

ROUGH SET THEORY BASED ON TWO RELATIONS AND ITS APPLICATIONS

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Abstract:

Rough set theory was introduced by Pawlak in the early 80's and has reached a level of high visibility and maturity. In recent years we have witness diverse as well as widespread research in rough sets theory and its applications worldwide.

In this paper, we introduce a new approximation space (biapproximation space) and define the lower and upper approximation based on two relations to take advantages of two relations in the same time. With this approximation and our definitions, we present two concepts to calculate the size of boundary region and discuss some of the basic properties of them. Examples are provided to illustrate the behavior of this new notion. Compared with Pawlak approximation space, our new approximation space is very efficient and settable when we have a lot of data for one case.

1. Introduction

The topological structure of a set is now considered as mathematical model for getting information from data [1, 2, 7]. The modeling process is based on relations obtained from a given data by one expert. Using two topologies help in discovering information using two points of view in the same time. Basic concepts of rough sets depends on a special type of topologies, namely quasi discrete topology, thus the general structures are generalizations of the quasi discrete topologies. Rough sets have been initiated by Pawlak [8, 9] in order to describe approximation knowledge of subsets of a given universe. In some sense this theory can be considered as a generalization of classical set theory and made a great success in knowledge acquisition in recent years and it has been applied in many applications such as knowledge discovery and machine learning [5, 6, 11, 14]. Unfortunately, it's based on complete information systems, an approximation is pair (U, R) , where U is a certain set called universe, and $R \subset U \times U$ is an equivalence relation based on ability to classify objects (elements of U) and it takes the form as real things, states, abstract concepts, moment of time, ..., etc and has equivalence classes and the set of all equivalence classes denoted by U/R , so the least (greatest) union of equivalence classes containing (contained in) set X is the upper (lower) approximation which denoted by $H(X)$ ($L(X)$). Now, any set in the approximation space is an exact or rough. By the use of two exact sets (lower and upper approximation), any rough set can be defined approximately in the approximation space.

2. Biapproximation Space

The aim of this section is to introduce the concept of a biapproximation space based on Pawlak approximation space and give examples to illustrate the behavior of this new notion. Suppose R is a binary relation on a universe U . Yao [14] defined a pair of approximation operations. $L(R), H(R): P(U) \rightarrow P(U)$ as follows:

$$L(R)(X) = \{x : \forall y, [xRy \Rightarrow y \in X]\} = \{x : RN(x) \subseteq X\},$$

$$H(R)(X) = \{x : \exists [xRy, y \in X]\} = \{x : RN(x) \cap X \neq \phi\},$$

Where $RN(X) = \{y \in U : xRy\}$. They are called the lower approximation operation and the upper approximation operation, respectively.

Definition 2.1. Let U be a finite set and $X \subset U$, R_1, R_2 be two equivalence relations on U . Then

(i) $\pi = (U, R_1, R_2)$ is called a biapproximation space.

(ii) The bi-lower approximation of X on U is defined as: $L_\pi(X) = L_{R_1}(X) \cup L_{R_2}(X)$, where

$$L_{R_i}(X) = \cup\{Y \in U/R, Y \subset X\}, i \in \{1, 2\}.$$

(iii) The bi-upper approximation of X on U is defined as: $H_\pi(X) = H_{R_1}(X) \cap H_{R_2}(X)$,

$$\text{Where } H_{R_i}(X) = \cup\{Y \in U/R, Y \cap X \neq \phi\}, i \in \{1, 2\}.$$

Proposition 2.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. Then the space (U, R) which given by $U/R = \{X \cap Y : X \in U/R_1, Y \in U/R_2\}$, forms an expansion of both (U, R_1) and (U, R_2) .

Proof. Since (U, R) is finer than (U, R_1) and (U, R_2) , then the proof is obvious.

Lemma 2.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and (U, R) be the expansion of (U, R_1) and (U, R_2) . Then the following hold:

$$(i) L_\pi(X) \subseteq L(X), \quad (ii) H(X) \subseteq H_\pi(X),$$

Where $L(X)$ and $H(X)$ are the lower and upper approximation of X in the expansion (U, R) .

Proof. (i) Since (U, R) is finer than (U, R_1) and (U, R_2) . Then $L_{R_1}(X) \subseteq L(X)$ and $L_{R_2}(X) \subseteq L(X)$, then $L_{R_1}(X) \cup L_{R_2}(X) \subseteq L(X)$. Thus $L_\pi(X) \subseteq L(X)$.

(ii) Follows from **Definition 2.1**, and **Proposition 2.1**

Remark 2.1. $L_{R_i}(X) \subseteq L_\pi(X) \subseteq L(X)$ and $H(X) \subseteq H_\pi(X) \subseteq H_{R_i}(X), i = 1, 2$.

Proposition 2.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and (U, R) be the expansion.

Then for $X \subset U$. The accuracy of X with respect to the expansion which given by $\frac{|L(X)|}{|H(X)|}$ is

greater than the accuracy of X with respect to π which given by $\frac{|L_\pi(X)|}{|H_\pi(X)|}$.

Proof. The proof is directly derivable from **Lemma 2.1** and **Remark 2.1**.

Example 2.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space where $U = \{a, b, c, d, e, f, g, h\}$, $U/R_1 = \{\{a, b, f\}, \{c, d\}, \{e, g, h\}\}$, $U/R_2 = \{\{a, b, d\}, \{c, e, h\}, \{f, g\}\}$ and $U/R = \{\{a, b\}, \{c\}, \{d\}, \{e, h\}, \{f\}, \{g\}\}$. Since $X = \{a, b, d, g\}$, then $L(X) = \{a, b, d, g\}$, $L_\pi(X) = \{a, b, d\}$, $H(X) = \{a, b, d, g\}$ and $H_\pi(X) = \{a, b, d, f, g\}$. Mean that $L_\pi(X) \subseteq L(X)$ and $H(X) \subseteq H_\pi(X)$.

The boundary region of any subset is contracted if it measured with respect to the biapproximation space (expansion of biapproximation space) and thus the degree of accuracy of two Pawlak approximation space increases. Also, we have the best results when we use the expansion of biapproximation space (Proposition 2.2).

3. Relative Biapproximation Space

The aim of this section is to define the concept of lower and upper approximation of one classification of biapproximation space with respect to another classification and introduce new definitions indicate the useful of the relative biapproximation space.

Definition 3.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and $U / R_1 = \{X_i\}$ and

$U / R_2 = \{X_j\}$, where $i, j, \in \{1, 2, \dots, n\}$. Then

(i) The lower approximation of X on U / R_1 by using U / R_2 is

$$L_{R_1}(X_i) = \cup \{X_j \in U / R_2, X_j \subset X_i\}$$

(ii) The upper approximation of X on U / R_1 by using U / R_2 is

$$H_{R_1}(X_i) = \cup \{X_j \in U / R_2, X_j \cap X_i \neq \phi\}.$$

Definition 3.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space where $U / R_i = \{X_1, X_2, \dots, X_n\}$, $i \in \{1, 2\}$ and $L_{R_i}(X_j) = \{L_{R_i}(X_1), L_{R_i}(X_2), \dots, L_{R_i}(X_j)\}$ be the lower approximation of members of U / R_i also the upper approximation of members of U / R_i given by $H_{R_i}(X_j) = \{H_{R_i}(X_1), H_{R_i}(X_2), \dots, H_{R_i}(X_j)\}$, $i, j \in \{1, 2, \dots, n\}$. Then,

(i) The accuracy of U / R_1 by U / R_2 defined as $\alpha_{R_2}(U, R_1) = \frac{\sum |L_{R_i}(X_j)|}{\sum |H_{R_i}(X_j)|}$,

(ii) The quality of U / R_1 by U / R_2 defined by $\gamma_{R_2}(U, R_1) = \frac{\sum |L_{R_i}(X_j)|}{|U|}$,

Where $j \in \{1, 2, \dots, n\}$.

Remark 3.1. The accuracy of U / R_1 expresses the percentage of possible correct decision when classifying objects employing the relation R_2 while the quality of U / R_1 expresses percentage of objects which can correctly classified to classes employing relation R_2 .

Example 3.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space, where $U = \{a, b, c, d, e, f, g, h\}$, $U / R_1 = \{\{a, c, e\}, \{b, d\}, \{f, g, h\}\}$ and $U / R_2 = \{\{a, b, d\}, \{c, e, h\}, \{f, g\}\}$, then

$$\alpha_{R_2}(U, R_1) = \frac{0+0+2}{6+3+5} = \frac{2}{14} = \frac{1}{7} \approx 14\%,$$

$$\gamma_{R_2}(U, R_1) = \frac{0+0+2}{8} = \frac{2}{8} = \frac{1}{4} = 25\%,$$

$$\alpha_{R_1}(U, R_2) = \frac{2+0+0}{5+6+3} = \frac{2}{14} = \frac{1}{7} \approx 14\%,$$

and

$$\gamma_{R_1}(U, R_2) = \frac{2+0+0}{8} = \frac{2}{8} = \frac{1}{4} = 25\%.$$

Example 3.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space, where $U = \{a, b, c, d, e, f\}$, $U / R_1 = \{\{a\}, \{b\}, \{c, e\}, \{d\}, \{f\}\}$ and $U / R_2 = \{a, b\}, \{c\}, \{d\}, \{e, f\}$, then

$$\alpha_{R_2}(U, R_1) = \frac{0+0+1+1+0}{2+2+3+1+2} = \frac{2}{10} = \frac{1}{5} = 20\%,$$

$$\gamma_{R_2}(U, R_1) = \frac{0+0+1+1+0}{6} = \frac{2}{6} = \frac{1}{3} = 33\%,$$

$$\alpha_{R_1}(U, R_2) = \frac{2+0+1+1}{2+2+1+3} = \frac{4}{7} \approx 57\%,$$

and

$$\gamma_{R_1}(U, R_2) = \frac{2+0+1+1}{6} = \frac{4}{6} = \frac{2}{3} \approx 66\%.$$

Proposition 3.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and $U/R_1 = \{X_1, X_2, \dots, X_n\}$ $i \in \{1, 2\}$. Then the following hold:

- (i) If there exists $j \in \{1, 2, \dots, n\}$ such that $L_{R_1}(X_j) \neq \phi$ then for each $k \neq j$ and $k \in \{1, 2, \dots, n\}$, $H_{R_1}(X_k) \neq U$ (the opposite is not true, if $L_{R_1}(X_j) = \phi$ is not true, $H_{R_1}(X_k) = U$).
- (ii) If there exists $j \in \{1, 2, \dots, n\}$ such that $H_{R_1}(X_j) = U$ then for each $k \neq j$ and $k \in \{1, 2, \dots, n\}$, $L_{R_1}(X_k) = \phi$ (the opposite is not true).
- (iii) If for each $j \in \{1, 2, \dots, n\}$, $L_{R_1}(X_j) \neq \phi$ holds, then $H_{R_1}(X_j) \neq U$ for each $j \in \{1, 2, \dots, n\}$ (the opposite is not true).
- (iv) If for each $j \in \{1, 2, \dots, n\}$, $H_{R_1}(X_j) \neq \phi$ holds, then $L_{R_1}(X_j) = \phi$, for each $j \in \{1, 2, \dots, n\}$ (the opposite is not true).

Proof (i) If $L_{R_1}(X_j) \neq \phi$, then there exists $X_1 \in U/R_2$ ($L_{R_1}(X_j) = \cup\{X_1 \in U/R_2, X_1 \subset X_j\}$). For each $X_k \neq X_j$, $H_{R_1}(X_k) = \cup\{X_i \in U/R_2, X_i \cap X_k \neq \phi\}$, $i \in \{1, 2, \dots, n\}$, then $H_{R_1}(X_k) = \cap X_i = \phi$ and consequently $H_{R_1}(X_k) \neq U$ for each $j \neq k$. Or, if $L_{R_1}(X_j) \neq \phi$, then there exists $x \in X$ such that $x \in X \subset X_j$ which implies $X_1 \cap X_k = \phi$ for each $j \neq k$. These yields $H_{R_1}(X_k \cap X_j) = \phi$ and $H_{R_1}(X_k \neq U)$, for each $j \neq k$.

(ii) If $U_{R_1}(X_j) = U$ ($H_{R_1}(X_j) = \cup\{X_i \in U/R_2, X_j \cap X_i \neq \phi\}$) that means $X_j \cap X_i \neq \phi$ for each $X_i \in U/R_2$, $i \in \{1, 2, \dots, n\}$, then $X_i \subset X_k$ doesn't hold or each $j \neq k$, ($\phi \subset X_k$)

$L_{R_1}(X_k) = \cup\{\phi \in U/R_2, \phi \subset X_k\}$, then $L_{R_1}(X_k) = \phi$.

(iii) and (iv) Follows similarly as in (i) and (ii).

Example 3.3. Let $\pi = (U, R_1, R_2)$ be a biapproximation space, where $U = \{a, b, c, d, e, f, g, h\}$, $U/R_1 = \{\{a, c, e\}, \{b, d\}, \{f, g, h\}\}$ and $U/R_2 = \{\{a, b, d\}, \{c, e, h\}, \{f, g\}\}$, then $L_{R_1}(\{a, b, d\}) = \{b, d\} \neq \phi$ and $H_{R_1}(\{c, e, h\}) = \{a, b, d, f, g\} \neq U$ and $H_{R_1}(\{f, g\}) = \{f, g, h\} \neq U$.

Example 3.4. Let $\pi = (U, R_1, R_2)$ be a biapproximation space, where $U = \{a, b, c, d, e, f, g, h\}$, with $U/R_1 = \{\{a, c, e\}, \{b, d\}, \{f, g, h\}\}$ and $U/R_2 = \{\{a, b, f\}, \{c, d\}, \{e, g, h\}\}$, then $U_{R_1}(\{a, b, f\}) = U$, $L_{R_1}(\{c, d\}) = \phi$ and $L_{R_1}(\{e, g, h\}) = \phi$.

4. Bi-Equal and Bi-Inclusion

The aim of this section is to define the concept of bi-equal and bi-inclusion of one classification of biapproximation space with respect to another classification.

Definition 4.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and $X, Y \subset U$.

Then the following hold:

- (i) X, Y are bottom equal on U/R_2 by U/R_1 if $L_{R_1}(X) = L_{R_1}(Y)$.
- (ii) X, Y are top equal U/R_2 by U/R_1 if $H_{R_1}(X) = H_{R_1}(Y)$.
- (iii) X, Y are bi-equal on U/R_2 by U/R_1 if $L_{R_1}(X) = L_{R_1}(Y)$ and $H_{R_1}(X) = H_{R_1}(Y)$.
- (iv) X is bottom inclusion in Y on U/R_2 by U/R_1 if $L_{R_1}(X) \subseteq L_{R_1}(Y)$.
- (v) X is top inclusion in Y on U/R_2 by U/R_1 if $H_{R_1}(X) \subseteq H_{R_1}(Y)$.
- (vi) X, Y is bi-inclusion in Y on U/R_2 by U/R_1 if $L_{R_1}(X) \subseteq L_{R_1}(Y)$ and $H_{R_1}(X) \subseteq H_{R_1}(Y)$.

Definition 4.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and R_1 and R_2 are equivalence relations. Then R_1 is a covering of R_2 if and only if R_2 depends on R_1 and R_1 is minimal (or, R_1 is covering of R_2 if and only if R_2 depends on R_1 and no proper subset R_1^* of R_1 exists such that R_2 depends on R_1^*).

In other words, R_1 and R_2 are equivalence if $U/R_1 = U/R_2$ (or, R_1 and R_2 have the same equivalence classes), and R_1 is finer than R_2 if $U/R_1 \subset U/R_2$ or U/R_2 is coarser than R_1 .

Example 4.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space where $U = \{a, b, c, d, e, f\}$, with $U/R_1 = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}\}$ and $U/R_2 = \{\{a, b\}, \{c, e, f\}, \{d\}\}$, then R_1 is covering of R_2 because $U/R_1 \subset U/R_2$.

5. Uniqueness of Binary Relations to Generate Rough Sets

Theorem 5.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space and $X \subset U$. Then:

- (i) $L(R_1 \cup R_2)(X) = L(R_1)(X) \cap L(R_2)(X)$ and
- (ii) $H(R_1 \cup R_2)(X) = H(R_1)(X) \cup H(R_2)(X)$.

Proof. (i) For any $X \subset U$, $L(R_1 \cup R_2)(X) = \{x : \forall y \in U, x(R_1 \cup R_2)y \Rightarrow y \in X\} = \{x : \forall y \in U, xR_1y \text{ or } xR_2y \Rightarrow y \in X\} = \{x : \forall y \in U, xR_1y \Rightarrow y \in X\} \cap \{x : \forall y \in U, xR_2y \Rightarrow y \in X\} = L(R_1)(X) \cap L(R_2)(X)$.

(ii) For any $X \subset U$, $H(R_1 \cup R_2)(X) = \{x : \exists y \in U, x(R_1 \cup R_2)y\} = \{x : \exists y \in X, xR_1y \text{ or } xR_2y\} = \{x : \exists y \in X, xR_1y\} \cup \{x : \exists y \in X, xR_2y\} = H(R_1)(X) \cup H(R_2)(X)$.

Proposition 5.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. If $R_1 \subset R_2$ then $L(R_2) \subseteq L(R_1)$ and $H(R_1) \subseteq H(R_2)$. Then we consider the situation for $R_1 \cap R_2$.

Theorem 5.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. Then:

- (i) $L(R_1)(X) \cup L(R_2)(X) \subseteq L(R_1 \cap R_2)(X)$ and
- (ii) $H(R_1 \cap R_2)(X) \subseteq H(R_1)(X) \cap H(R_2)(X)$.

Proof. It is easy to proof this theorem by **Proposition 5.1**.

The equalities in the above theorem is not hold generally as shown by the following example.

Example 5.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space, where $U = \{a, b, c\}$, $R_1 = \{\{a, a\}, \{a, b\}, \{b, b\}\}$, and $R_2 = \{\{a, a\}, \{a, c\}, \{c, a\}, \{c, b\}, \{c, c\}\}$. Then we have

$$\begin{aligned} RN_{R_1}(\{a\}) &= \{a, b\}, RN_{R_1}(\{b\}) = \{b\}, RN_{R_1}(\{c\}) = \phi, \\ RN_{R_2}(\{a\}) &= \{a, b\}, RN_{R_2}(\{b\}) = \phi, RN_{R_2}(\{c\}) = \{a, b, c\}. \\ R_1 \cap R_2 &= \{\{a, a\}\}, \text{ and} \end{aligned}$$

$$RN_{R_1 \cap R_2}(\{a\}) = \{a\}, RN_{R_1 \cap R_2}(\{b\}) = \phi, RN_{R_1 \cap R_2}(\{c\}) = \phi.$$

For $X = \{a\}$ and $Y = \{b\}$, we have $L(R_1)(X) = \{c\}$, $H(R_1)(Y) = \{a, b\}$.

$$L(R_2)(X) = \{b\}, H(R_2)(Y) = \{c\}, \text{ and}$$

$$L(R_1 \cap R_2)(X) = \{a, b, c\}, H(R_1 \cap R_2)(Y) = \phi.$$

Thus, $L(R_1)(X) \cup L(R_2)(X) = L(R_1 \cap R_2)(X)$ and

$$H(R_1 \cap R_2)(Y) \subseteq H(R_1)(Y) \cap H(R_2)(Y).$$

Proposition 5.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. If $H(R_1) \subseteq H(R_2)$, then $R_1 \subseteq R_2$.

Proof. For each $x, y, \in U$, if $(x, y) \in R_1$, $y \in RN_{R_1}(x)$, $x \in H(R_1)(\{y\}) \subseteq H(R_2)(\{y\})$, so $RN_{R_2}(x) \cap \{y\} \neq \phi$, that means $(x, y) \in R_2$, thus $R_1 \subseteq R_2$.

Corollary 5.1. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. If $H(R_1) = H(R_2)$, then $R_1 = R_2$.

Theorem 5.3. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. Then $H(R_1) = H(R_2)$, if and only if $R_1 = R_2$.

Proof. It comes from **Proposition 5.1** and **Corollary 5.1**.

By the duality between $H(R)$ and $L(R)$, we have the following result about $L(R)$.

Proposition 5.3. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. If $L(R_1) \subseteq L(R_2)$, then $R_2 \subseteq R_1$.

Corollary 5.2. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. If $L(R_1) = L(R_2)$, if and only if $R_1 = R_2$.

Theorem 5.4. Let $\pi = (U, R_1, R_2)$ be a biapproximation space. Then $L(R_1) = L(R_2)$, if and only if $R_1 = R_2$.

6. Conclusions and future works

Pawlak approximation space is considered as mathematical model for getting information from data. The modeling process is based on equivalence relation obtained from a given data by one expand (view). Using two Pawlak approximation spaces (Biapproximation space) help in discovering information using two points of views in the same time and the new of views in the same and the new approximation focus on the expansion of the original model proposed by Pawlak. So the purpose of this paper is to extend the concept of lower and upper approximation. Pawlak approximation space introduced the diagnosis (solution) of some problems in Math., Chemistry... etc but when we have two (any) diagnosis's for any problem we use biapproximation space to find the best diagnosis or study one of these diagnosis's by using another one. Compared with Pawlak approximation space, our new approximation space is very efficient and settable when we have a lot of data for one case.

We proved that two different binary relations will generate two different lower approximation operations and two different upper approximation operations. As far as the applications of binary relation based rough sets to knowledge discovery from database are concerned, the reader is referred to [3, 4, 10]. In this future, we will explore the

relationships between binary relation based on rough sets and covering based on rough sets [15]. Another future research topic is to apply binary relation based rough set theory to the computational theory of linguistic dynamic systems [12] and security [16, 17]. In the future works, we will introduce many topological applications in rough context and also many real life applications by using suggested structures in this paper.

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