcal{I}_s$-open.*

PROPOSITION 3.6. *Every open set of an ideal topological space is an α-$\mathcal{I}_s$-open set.*

Proof. Let A be any open set. Then we have $A = int(A) \subseteq int(((int(A))_* \cup int(A)) = int(cl^{*s}(int(A))$. This shows that A is an α-$\mathcal{I}_s$-open set.

REMARK 3.7. *Converse of the Proposition 3.6 need not be true as seen from the following example.*

EXAMPLE 3.8. *Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\phi\}$. Set $A = \{a, b\}$. Then A is an α-$\mathcal{I}_s$-open set but $A \notin \tau$.*

PROPOSITION 3.9. *Every α-$\mathcal{I}_s$-open set is both pre-$\mathcal{I}_s$-open and semi-$\mathcal{I}_s$-open set.*

Proof. The proof is obvious.

REMARK 3.10. *Converse of the Proposition 3.9 need not be true as seen from the following example.*

EXAMPLE 3.11. *Let* $X = \{a, b, c, d\}, \tau = \{\phi, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ *and* $\mathcal{I} = \{\phi, \{d\}, \{c\}, \{c, d\}\}$. *Set* $A = \{a\}$ *is pre-*$\mathcal{I}_s$*-open but it is not an* α*-*$\mathcal{I}_s$*-open set. In Example 3.4,* $A = \{a,\ b,\ c\}$ *is semi-*$\mathcal{I}_s$*-open but it is not an* α*-*$\mathcal{I}_s$*-open set.*

PROPOSITION 3.12. *For a subset of an ideal topological space the following hold.*

(1) *Every* α*-*$\mathcal{I}_s$*-open set is an* α*-I-open.*
(2) *Every semi-*$\mathcal{I}_s$*-open set is semi-I-open.*
(3) *Every pre-*$\mathcal{I}_s$*-open set is pre-I-open.*

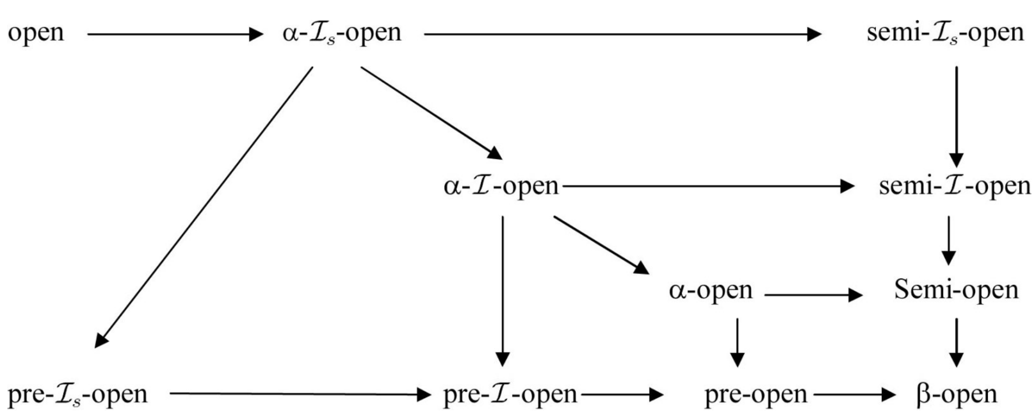
Proof. The proof is obvious.

REMARK 3.13. *Converse of the Proposition 3.12 need not be true as seen from the following example.*

EXAMPLE 3.14. *In Example 3.4,* $A = \{a,\ b,\ c\}$ *is an* α*-*$\mathcal{I}$*-open but it is neither* α*-*$\mathcal{I}_s$*-open nor pre-*$\mathcal{I}_s$*-open. In Example 3.11,* $A = \{a,\ b,\ c\}$ *is semi-*$\mathcal{I}$*-open but it is not semi-*$\mathcal{I}_s$*-open. In Example 3.5,* $A = \{a,\ b,\ c\}$ *is pre-*$\mathcal{I}$*-open but it is not pre-*$\mathcal{I}_s$*-open.*

EXAMPLE 3.15. *In Example 3.4,* $A = \{a,\ b,\ d\}$ *is* α*-*$\mathcal{I}_s$*-open but it is not* $\mathcal{I}$*-open.*

PROPOSITION 3.16. *We have the following diagram among several sets defined above.*



PROPOSITION 3.17. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and* A *an open subset of* X. *Then the following hold, if* $\mathcal{I} = \{\phi\}$, *then*

(1) *A is α-$\mathcal{I}_s$-open if and only if A is α-open.*
(2) *A is semi-$\mathcal{I}_s$-open if and only if A is semi-open.*
(3) *A is pre -$\mathcal{I}_s$-open if and only if A is pre -open.*

Proof. If $\mathcal{I} = \{\phi\}$, then $A_* = scl(A)$ for any subset A of X and hence $cl^{*s}(A) = A_* \cup A = scl(A)$.

(1) By Proposition 3.2, every α-$\mathcal{I}_s$-open set is an α-open. Conversely, if A is α-open then $A \subseteq int(cl(int(A))) = scl(int(A)) = cl^{*s}(int(A))$. Hence $A = int(A) \subseteq int(cl^{*s}(int(A)))$. Therefore A is α-$\mathcal{I}_s$-open. Thus A is α-$\mathcal{I}_s$-open if and only if A is α-open.
(2) By Proposition 3.2, every semi-$\mathcal{I}_s$-open set is semi-open. Conversely, if A is semi-open then $A \subseteq cl(int(A))$. Hence $A = int(A) \subseteq int(cl(int(A))) = scl(int(A)) = cl^{*s}(int(A))$. Therefore A is semi-$\mathcal{I}_s$-open. Thus A is semi-$\mathcal{I}_s$-open if and only if A is semi-open.
(3) By Proposition 3.2, every pre -$\mathcal{I}_s$-open set is pre -open. Conversely, if A is pre -open then $A \subseteq int(cl(A)) = scl(A) = cl^{*s}(A)$. Hence $A = int(A) \subseteq int(cl^{*s}(A))$. Therefore A is pre-$\mathcal{I}_s$-open. Thus A is pre -$\mathcal{I}_s$-open if and only if A is pre-open.

4. $B_{\mathcal{I}s}$-SETS, $C_{\mathcal{I}s}$-SETS AND $S_{\mathcal{I}s}$-SETS

In this section we introduce $B_{\mathcal{I}s}$-sets, $C_{\mathcal{I}s}$-sets and $S_{\mathcal{I}s}$-sets and studied some of their properties.

DEFINITION 4.1. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

(1) t-$\mathcal{I}_s$-set if $int(cl^{*s}(A)) = int(A)$,
(2) α^*-$\mathcal{I}_s$-set if $int(cl^{*s}(int(A))) = int(A)$,
(3) s-$\mathcal{I}_s$-set if $cl^{*s}(int(A)) = int(A)$.

DEFINITION 4.2. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

(1) $B_{\mathcal{I}s}$-set if $A = U \cap V$, where $U \in \tau$ and V is a t-$\mathcal{I}_s$-*set*,
(2) $C_{\mathcal{I}s}$-set if $A = U \cap V$, where $U \in \tau$ and V is an α^*-$\mathcal{I}_s$-*set*,
(3) $S_{\mathcal{I}s}$-set if $A = U \cap V$, where $U \in \tau$ and V is a s -$\mathcal{I}_s$-*set*.

PROPOSITION 4.3. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A be a subset of X. Then the following holds:*

(1) *If A is a t-set then A is a t-$\mathcal{I}_s$-set.*
(2) *If A is an α^*-set then A is an α^*-$\mathcal{I}_s$-set.*
(3) *If A is a semi-$*$-perfect then A is a t-$\mathcal{I}_s$-set.*
(4) *If A is a t-$\mathcal{I}_s$-set then A is an α^*-$\mathcal{I}_s$-set.*

Proof.

(1) Let A be a t-set. Then we have $int(cl^{*s}(A)) = int(A_* \cup A) \subseteq int(scl(A) \cup A) \subseteq int(cl(A) \cup A) = int(cl(A)) = int(A)$. Now $A \subseteq cl^{*s}(A)$ and $int(A) \subseteq int(cl^{*s}(A))$. Therefore we obtain $int(cl^{*s}(A)) = int(A)$.
(2) Let A be an α^*-set. Then we have $int(cl^{*s}(int(A))) = int((int(A))_* \cup int(A)) \subseteq int(scl(int(A)) \cup int(A)) \subseteq int(cl(int(A)) \cup int(A)) = int(cl(int(A))) = int(A)$. Now $int(A) \subseteq cl^{*s}(int(A))$ and $int(A) \subseteq int(cl^{*s}(int(A)))$. Therefore we obtain $int(cl^{*s}(int(A))) = int(A)$.
(3) Let A be a semi-$*$-perfect. Then $int(cl^{*s}(A)) = int(A_* \cup A) = int(A)$.
(4) Let A be a t -$\mathcal{I}_s$-set. Then we have $int(A) = int(cl^{*s}(A)) \supseteq int(cl^{*s}(int(A))) \supseteq int(A)$ and hence $int(cl^{*s}(int(A))) = int(A)$.

REMARK 4.4. *Converse of the Proposition 4.3 need not be true as seen from the following example.*

EXAMPLE 4.5. *In Example 3.4, $A = \{a,\ b,\ d\}$ is both t-$\mathcal{I}_s$-set and α^*-$\mathcal{I}_s$-set but it is neither t-set nor α^*-set*

EXAMPLE 4.6. *Let $X = \{a, b, c, d\}, = \{\phi, \{c\}, \{a, b\}, \{a, b, c\}, X\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Set $A = \{b,\ c,\ d\}$ is α^*-$\mathcal{I}_s$-set but it is not t-$\mathcal{I}_s$-set also $B = \{a,\ c,\ d\}$ is t-$\mathcal{I}_s$-set but it is not semi-$*$-perfect.*

PROPOSITION 4.7. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A be a subset of X. Then the following holds:*

(1) *If A is a t-$\mathcal{I}_s$-set then A is a $B_{\mathcal{I}s}$-set.*
(2) *If A is an α^*-$\mathcal{I}_s$-set then A is a $C_{\mathcal{I}s}$-set.*
(3) *If A is a s-$\mathcal{I}_s$-set then A is a $S_{\mathcal{I}s}$-set.*

Proof.

(1) Let A be a t -$\mathcal{I}_s$-set. If we take $U = X \in \tau$, then $A = U \cap A$ and hence A is a $B_{\mathcal{I}s}$-set.
(2) This is obvious.
(3) Trivial.

REMARK 4.8. *We have the following diagram among several sets defined above.*

$$\begin{array}{ccccccc}
\alpha^*\text{-set} & \longrightarrow & C\text{-set} & \longleftarrow & B\text{-set} & \longleftarrow & t\text{-set} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\alpha^*\text{-}\mathcal{I}_s\text{-set} & \longrightarrow & C_{\mathcal{I}s}\text{-set} & \longleftarrow & B_{\mathcal{I}s}\text{-set} & \longleftarrow & t\text{-}\mathcal{I}_s\text{-set} \\
\uparrow & & \uparrow & & & & \\
s\text{-}\mathcal{I}_s\text{-set} & \longrightarrow & S_{\mathcal{I}s}\text{-set} & & & &
\end{array}$$

EXAMPLE 4.9. *In Example 4.6, $A = \{b,\ c,\ d\}$ is s-$\mathcal{I}_s$-set but it is not t-$\mathcal{I}_s$-set. Therefore A is a $S_{\mathcal{I}s}$-set but not a $B_{\mathcal{I}s}$-set. For $cl^{*s}(int(A)) = cl^{*s}(\{c\}) = \{c\} \cup \{c\} = int(A)$ and hence A is a $S_{\mathcal{I}s}$-set. Since $A_* = X, int(cl^{*s}(A)) = int(A_* \cup A) = X \neq \{c\} = int(A)$ and hence A is not a t-$\mathcal{I}_s$-set and A is not a $B_{\mathcal{I}s}$-set.*

EXAMPLE 4.10. *Let $X = \{a, b, c, d\}, = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, X\}$ and $\mathcal{I} = \phi$, set $A = \{b\}$ is a t-$\mathcal{I}_s$-set and hence a $B_{\mathcal{I}s}$-set. But $A = \{b\}$ is not a $S_{\mathcal{I}s}$-set. For $cl^{*s}(int(A)) = cl^{*s}(\{b\}) = \{b\} \cup \{a, b\} = \{a, b\} \neq a = int(A)$ and hence A is not a s-$\mathcal{I}_s$-set .*

EXAMPLE 4.11. *In Example 3.4, $A = \{a\}$ is both α-$\mathcal{I}_s$-open and semi-$\mathcal{I}_s$-open but it is neither $C_{\mathcal{I}s}$-set nor $S_{\mathcal{I}s}$-set.*

REMARK 4.12. *The notion of pre-$\mathcal{I}_s$-openness (resp. α-$\mathcal{I}_s$-openness, semi-$\mathcal{I}_s$-openness) is different from $B_{\mathcal{I}s}$-sets (resp. $C_{\mathcal{I}s}$-sets, $S_{\mathcal{I}s}$-sets).*

EXAMPLE 4.13. *In Example 3.4, $A = \{a,\ b,\ c\}$ is a pre-$\mathcal{I}_s$-open, but it is not a $B_{\mathcal{I}s}$-set. Since A is not a t-$\mathcal{I}_s$-set. $B = \{b,\ c\}$ is a t-$\mathcal{I}_s$-set and hence B is a $B_{\mathcal{I}s}$-set but it is not a pre-$\mathcal{I}_s$-open.*

EXAMPLE 4.14. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}, X\}$ and $\mathcal{I} = \{\phi, \{d\}\}$. Set $A = \{a, b, c\}$ is a semi-$\mathcal{I}_s$-open, but it is not a $S_{\mathcal{I}s}$-set. Since A is not a s-$\mathcal{I}_s$-set. $B = \{b,\ c\}$ is a s-$\mathcal{I}_s$-set and hence B is a $S_{\mathcal{I}s}$-set but it is not a semi-$\mathcal{I}_s$-open.*

EXAMPLE 4.15. *Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\phi, \{c\}\}$. Set $A = \{a, b, c\}$ is an α-$\mathcal{I}_s$-open, but it is not a $C_{\mathcal{I}s}$-set. Since A is not a α^*-$\mathcal{I}_s$-set. $B = \{b,\ c\}$ is a $S_{\mathcal{I}s}$-set and hence B is a $C_{\mathcal{I}s}$-set but it is not a α-$\mathcal{I}_s$-open.*

PROPOSITION 4.16. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A be a subset of X. Then the following conditions are equivalent:*

1. *A is open.*
2. *A is pre-$\mathcal{I}_s$-open and $B_{\mathcal{I}s}$-set.*
3. *A is α-$\mathcal{I}_s$-open and $C_{\mathcal{I}s}$-set.*
4. *A is semi-$\mathcal{I}_s$-open and $S_{\mathcal{I}s}$-set.*

Proof. $(a) \Rightarrow (b), (a) \Rightarrow (c)$ and $(a) \Rightarrow (d)$ are obvious.
$(b) \Rightarrow (a)$: By the pre-$\mathcal{I}_s$-openness of A, $A \subseteq int(cl^{*s}(A)) = int(cl^{*s}(U \cap V))$, where $U \in \tau$ and V is a t-$\mathcal{I}_s$-set. Hence $A \subseteq int(cl^{*s}(U)) \cap int(cl^{*s}(V))$. Now $A \subseteq U \cap A \subseteq U \cap [int(cl^{*s}(U)) \cap int(V)] = U \cap int(V) = int(A)$. This shows that A is open.
$(c) \Rightarrow (a)$:
By the α-$\mathcal{I}_s$-openness of A, $A \subseteq int(cl^{*s}(int(A))) = int(cl^{*s}(int(U \cap V)))$, where $U \in \tau$ and V is a α^*-$\mathcal{I}_s$-set. Hence $A \subseteq int(cl^{*s}(int(U))) \cap int(cl^{*s}(int(V)))$. Now $A \subseteq U \cap A \subseteq U \cap [int(cl^{*s}(int(U))) \cap int(V)] = U \cap int(V) = int(A)$. This shows that A is open.
$(d) \Rightarrow (a)$:
By the semi-$\mathcal{I}_s$-openness of A, $A \subseteq cl^{*s}(int(A)) = cl^{*s}(int(U \cap V))$, where U $\in \tau$ and V is a s-$\mathcal{I}_s$-set. Hence $A \subseteq cl^{*s}(int(U)) \cap cl^{*s}(int(V))$. Now $A \subseteq U \cap A \subseteq U \cap [cl^{*s}(int(U)) \cap int(V)] = U \cap int(V) = int(A)$. This shows that A is open

5. Decomposition of Continuity

In this section we introduce α-$\mathcal{I}_s$-continuous, semi -$\mathcal{I}_s$-continuous, pre -$\mathcal{I}_s$-continuous and studied some of their properties.

DEFINITION 5.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be α-$\mathcal{I}$-continuous [5] (resp. semi -$\mathcal{I}$-continuous[5], pre -$\mathcal{I}$-continuous[3]) if for every $V \in \sigma$, $f^{-1}(V)$ is an α-$\mathcal{I}$-set (resp. semi -$\mathcal{I}$-set, pre -$\mathcal{I}$-set) of $(X, \tau, \mathcal{I})$.

DEFINITION 5.2. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be α-continuous[12] (resp. semi -continuous[10], pre -continuous[11] if for every $V \in \sigma, f^{-1}(V)$ is an α-open (resp. semi-open, pre-open) of (X, τ).

DEFINITION 5.3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be α-$\mathcal{I}_s$-continuous(resp. semi -$\mathcal{I}_s$-continuous, pre -$\mathcal{I}_s$-continuous) if for every $V \in \sigma, f^{-1}(V)$ is an α-$\mathcal{I}_s$-set (resp. semi-$\mathcal{I}_s$-set, pre -$\mathcal{I}_s$-set) of $(X, \tau, \mathcal{I})$.

PROPOSITION 5.4. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be α-$\mathcal{I}_s$-continuous(resp. semi -$\mathcal{I}_s$-continuous, pre -$\mathcal{I}_s$-continuous) then f is α-continuous (resp. semi-continuous, pre-continuous).*

Proof. This is an immediate consequence of Proposition 3.2

PROPOSITION 5.5. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be α-$\mathcal{I}_s$-continuous(resp. semi -$\mathcal{I}_s$-continuous, pre -$\mathcal{I}_s$-continuous) then f is α-$\mathcal{I}$-continuous (resp. semi -$\mathcal{I}$-continuous, pre -$\mathcal{I}$-continuous).*

Proof. This is an immediate consequence of Proposition 3.12

DEFINITION 5.6. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be B-continuous[13] (resp. C-continuous[4]) if for every $V\in\sigma, f^{-1}(V)$ is a B-set (resp. C-set) of (X,τ).

DEFINITION 5.7. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be $B_{\mathcal{I}}$-continuous [5] (resp. $C_{\mathcal{I}}$-continuous[5], $S_{\mathcal{I}}$-continuous[5]) if for every $V\in\sigma, f^{-1}(V)$ is an $B_{\mathcal{I}}$-set (resp. $C_{\mathcal{I}}$-set, $S_{\mathcal{I}}$-set) of (X,τ,I).

DEFINITION 5.8. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be $B_{\mathcal{I}s}$-continuous (resp. $C_{\mathcal{I}s}$-continuous, $S_{\mathcal{I}s}$-continuous) if for every $V\in\sigma, f^{-1}(V)$ is an $B_{\mathcal{I}s}$-set (resp. $C_{\mathcal{I}s}$-set, $S_{\mathcal{I}s}$-set) of (X,τ,I).

PROPOSITION 5.9. *If a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be B-continuous(resp. C-continuous) then f is $B_{\mathcal{I}s}$-continuous (resp. $C_{\mathcal{I}s}$-continuous).*

Proof. Proof is obvious.

THEOREM 5.10. *Let $(X,\tau,\mathcal{I})$ be an ideal topological space for a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ the following conditions are equivalent:*

(1) *f is continuous.*
(2) *f is pre -$\mathcal{I}_s$-continuous and $B_{\mathcal{I}s}$-continuous.*
(3) *f is α-$\mathcal{I}_s$-continuous and $C_{\mathcal{I}s}$-continuous.*
(4) *f is semi -$\mathcal{I}_s$-continuous and $S_{\mathcal{I}s}$-continuous.*

Proof. Proof is trivial from Proposition 4.16

COROLLARY 5.11. *Let $(X,\tau,\mathcal{I})$ be an ideal topological space and $\mathcal{I}=\{\phi\}$ and A is open. For a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ the following conditions are equivalent:*

(1) *f is continuous.*
(2) *f is pre-continuous and B-continuous.*
(3) *f is α-continuous and C-continuous.*

Proof. Since $\mathcal{I}=\{\phi\}$, we have $A_*=scl(A)$ and $cl^{*s}(A)=A_*\cup A=scl(A)$ for any open subset A of X. Therefore we obtain (a)α-$\mathcal{I}_s$-open (resp. pre -$\mathcal{I}_s$-open) if and only if it is α-open (resp. Pre-open) (b) A is a $C_{\mathcal{I}s}$-set (resp. $B_{\mathcal{I}s}$-set) if and only if it is a C-set (resp. B-set). The proof follows from Theorem 3.10

References

[1] M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud, *β-open sets and β-continuous mappings,* Bull. Fac. Sci. Assiut Univ. , 12 (1983), 77-90.
[2] M.E. Abd El-Monsef, E.F. Lashien and A.A. Nasef, *Some topological operators via ideals*, Kyungpook Math. J., 32, No. 2 (1992), 273-284.

[3] J. Dontchev, *Idealization of Ganster-Reilly decomposition theorems*, http://arxiv.org/abs/ Math. GN/9901017, 5 Jan. 1999(Internet).
[4] E. Hatir, T.Noiri and S. Yuksel, *A decomposition of continuity*, Acta. Math. Hungar. 70(1996), 145-150.
[5] E. Hatir and T.Noiri, *On decompositions of continuity via idealization*, Acta. Math. Hungar. 96(4)(2002), 341-349.
[6] D. Jankovic and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4) (1990), 295-310.
[7] M. Khan and T. Noiri, *Semi-local functions in ideal topological spaces*, J. Adv. Res. Pure Math., 2(1) (2010), 36-42.
[8] M. Khan and T. Noiri, *On gI-closed sets in ideal topological spaces*, J. Adv. Stud. in Top., 1(2010),29-33.
[9] K. Kuratowski, *Topology*, Vol. I, Academic press, New York, 1966.
[10] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36-41.
[11] A. S. Mashhour, M. E. Abd. El-Monsef and S. N. El-deeb, *On pre-continuous and weak pre-continuous mapping*, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47-53.
[12] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15 (1965), 961-970.
[13] J. Tong, *On decompositions of continuity in topological space*, Acta. Math. Hungar. 54(1989), 51-55.
[14] R. Vaidyanathaswamy, *Set Topology*, Chelsea Publishing Company, 1960.

R. Santhi, Department of Mathematics, NGM College, Pollachi-642001, Tamil Nadu, India

M. Rameshkumar, Department of Mathematics, P. A College of Engineering and Technology, Pollachi-642002, Tamil Nadu, India