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NEW EXACT SOLUTIONS FOR SOME NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper we propose a new form of Pade′-II equation, namely, a modified Pade′-II equation. As the mapping method is a promising method to solve nonlinear evolution equations. We apply it, to solve the Pade′-II, modified Pade′-II equations and the (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functions and elliptic functions.

1. Introduction

In recent years, directly searching for exact solutions of nonlinear partial differential equations (PDE's) has become more and more attractive field in different branches of physics and applied mathematics. These equations appear in condensed matter, solid state physics, fluid mechanics, chemical kinetics, plasma physics, nonlinear optics, propagation of fluxions in Josephson junctions, theory of turbulence, ocean dynamics, biophysics and star formation and many others. In order to get exact solutions directly, many powerful methods have been introduced such as the $\left(\frac{G'}{G}\right)$-expansion method [1], inverse scattering method [2,4], Hirota's bilinear method [5,6], the tanh method [7,8], the sine-cosine method [9,10], Backlund transformation method [11,12], the homogeneous balance [13,14], Darboux transformation [15], the Jacobi elliptic function expansion method [16], A modified tanh-coth method [17,18].

Recently, Yan-Ze Peng [19] introduced a new approach, namely, the mapping method for a reliable treatment of the nonlinear wave equations. The useful mapping method is then widely used by many authors [20,21] and others.

2. Description of the method

Consider the general nonlinear PDE's, say, in two variables,

$$P(u, u_x, u_t, u_{xx}, u_{xt}, ...) = 0. \tag{1}$$

Let $u(x,t) = u(\xi)$, where $\xi = \lambda\,(x - ct)$, then equation (1) reduces to a nonlinear ordinary differential equation (ODE)

$$Q(u, u', u'', ...) = 0. \tag{2}$$

Key words and phrases. Mapping method; (KD) equation; Pade ′-II equation; modified Pade ′-II equation; (2+1) dimensional Konopelchenko-Dubrovsky equation.

Assume the solution of equation (2) takes the form

$$u(x,t)=u(\xi)=a_0+\sum_{i=1}^{m}a_i\left(f(\xi)\right)^i+b_i\left(f(\xi)\right)^i, \tag{3}$$

where the coefficients $a_0, a_i, b_i(i=0,1,2,...,m\ ), \lambda, c$ are constants to be determined, and $f=f(\xi)$ satisfies a nonlinear ODE

$$\frac{df(\xi)}{d\xi}=\sqrt{pf^2(\xi)+\frac{1}{2}qf^4(\xi)+r}, \tag{4}$$

where $p,q,r\in R$.

Substituting (3) into (2), using (4) repeatedly, the parameter m will be found by balancing the highest-order nonlinear term with the highest-order partial derivative term in the resulting equation. Setting the coefficients of the each order of $f^i(\xi)$ and $f^i(\xi)\sqrt{pf^2(\xi)+\frac{1}{2}qf^4(\xi)+r}$ to zero, we obtain a set of nonlinear algebraic equations for $a_0, a_i, b_i(i=1,2,...n\ ), \lambda, c$. With the aid of Maple, a symbolic computer program, we can solve the set of nonlinear algebraic equations and obtain all the constants $a_0, a_i, b_i(i=1,2,...n\ ), \lambda$ and c.

The ODE (4) has the following solutions:

when $p=1, q=-2, r=0$, then

$$f(\xi)=\mathrm{sech}(\xi) \tag{5}$$

when $p=-2, q=2, r=1$, then

$$f(\xi)=\tanh(\xi) \tag{6}$$

when $p=\frac{1}{2}, q=\frac{1}{2}r=\frac{1}{4}$, then

$$f(\xi)=\tan(\xi)\pm\sec(\xi) \tag{7}$$

when $p=-\left(1+k^2\right), q=2k^2, r=1$, then

$$f(\xi)=\mathrm{sn}(\xi) \tag{8}$$

when $p=2k^2-1, q=-2k^2, r=1-k^2$, then

$$f(\xi)=\mathrm{cn}(\xi) \tag{9}$$

when $p=2-k^2, q=-2, r=-\left(1-k^2\right)$, then

$$f(\xi)=\mathrm{dn}(\xi) \tag{10}$$

when $p=2-k^2, q=2, r=1-k^2$, then

$$f(\xi)=\mathrm{cs}(\xi) \tag{11}$$

when $p=-\left(1+k^2\right), q=2, r=k^2$, then

$$f(\xi)=\mathrm{dc}(\xi) \tag{12}$$

when $p=-1+2k^2, q=2, r=-k^2\left(1-k^2\right)$, then

$$f(\xi)=\mathrm{ds}(\xi) \tag{13}$$

when $p=0, q=2, r=0$ and c is an arbitrary constant, then

$$f(\xi)=\frac{c}{\xi}, \tag{14}$$

when $p=1, q=0, r=0$, then

$$f(\xi)=e^{\xi} \tag{15}$$

3. Applications

3.1. Pade′-II Equation. We consider the Pade′-II equation in the form

$$u_t+u_x+uu_x+au_{xxx}+bu_{xxt}=0, \tag{16}$$

where $u=u\left(x,t\right)$, a,b are real numbers.

Pade′-II equation is a nonlinear wave equation modeling unidirectional propagation of long wave in dispersive media. It is originally derived by using a Pade$'$ $(2,2)$ approximation of the phase velocity that arises in linear wave theory [22].

We will apply the mapping method to solve the nonlinear general Pade′-II equation. Substituting $u(x,t)=u(\xi)$, where $\xi=\lambda\left(x-ct\right)$, into Eq. (16) and integrating once yields

$$\left(1-c\right)u+\frac{u^2}{2}+\lambda^2\left(a-bc\right)u''=0. \tag{17}$$

Balancing the order of the nonlinear term u^2 with the highest derivative u'' gives $2m=m+2$ that gives $m=2$. Thus, the solution of Eq. (17) has the form

$$u\left(\xi\right)=a_0+a_1f\left(\xi\right)+a_2\left(f\left(\xi\right)\right)^2+b_1\left(f\left(\xi\right)\right)^{-1}+b_2\left(f\left(\xi\right)\right)^{-2}. \tag{18}$$

Substituting (18) in (17) and using (4), collecting the coefficients of each power of $f^i, 0\leq i\leq 8$, setting each coefficient to zero, and solving the resulting system, obtain three sets of solutions.

(A) $a_0=-2+2c$, $a_1=a_2=0$, $\lambda=\lambda$
(B) $a_0=\left(1-c\right)\left(4\alpha p-1\right), a_1=0$, $a_2=-6\alpha\left(c-1\right)q$, $\lambda=\pm\sqrt{\frac{c-1}{a-bc}}\beta$
(C) $a_0=\left(c-1\right)\left(4\alpha p+1\right), a_1=0$, $a_2=6\alpha\left(c-1\right)q$, $\lambda=\pm\sqrt{\frac{1-c}{a-bc}}\beta$

where $\alpha=\sqrt{\frac{1}{16p^2-24rq}}$ and $\beta=\sqrt{\frac{2}{2p^2-3rq}}$. Note that $b_1=b_2=0$ for all sets of solutions.

- Using (5), a solution of the ODE (4), and (A), (B) and (C), Eq. (18) gives

$$u_1(x,t)=-2+2c \qquad \text{(constant solution)}.$$

for $c>1$

$$u_2(x,t)=3\,(c-1)\,\mathrm{sech}^2\left[\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}\,(x-ct)\right],$$

$$u_3(x,t)=-2+2c+3\,(1-c)\sec^2\left[\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}\,(x-ct)\right].$$

respectively, for $c<1$

$$u_4(x,t)=3\,(c-1)\sec^2\left[\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}\,(x-ct)\right],$$

$$u_5(x,t)=-2+2c+3\,(1-c)\,\mathrm{sech}^2\left[\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}\,(x-ct)\right].$$

- Using (6) and (B) and (C), Eq. (18) gives u_2 and u_3 for $c>1$ and u_4 and u_5 for $c<1$, respectively.
- Using (8) and (B) and (C), Eq. (18) gives

$$u_{6,7}\,(x,t)=a_0+a_2\,\mathrm{sn}^2\,(\lambda\xi)\,,$$

respectively, where a_0, a_2 are defined in (B) and (C).

When $k\to 0$, $u_{6,7}$ become u_1, when $k\to 1$, we obtain u_2 and u_3 for $c>1$ and u_4 and u_5 for $c<1$, respectively.

- Using (9) and (B) and (C), Eq. (18) gives

$$u_{8,9}\,(x,t)=a_0+a_2\,\mathrm{cn}^2\,(\lambda\xi)\,,$$

respectively, where a_0, a_2 are defined in (B) and (C). As $k\to 0$, $u_{8,9}$ become u_1 and as $k\to 1$ we obtain u_2 and u_3.

- Using (10), and (B) and (C), Eq. (18) gives

$$u_{10,11}\,(x,t)=a_0+a_2\,\mathrm{dn}^2\,(\lambda\xi)\,,$$

respectively, where a_0, a_2 are defined in (B) and (C). As $k\to 0$, we obtain constant solution, as $k\to 1$, we obtain u_1.

- Using (11), and (B) and (C), Eq. (18) gives

$$u_{12,13}\,(x,t)=a_0+a_2\,\mathrm{cs}^2\,(\lambda\xi)\,,$$

respectively, where a_0, a_2 are defined in (B) and (C). As $k\to 0$ and $c>1$, $u_{12,13}$ become

$$u_{14}\,(x,t)=1-c+3\,(1-c)\cot^2\left[\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}\,(x-ct)\right],$$

$$u_{15}\,(x,t)=3c-3-3\,(c-1)\coth^2\left[\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}\,(x-ct)\right].$$

and for $c<1$, we obtain

$$u_{16}(x,t)=1-c-3(1-c)\coth^2\left[\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right],$$

$$u_{17}(x,t)=3c-3+3(c-1)\cot^2\left[\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right].$$

As $k\to 1$, we obtain $u_{14},...,u_{17}$, for $c>0$ and $c<0$, respectively.

- Using (12), and (B) and (C), Eq. (18) gives

$$u_{18,19}(x,t)=a_0+a_2\,\mathrm{ns}^2(\lambda\xi),$$

respectively, where a_0,a_2 are defined in (B) and (C). As $k\to 1$, $u_{18,19}$ become u_{14} and u_{15} and as $k\to 0$, we obtain $u_{14},...,u_{17}$, for $c>0$ and $c<0$, respectively.

- Using (7), and (B) and (C), respectively, Eq. (18) gives

for $c>1$

$$u_{20}(x,t)=1-c+3(1-c)\left(\tan\sqrt{\frac{c-1}{a-bc}}(x-ct)\pm\sec\sqrt{\frac{c-1}{a-bc}}(x-ct)\right)^2,$$

$$u_{21}(x,t)=3(c-1)\left(1+\left(i\tanh\sqrt{\frac{c-1}{a-bc}}(x-ct)\pm\mathrm{sech}\sqrt{\frac{c-1}{a-bc}}(x-ct)\right)^2\right).$$

for $c<1$

$$u_{22}(x,t)=1-c+3(1-c)\left(i\tanh\sqrt{\frac{1-c}{a-bc}}(x-ct)\pm\mathrm{sech}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)^2,$$

$$u_{23}(x,t)=3(c-1)\left(1+\left(\tan\sqrt{\frac{1-c}{a-bc}}(x-ct)\pm\sec\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)^2\right).$$

3.2. **Modified Pade′-II Equation.** In this section, we browse our proposed equation, namely, a modified Pade′-II equation as the form

$$u_t+u_x+u^2u_x+au_{xxx}+bu_{xxt}=0, \tag{19}$$

where $u=u(x,t)$, a,b are real numbers.

Now, we apply the mapping method to solve our equation, as a consequence, we get the original solutions for our new equation, as the follows

Substituting $u(x,t)=u(\xi)$, $\xi=\lambda(x-ct)$, into Eq. (19) and integrating once yields

$$(1-c)u+\frac{u^3}{3}+\lambda^2(a-bc)u''=0. \tag{20}$$

Balancing the order of the nonlinear term u^3 with the highest derivative u'' gives $3m = m+2$ that gives $m = 1$. Thus, the solution of (20) has the form

$$u(\xi) = a_0 + a_1 f(\xi) + b_1 (f(\xi))^{-1} \tag{21}$$

Substituting (21) in (20) and using (4), collecting the coefficients of each power of $f^i, 0 \leq i \leq 6$, setting each coefficient to zero, and solving the resulting system, obtain the following sets of solutions.

(1) $a_0 = \sqrt{-3+3c}\ , a_1 = b_1 = 0\ , \lambda = \lambda$

(2) $a_0 = 0\ , a_1 = \sqrt{3\frac{q}{p}(1-c)}\ , b_1 = 0\ , \lambda = \pm\sqrt{\frac{c-1}{p(a-bc)}}$

(3) $a_0 = 0\ , a_1 = -\sqrt{3\frac{q}{p}(1-c)}\ , b_1 = 0\ , \lambda = \pm\sqrt{\frac{c-1}{p(a-bc)}}$

(4) $a_0 = 0\ , a_1 = 0\ , b_1 = \sqrt{\frac{6r}{p}(1-c)}\ , \lambda = \pm\sqrt{\frac{c-1}{p(a-bc)}}$

(5) $a_0 = 0\ , a_1 = 0\ , b_1 = -\sqrt{\frac{6r}{p}(1-c)}\ , \lambda = \pm\sqrt{\frac{c-1}{p(a-bc)}}$

(6) $a_0 = 0, a_1 = \pm\frac{(c-1)\left[p(a-bc)\left(3\sqrt{2qr}-p\right)-1\right]}{(a-bc)(18qr-p^2)\sqrt{6r(c-1)\frac{3\sqrt{2qr}-p}{p^2-18qr}}},$

$b_1 = \mp\sqrt{6r(c-1)\frac{3\sqrt{2qr}-p}{p^2-18qr}}, \lambda = \frac{\sqrt{18qr(c-1)(a-bc)(p^2-1)\left(p-3\sqrt{2qr}\right)}}{(a-bc)(18qr-p^2)}$

(7) $a_0 = 0, a_1 = \pm\frac{(c-1)\left[p(a-bc)\left(3\sqrt{2qr}-p\right)-1\right]}{(a-bc)(18qr-p^2)\sqrt{6r(c-1)\frac{3\sqrt{2qr}-p}{p^2-18qr}}},$

$b_1 = \mp\sqrt{6r(c-1)\frac{3\sqrt{2qr}-p}{p^2-18qr}}, \lambda = -\frac{\sqrt{18qr(c-1)(a-bc)(p^2-1)\left(p-3\sqrt{2qr}\right)}}{(a-bc)(18qr-p^2)}$

Using (21), the solution of Eq. (4) when $p = 1, q = -2, r = 0$, and the sets of solutions 2-7, we get

$u_1(x,t) = \pm\sqrt{-3+3c}.$

for $c > 1$ and $a > bc$

$u_2(x,t) = \pm\sqrt{-6+6c}\,\text{sech}\sqrt{\frac{c-1}{a-bc}}(x-ct).$

for $c < 1$ and $a < bc$

$u_3(x,t) = \pm i\sqrt{6-6c}\,\text{sech}\sqrt{\frac{1-c}{bc-a}}(x-ct).$

for $c < 1$ and $a > bc$

$u_4(x,t) = \pm i\sqrt{6(1-c)}\sec\sqrt{\frac{1-c}{a-bc}}(x-ct).$

for $c > 1$ and $a < bc$

$u_5(x,t) = \pm\sqrt{6(c-1)}\sec\sqrt{\frac{c-1}{bc-a}}(x-ct).$

Using Eq. (21), the solution of (4) when $p = -2, q = 2, r = 1$, and the sets of solutions 2-7, we get for $c < 1$ and $a < bc$

$u_6(x,t) = \pm\sqrt{3-3c}\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}(x-ct)\right),$

$u_7(x,t) = \pm\sqrt{3-3c}\cot\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}(x-ct)\right),$

$u_8(x,t) = \pm\sqrt{\frac{3}{2}(1-c)}\left[\tanh\left(\frac{1}{2}\sqrt{\frac{1-c}{bc-a}}(x-ct)\right) - \coth\left(\frac{1}{2}\sqrt{\frac{1-c}{bc-a}}(x-ct)\right)\right],$

$u_9(x,t) = \pm\frac{1}{2}\sqrt{3(1-c)}\left[\tan\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{1-c}{bc-a}\right)}(x-ct)\right) - \cot\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{1-c}{bc-a}\right)}(x-ct)\right)\right].$

for $c>1$ and $a>bc$

$u_{10}(x,t)=\pm i\sqrt{-3+3c}\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}(x-ct)\right),$

$u_{11}(x,t)=\pm i\sqrt{-3+3c}\cot\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}(x-ct)\right),$

$u_{12}(x,t)=\pm i\sqrt{\frac{3}{2}(c-1)}\left[\tanh\left(\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}(x-ct)\right)-\coth\left(\frac{1}{2}\sqrt{\frac{c-1}{a-bc}}(x-ct)\right)\right],$

$u_{13}(x,t)=\pm\frac{1}{2}i\sqrt{3(c-1)}\left[\tan\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{c-1}{a-bc}\right)}(x-ct)\right)-\cot\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{c-1}{a-bc}\right)}(x-ct)\right)\right].$

for $c>1$ and $a<bc$

$u_{14}(x,t)=\pm\sqrt{-3+3c}\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right),$

$u_{15}(x,t)=\pm\sqrt{-3+3c}\coth\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right),$

$u_{16}(x,t)=\pm\sqrt{\frac{3}{2}(c-1)}\left[\tan\left(\frac{1}{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)+\cot\left(\frac{1}{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right],$

$u_{17}(x,t)=\pm\frac{1}{2}\sqrt{3(c-1)}\left[\tanh\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{c-1}{bc-a}\right)}(x-ct)\right)+\coth\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\frac{c-1}{bc-a}\right)}(x-ct)\right)\right].$

for $c<1$ and $a>bc$

$u_{18}(x,t)=\pm i\sqrt{3-3c}\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right),$

$u_{19}(x,t)=\pm i\sqrt{3-3c}\coth\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right),$

$u_{20}(x,t)=\pm i\sqrt{\frac{3}{2}(1-c)}\left[\tan\left(\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)+\cot\left(\frac{1}{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right],$

$u_{21}(x,t)=\pm\frac{1}{2}i\sqrt{3(1-c)}\left[\tanh\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)+\coth\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right].$

Using Eq.. (21), the solution of (4) when $p=-(k^2+1), q=2k^2, r=1$, and the sets of solutions 2-7, we get

$u_{22,23,\ldots,27}(x,t)=a_0+a_1\operatorname{sn}\lambda\xi+b_1\operatorname{ns}\lambda\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 2-7.

Note that, when $k\to 0$ we obtain constant solution, when $k\to 1$ we get, $[u_6, u_7, \ldots, u_{21}]$.

Using Eq. (21), the solution of (4) when $p=2k^2-1, q=-2k^2, r=1-k^2$, and the sets of solutions 2-7, we get

$u_{28,29\ldots,33}(x,t)=a_0+a_1\operatorname{cn}\lambda\xi+b_1\operatorname{nc}\lambda\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 2-7.

When $k\to 0$ we obtain constant solution, when $k\to 1$ we get, $[u_2, u_3, \ldots, u_5]$.

Using Eq. (21), the solution of (4) when $p=2-k^2, q=-2, r=-(1-k^2)$, and the sets of solutions 2-7, we get

$u_{34,35,\ldots,39}(x,t)=a_0+a_1\operatorname{dn}\lambda\xi+b_1\operatorname{nd}\lambda\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 2-7.

As $k\to 0$ we obtain constant solutions, as $k\to 1$ we get, $[u_2, u_3, \ldots, u_5]$.

Using Eq. (21), the solution of (4) when $p=2-k^2, q=2, r=1-k^2$, and the sets of solutions 2-7, we get

$u_{40,41,\ldots,45}(x,t)=a_0+a_1\operatorname{cs}\lambda\xi+b_1\operatorname{sc}\lambda\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 2-7.

When $k\to 0$ we obtain, $[u_6, u_7\ldots, u_{21}]$, when $k\to 1$ we get constant solution.

Using Eq. (21), the solution of (4) when $p=-\left(1+k^2\right), q=2, r=k^2$, and the sets of solutions 2-7, we get

$u_{46,47,\ldots,51}\left(x,t\right)=a_0+a_1\operatorname{ns}\lambda\xi+b_1\operatorname{sn}\lambda\xi$, where a_0, a_1 and b_1 are defined in the sets of solutions 2-7.

As $k\to 0$ we obtain constant solution, as $k\to 1$ we get, $[u_6, u_7, ..., u_{21}]$.

Using Eq. (21), the solution of (4) when $p=\frac{1}{2}, q=\frac{1}{2}, r=\frac{1}{4}$, and the sets of solutions 2-7, we get

for$c<1$ and $a<bc$

$$u_{52}\left(x,t\right)=\sqrt{3-3c}\left(\tan\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right),$$

$$u_{53}\left(x,t\right)=-\sqrt{3-3c}\left(\tan\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right),$$

$$u_{54}\left(x,t\right)=\frac{\sqrt{3-3c}}{\left(\tan\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)},$$

$$u_{55}\left(x,t\right)=\frac{-\sqrt{3-3c}}{\left(\tan\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)},$$

$$u_{56}\left(x,t\right)=-\sqrt{\frac{3}{2}}\sqrt{1-c}\left(\tanh\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\mp i\operatorname{sech}\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)+$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{1-c}}{\tanh\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\mp i\operatorname{sech}\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)},$$

$$u_{57}\left(x,t\right)=\sqrt{\frac{3}{2}}\sqrt{1-c}\left(\tanh\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\mp i\operatorname{sech}\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)-$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{1-c}}{\tanh\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\mp i\operatorname{sech}\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)},$$

$$u_{58}\left(x,t\right)=\frac{\sqrt{3}}{2}\sqrt{1-c}\left(\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)-$$

$$\frac{\frac{\sqrt{3}}{2}\sqrt{1-c}}{\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)},$$

$$u_{59}\left(x,t\right)=-\frac{\sqrt{3}}{2}\sqrt{1-c}\left(\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\right)+$$

$$\frac{\frac{\sqrt{3}}{2}\sqrt{1-c}}{\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)}.$$

for $c>1$ and $a>bc$

$$u_{60}\left(x,t\right)=i\sqrt{-3+3c}\left(\tan\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right),$$

$$u_{61}\left(x,t\right)=-i\sqrt{-3+3c}\left(\tan\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right),$$

$$u_{62}\left(x,t\right)=\frac{i\sqrt{-3+3c}}{\left(\tan\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)},$$

$$u_{63}\left(x,t\right)=\frac{-i\sqrt{-3+3c}}{\left(\tan\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\sqrt{2}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)},$$

$$u_{64}\left(x,t\right)=\sqrt{\frac{3}{2}}\sqrt{-1+c}\left(i\tanh\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\operatorname{sech}\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)+$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{-1+c}}{i\tanh\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\operatorname{sech}\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)},$$

$$u_{65}\left(x,t\right)=-\sqrt{\frac{3}{2}}\sqrt{-1+c}\left(i\tanh\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\operatorname{sech}\left(\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)-$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{-1+c}}{i\tanh\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)\pm\operatorname{sech}\left(\sqrt{\frac{1-c}{bc-a}}\left(x-ct\right)\right)},$$

$$u_{66}\left(x,t\right)=i\frac{\sqrt{3}}{2}\sqrt{c-1}\left(\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)-$$

$$\frac{i\frac{\sqrt{3}}{2}\sqrt{1-c}}{\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)},$$

$$u_{67}\left(x,t\right)=-i\frac{\sqrt{3}}{2}\sqrt{c-1}\left(\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\right)+$$

$$\frac{i\frac{\sqrt{3}}{2}\sqrt{c-1}}{\tan\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)\pm\sec\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{a-bc}}\left(x-ct\right)\right)}.$$

for $c > 1$ and $a < bc$

$$u_{68}(x,t) = \sqrt{-3+3c}\left(\tanh\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right),$$

$$u_{69}(x,t) = -\sqrt{-3+3c}\left(\tanh\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right),$$

$$u_{70}(x,t) = \frac{\sqrt{-3+3c}}{\left(\tanh\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm i\,\mathrm{sech}\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right)},$$

$$u_{71}(x,t) = \frac{-\sqrt{-3+3c}}{\left(\tanh\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm i\,\mathrm{sech}\left(\sqrt{2}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right)},$$

$$u_{72}(x,t) = \sqrt{\frac{3}{2}}\sqrt{-1+c}\left(\tan\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm \sec\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right) + \frac{\sqrt{\frac{3}{2}}\sqrt{-1+c}}{\tan\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm \sec\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)},$$

$$u_{73}(x,t) = -\sqrt{\frac{3}{2}}\sqrt{-1+c}\left(\tan\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm \sec\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right) - \frac{\sqrt{\frac{3}{2}}\sqrt{-1+c}}{\tan\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \pm \sec\left(\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)},$$

$$u_{74}(x,t) = -\frac{\sqrt{3}}{2}\sqrt{c-1}\left(\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right) - \frac{\frac{\sqrt{3}}{2}\sqrt{c-1}}{\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)},$$

$$u_{75}(x,t) = \frac{\sqrt{3}}{2}\sqrt{c-1}\left(\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)\right) + \frac{\frac{\sqrt{3}}{2}\sqrt{c-1}}{\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right) \mp i\,\mathrm{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{c-1}{bc-a}}(x-ct)\right)}.$$

for $c < 1$ and $a > bc$

$$u_{76}(x,t) = \sqrt{3-3c}\left(i\tanh\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right) \pm \mathrm{sech}\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right),$$

$$u_{77}(x,t) = -\sqrt{3-3c}\left(i\tanh\left(\sqrt{2}\sqrt{\frac{1-c}{bc-a}}(x-ct)\right) \pm \mathrm{sech}\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right),$$

$$u_{78}(x,t)=\frac{\sqrt{3-3c}}{\left(i\tanh\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)},$$

$$u_{79}(x,t)=\frac{-\sqrt{3-3c}}{\left(i\tanh\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\sqrt{2}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)},$$

$$u_{80}(x,t)=i\sqrt{\frac{3}{2}}\sqrt{1-c}\left(\tan\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\sec\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)+$$
$$\frac{i\sqrt{\frac{3}{2}}\sqrt{1-c}}{\tan\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\sec\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)},$$

$$u_{81}(x,t)=-i\sqrt{\frac{3}{2}}\sqrt{1-c}\left(\tan\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\sec\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)-$$
$$\frac{i\sqrt{\frac{3}{2}}\sqrt{1-c}}{\tan\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\sec\left(\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)},$$

$$u_{82}(x,t)=\frac{\sqrt{3}}{2}\sqrt{1-c}\left(i\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)-$$
$$\frac{\frac{\sqrt{3}}{2}\sqrt{1-c}}{i\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)},$$

$$u_{83}(x,t)=-\frac{\sqrt{3}}{2}\sqrt{1-c}\left(i\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\right)+$$
$$\frac{\frac{\sqrt{3}}{2}\sqrt{1-c}}{i\tanh\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)\pm\operatorname{sech}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{1-c}{a-bc}}(x-ct)\right)}.$$

3.3. **The (2+1) Dimensional KD Equation.** In this section, we will solve the nonlinear (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation of the form

$$\begin{aligned} u_y &= v_x, \\ u_t - u_{xxx} - 6buu_x + \frac{3}{2}a^2u^2u_x - 3v_y + 3au_xv &= 0, \end{aligned} \tag{22}$$

where $u = u(x,y,t)$, $v = v(x,y,t)$ and a, b are real numbers.

The nonlinear (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation is a nonlinear integrable evolution equation on two spatial dimensions and one temporal. In [15], this equation was investigated by the inverse scattering transform method. The F-expansion method is used in [23] to investigate the KD equation.

We apply the mapping method, to solve the nonlinear (2+1) dimensional Konopelchenko-Dubrovsky (KD) equation. Substituting $u(x,y,t)=u(\xi), v(x,y,t)=v(\xi), \xi=\lambda(x+y-\beta t)$, into Eq. (16) and integrating once yields

$$\begin{aligned} u &= v, \\ -\beta u-\lambda^2 u''-3bu^2+\frac{a^2}{6}u^3-3u+\frac{3}{2}au^2 &= 0. \end{aligned} \tag{23}$$

Balancing the order of the nonlinear term u^3 with the highest derivative u'' gives $3m=m+2$ that gives $m=1$. Thus, the solution of (23) has the form

$$u(\xi)=v(\xi)=a_0+a_1 f(\xi)+b_1(f(\xi))^{-1}. \tag{24}$$

Substituting (24) in (23) and using (4), collecting the coefficients of each power of $f^i, 0\le i\le 6$, setting each coefficient to zero, and solving the resulting system, obtain the following sets of solutions.

(1) $a_0=-\frac{a-2b}{a^2}, a_1=0, b_1=\pm\frac{\sqrt{\frac{-2r}{p}}(a-2b)}{a^2}, \lambda=\frac{\sqrt{\frac{-1}{2p}}(a-2b)}{a}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

(2) $a_0=-\frac{a-2b}{a^2}, a_1=0, b_1=\pm\frac{\sqrt{\frac{-2r}{p}}(a-2b)}{a^2}, \lambda=-\frac{\sqrt{\frac{-1}{2p}}(a-2b)}{a}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

(3) $a_0=-\frac{a-2b}{a^2}, a_1=\pm\frac{\sqrt{\frac{-q}{p}}(a-2b)}{a^2}, b_1=0, \lambda=\frac{\sqrt{\frac{-1}{2p}}(a-2b)}{a}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

(4) $a_0=-\frac{a-2b}{a^2}, a_1=\pm\frac{\sqrt{\frac{-q}{p}}(a-2b)}{a^2}, b_1=0, \lambda=-\frac{\sqrt{\frac{-1}{2p}}(a-2b)}{a}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

(5) $a_0=-\frac{a-2b}{a^2}, a_1=\pm\frac{\sqrt{\frac{pq+3q\sqrt{2qr}}{18qr-p^2}}(a-2b)}{a^2}, b_1=\pm\frac{1}{3}\frac{\frac{p(p+3\sqrt{2qr})(a-2b)}{18qr-p^2}+(a-2b)}{a^2\sqrt{\frac{pq+3q\sqrt{2qr}}{18qr-p^2}}},$

$\lambda=\frac{1}{\sqrt{2}a}\frac{\sqrt{(18qr-p^2)(p+3\sqrt{2qr})}(a-2b)}{18qr-p^2}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

(6) $a_0=-\frac{a-2b}{a^2}, a_1=\pm\frac{\sqrt{\frac{pq+3q\sqrt{2qr}}{18qr-p^2}}(a-2b)}{a^2}, b_1=\pm\frac{1}{3}\frac{\frac{p(p+3\sqrt{2qr})(a-2b)}{18qr-p^2}+(a-2b)}{a^2\sqrt{\frac{pq+3q\sqrt{2qr}}{18qr-p^2}}},$

$\lambda=-\frac{1}{\sqrt{2}a}\frac{\sqrt{(18qr-p^2)(p+3\sqrt{2qr})}(a-2b)}{18qr-p^2}, \beta=-4\frac{(a^2-ab+b^2)}{a^2}$

Using Eq.. (24), the solution of (4) when $p=1, q=-2, r=0$, and the above sets of solutions, we get

$$u_1(x,y,t)=a_0, \text{ (constant solution)},$$

$$u_2(x,y,t)=-\frac{a-2b}{a^2}\pm\frac{\sqrt{2}(a-2b)}{a^2}\sec\left(\frac{1}{\sqrt{2}}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right).$$

Note that $u(x,y,t)=v(x,y,t)$, for all cases.

Using Eq. (24), the solution of (4) when $p=-2, q=2, r=1$, and the sets of solutions 1-6, we get

$$u_3(x,y,t)=-\frac{a-2b}{a^2}\pm\frac{a-2b}{a^2}\tanh\left(\frac{1}{2}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right),$$

$$u_4(x,y,t)=-\frac{a-2b}{a^2}\pm\frac{a-2b}{a^2}\coth\left(\frac{1}{2}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right),$$

$$\begin{aligned} u_5(x,y,t) &= -\frac{a-2b}{a^2} \pm \frac{a-2b}{2a^2}\tanh\left(\frac{1}{4}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) \\ &\pm\frac{a-2b}{2a^2}\coth\left(\frac{1}{4}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right), \end{aligned}$$

$$\begin{aligned} u_6(x,y,t) &= -\frac{a-2b}{a^2} \pm \frac{\sqrt{2}(a-2b)}{2a^2}\tan\left(\frac{\sqrt{2}}{4}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) \\ &\pm\frac{\sqrt{2}(a-2b)}{2a^2}\cot\left(\frac{\sqrt{2}}{4}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right). \end{aligned}$$

Using Eq.. (24), the solution of (4) when $p=-(1+k^2), q=2k^2, r=1$, and the sets of solutions 1-6, we get

$$u_{7,\ldots,12}(x,y,t) = a_0 + a_1\,\mathrm{sn}(\lambda\xi) + b_1\,\mathrm{ns}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. Note that, as $k\to 0$, we obtain

$$u_{13}(x,y,t) = -\frac{a-2b}{a^2} \pm \frac{\sqrt{2}(a-2b)}{a^2}\csc\left(\frac{1}{\sqrt{2}}\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right).$$

As $k\to 1$, we get $u_3, u_4 ..., u_6$.

Using Eq. (24), the solution of (4) when $p=2k^2-1, q=-2k^2, r=1-k^2$ and the sets of solutions 1-6, we obtain

$$u_{14,\ldots,119}(x,y,t) = a_0 + a_1\,\mathrm{cn}(\lambda\xi) + b_1\,\mathrm{nc}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. When $k\to 0$, we obtain u_2, also we get u_2 when $k\to 1$.

Using Eq. (24), the solution of (4) when $p=2-k^2, q=-2, r=-(1-k^2)$, and above sets of solutions 1-6, we get

$$u_{20,\ldots,25}(x,y,t) = a_0 + a_1\,\mathrm{dn}(\lambda\xi) + b_1\,\mathrm{nd}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. As $k\to 0$, we get u_1 and v_1, as $k\to 1$, we obtain u_2 .

Using Eq. (24), the solution of (4) when $p=2-k^2, q=2, r=(1-k^2)$, and the sets of solutions 1-6, we get

$$u_{26,\ldots,31}(x,y,t) = a_0 + a_1\,\mathrm{cs}(\lambda\xi) + b_1\,\mathrm{sc}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. As $k\to 0$, we obtain, $[u_3, u_4 ..., u_6]$, as $k\to 1$, we get u_{13}.

Using Eq. (24), the solution of (4) when $p=-(1+k^2), q=2, r=k^2$, and the sets of solutions 1-6, we get

$$u_{32,\ldots,37}(x,y,t)=a_0+a_1\operatorname{dc}(\lambda\xi)+b_1\operatorname{cd}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. As $k\to 0$, we obtain u_2, as $k\to 1$, we get u_1.

Using Eq. (24), the solution of (4) when $p=-1+2k^2, q=2, r=-k^2(1-k^2)$ and the sets of solutions 1-6, we get

$$u_{38,\ldots,43}(x,y,t)=a_0+a_1\operatorname{ds}(\lambda\xi)+b_1\operatorname{sd}(\lambda\xi),$$

where a_0, a_1 and b_1 are defined in the sets of solutions 1-6. As $k\to 0$, we obtain u_{13}, as $k\to 1$, we get also u_{13} and.

Using Eq. (24), the solution of (4) when $p=0, q=2, r=0$, and the sets of solutions 1-6, we get u_1.

Using Eq. (24), the solution of (4) when $p=1, q=0, r=0$, and the sets of solutions 1-6, we obtain u_1.

Using Eq. (24), the solution of (4) when $p=\frac{1}{2}, q=\frac{1}{2}, r=\frac{1}{4}$, and the sets of solutions 1-6, we get

$$\begin{aligned}
u_{44}(x,y,t) &= -\frac{a-2b}{a^2}+\frac{(a-2b)}{\sqrt{2}a^2}\left(\pm\tan\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)\\
&\quad+\frac{(a-2b)}{\sqrt{2}a^2}\sec\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)+\\
&\frac{(a-2b)}{\sqrt{2}a^2\left(\pm\tan\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)+\sec\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)},\\
u_{45}(x,y,t) &= -\frac{a-2b}{a^2}-\frac{(a-2b)}{\sqrt{2}a^2}\left(\pm\tan\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)\\
&\quad-\frac{(a-2b)}{\sqrt{2}a^2}\sec\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\\
&-\frac{(a-2b)}{\sqrt{2}a^2\left(\pm\tan\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)+\sec\left(\frac{a-2b}{\sqrt{2}a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)},\\
u_{46}(x,y,t) &= -\frac{a-2b}{a^2}+\frac{(a-2b)}{a^2}\tanh\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\\
&\quad\pm\frac{i(a-2b)}{a^2}\operatorname{sech}\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right),\\
u_{47}(x,y,t) &= -\frac{a-2b}{a^2}-\frac{(a-2b)}{a^2}\tanh\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\\
&\quad\pm\frac{i(a-2b)}{a^2}\operatorname{sech}\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right),
\end{aligned}$$

$$u_{48}(x,y,t) = -\frac{a-2b}{a^2} + \frac{(a-2b)}{a^2\left(\pm\tanh\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) + i\,\mathrm{sech}\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)},$$

$$u_{49}(x,y,t) = -\frac{a-2b}{a^2} - \frac{(a-2b)}{a^2\left(\pm\tanh\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) + i\,\mathrm{sech}\left(\left(\frac{a-2b}{a}\right)\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)},$$

$$\begin{aligned} u_{50}(x,y,t) &= -\frac{a-2b}{a^2} - \frac{(a-2b)}{2a^2}\left(\pm\tanh\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right) \\ &\quad -\frac{(a-2b)}{2a^2} i\,\mathrm{sech}\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) \\ &\quad -\frac{(a-2b)}{2a^2\left(\pm\tanh\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) + i\,\mathrm{sech}\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)}, \end{aligned}$$

$$\begin{aligned} u_{51}(x,y,t) &= -\frac{a-2b}{a^2} + \frac{(a-2b)}{2a^2}\left(\pm\tanh\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right) \\ &\quad +\frac{(a-2b)}{2a^2} i\,\mathrm{sech}\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) \\ &\quad +\frac{(a-2b)}{2a^2\left(\pm\tanh\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right) + i\,\mathrm{sech}\left(\frac{a-2b}{2a}\left(x+y+4\frac{(a^2-ab+b^2)}{a^2}t\right)\right)\right)}. \end{aligned}$$

Note that, if we replace sec by $-\sec$ and sech by $-\mathrm{sech}$ in $u_{44}, ..., u_{51}$, we obtain also true solutions.

By comparing our results with the results in [17], it can be seen that some of the obtained results are new, and the rest solutions are the same.

4. Conclusion

In this paper, the mapping method has been successfully implemented to find new traveling wave solutions for three nonlinear PDE's, the Pade′-II, (2+1) dimensional Konopelchenko-Dubrovsky (KD) and our new proposed equation namely, a modified Pade′-II equation. The results show that this method is a powerful Mathematical tool for obtaining exact solutions for the Pade′-II, KD and our proposed equation. It is also a promising method to solve other nonlinear partial differential equations.

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