ASIAN JOURNAL OF MATHEMATICS AND APPLICATIONS
Volume 2018, Article ID ama0449, 14 pages
ISSN 2307-7743
http://scienceasia.asia

MULTI-OBJECTIVE OPTIMIZATION MODEL FOR IRRIGATION WATER ALLOCATION: A CASE STUDY OF NDURUMA CATCHMENT-ARUSHA, TANZANIA

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Abstract. In Tanzania, management of land and water resources is considered an absolutely strategic priority for agricultural development. In this study, a Multi-Objective optimization (MOO) model was formulated to utilize the available water by identifying the best crop patterns which maximize the farm total net benefit and minimize the total variable costs. The data for 6 crops collected from Nkoanrua region and FAO were used for the model analysis. The Subdivision Algorithm which is a set oriented numerical method was used to analyse the model. After 54 subdivision steps, the model proposed that, the amount of land allocated to crops which are less profitable depends on the minimum requirement constraints, while, for more profitable crops, the allocation is based on costs of production, minimum and maximum requirement constraints, Benefit, and water requirements. Lastly, the sensitivity analysis shows that, the price and production costs for carrots and maize has no impact on the model solutions.

1. Introduction

Land and water are essential resources to sustainable agricultural development, and are basically linked to global challenges of food insecurity. Pressure from population explosion, urbanization, extravagant lifestyles, climate change, intensive agriculture and industrialization makes water and land to be under threat. Planning and managing agricultural water in the world is an important field of study for the assurance of food supply and to minimize water stress in the future.

Key words and phrases. Multi-Objective Optimization Model, Irrigation Water Allocation, Tanzania.

There have been extensive efforts that had fairly succeeded in introducing the concept of water rights which lead to the formation of different water board organizations at different levels [5], meanwhile, most of these organizations have not kept pace with the growing intensity of agricultural development and the increasing level of interdependence and competition over land and water resources. The needs for an optimization model is crucial tool to support reallocation of the limited water resources based on a fair, effective and sustainable standard [24]. To address this challenge, global optimization techniques are essential field of study. In agriculture the aim is to achieve maximum crop yield under limited water and land supply [20]. When many uncertainty factors are involved in irrigation practice, management of water for irrigation becomes a complex tasks. The factors such as crop planting area, irrigation water use efficiency, water supply, groundwater resources, minimum and maximum resource constraints, and economic constraints may have characteristics of an uncertainty that may affect the efficiency of the model.

There are a number of models that has been formulated to ensure the efficient utilization of available water and land in agriculture. [22] formulated a model for maximizing benefits for irrigation project in India, Genetic Algorithm (GA) and Linear Programming (LP) techniques were applied to solve the model and the results were compared. The comparison showed that, GA is an effective optimization tool for irrigation planning and can be used for more complicated systems that involve non-linear optimization. [2] developed a non-linear optimization (NLP) and Linear Programming (LP) model for the determination of optimum cropping pattern, water amount and farm income under sufficient and limited water supply environments. The NLP model provide higher farm income values than the LP model under deficit irrigation conditions.

[25] proposed a multi-criteria decision-making (MCDM) framework to analyse production targets under physical, biological, economic and environmental constraints. The Goal Programming (GP) and simple LP techniques were employed to analyse the model, the obtained results showed that GP solutions were better than of the simple LP. This supports [4] claim about the effectiveness of GP in analysing multi-objective, nevertheless, there is an argument on how to select the target values and weights for the different goals.

[17] formulated a model for optimal irrigation planning with the area for cultivation as decision variable, the model comprises of three conflicting objectives. To obtain individual solutions of the objective functions, Crisp Linear Programming (CLP) was used, then, the crisp solutions were then solved together to obtain the solution under fuzzy multi-objective environment with maximum-minimum operators. The two-phase Multi- Objective Fuzzy Linear Programming (MOFLP) approach didn't show any significant improvement over the maximum-minimum MOFLP.

In the study conducted by [15], a multi-objective irrigation water resources optimal allocation model (ICSIMP) was developed, the model was integrated with Interval Linear

Programming (ILP), the Chance-Constrained Programming(CCP), Semi-Infinite Programming (SIP) and Integer Programming (IP). The obtained solutions offered a scientific basis for local and and similar area with water resources optimal allocation system.

Other studies which proposed the model for irrigation water management: [18, 19],[14, 23] and [1, 9]. Prominent methods used for multi-objective model analysis includes; Genetic Algorithm (GA), Non-Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Differential Evolution (MODE) respectively.

Despite the successes of the previous studies on optimizing the use of water for irrigation by using either traditional mathematical programming methods or evolutionary algorithms to determine the solutions of the problem, it is true that, the use of traditional programming methods does not offer Pareto optimal solutions in a single run and are very sensitive to the shape of the Pareto front [12]. Furthermore, there is a question if there is a sufficient and commonly accepted delineations of the quantitative performance metrics for the multi-objective optimizers when using Evolutionary Algorithms [11]. For the purpose of the contribution of the body of knowledge in the field of irrigation water optimization, the paper proposed a model for optimizing irrigation water allocation by identifying the best crop patterns for maximizing the farm net benefit and minimizing the crops production costs. The Subdivision Algorithm proposed by [6], which is a set oriented numerics method was used to analyse the model, by using this method, the Pareto set is approximated by a nested sequence of increasingly refined box coverings.

2. Problem formulation

The multi-objective programming algorithm was taken into consideration in model formulation. The algorithm is one of the Multi-objective Decision Making (MODM) technique in which problem with multiple conflicting objectives are considered [10]. Planning the use of available water and land for irrigation was structured as a Multi-objective Optimization Problem (MOOP) with multiple conflicting objectives. The model consists of the uncertainties of the problem (decision variables), the known data (parameters), the constraints (the relationships that describe and control the system) and the drivers of the optimization (two objective functions).

2.1. **Decision Variables of the Model.** The decision variables for the model are the amounts of land in hectares that will be allocated to different crop varieties. The allocation should be determined in a way that provide the maximum value of total net benefit and Minimum value of the total variable costs through proper utilization of available water and land. Defined as A_i for $i = 1, 2, ..., I$.

2.2. **Parameters of the Model.** The parameters are the known values (data) that are required by the model as an input to calculate the decision variables. These parameters are as defined in Table 1

TABLE 1. Parameters and description.

Parameters	**Description**
VC_i	Variable cost (TZS) in a single cycle of crop i production, represented as per unit hectare.
P_i	Market price (TZS) of crop i represented as per unit kilogram (Kg).
Ym_i	Maximum crop yield (Kg), represented as per unit hectare.
ETa	Actual evapotranspiration (mm).
ETm	Maximum (crop) evapotranspiration (mm).
$\lambda_{i,j}$	Sensitivity index of crop i to water stress at growth stage j.
k_i	Crop i yield response factor.
$IWRi$	Amount of irrigation water needed by the crop i to grow optimally, represented as per unit hectare
TAW	Total available water for Irrigation supply in the region (mm)
A^{max}	Maximum crop i area (Ha)
A^{min}	Minimum crop i area (Ha)
J	Number of growth stages
I	Number of crops
CP_i	Crop i Production Costs, the sum of fixed costs (FC) and variable costs (VC)
$w_{i,j}$	Amount of water required by crop i at growth stage j
P_e	Effective precipitation (mm)
P_T	Total precipitation (mm)

2.3. **Objective Function.** The multi-objective optimization problem designated in this research consists of two objective functions, namely: Maximizing Total Net Benefit ($\max TNB$) from planning region under different crops after meeting the costs of seeds, fertilizer, labor, transportation, surface water and plant protection, and Minimizing the Total related Variable Costs ($\min TVC$) in irrigation and crops growing.

1. **Total Net Benefit**
The Total Net Benefit (TNB) from planning region under different crops after meeting the costs of seeds, transport, fertilizer, labour, and plant protection are to be maximized.

$$\underset{A_i}{Maximize}\ TNB = \sum_{i=1}^{I} (Ya_i(w_{i,j})P_i - PC_i)A_i \tag{1}$$

where $Ya_i(w_{i,j})$ is expressed from [13] crop production function as:

$$Ya_i(w_{i,j}) = Ym_i \prod_{j=1}^{J} \left(\frac{ETa_{i,j}}{ETm_{i,j}} \right)^{\lambda_{i,j}} \tag{2}$$

Upon substitution, equation 2 into 1, the objective is expressed as follows:

$$\underset{A_i}{Maximize}\ TNB = \sum_{i=1}^{I} \left(P_i Ym_i \prod_{j=1}^{J} \left(\frac{ETa_{i,j}}{ETm_{i,j}} \right)^{\lambda_{i,j}} - PC_i \right) A_i \tag{3}$$

The calculation of ETa is given by [8]:

$$ETa(z,t) = g(z)ETm(t) \tag{4}$$

$$g(z) = \begin{cases} 1 & (1-p)TASW < z \leq TASW \\ \frac{z}{[(1-p)TASW]} & 0 < z \leq (1-p)TASW \end{cases} \tag{5}$$

where, z = level of soil moisture in the root zone, t = time interval, $g(z)$ = soil water depletion function, $TASW$ = total available soil water, p = soil moisture depletion factor for no stress and $ETm(t)$ = maximum evapotranspiration.
Since the only decision variables are the optimal allocation of cropped areas, the actual yield is taken to be equal to the maximum yield, hence: $g(z) = 1$ [7].

2. **Variable Costs Minimization**
Minimization of all variable costs incurred in crop production (TZS). The variable costs in crop production includes: Labour Costs (LC); Plant Protection Costs (PPC); Fertilizer Costs (FC); Seeds Cost (SC) and Transport Costs (TRC).

$$VC = LC + PPC + FC + SC + TRC \tag{6}$$

since the variable costs for crop production is proportional to the land allocated for cultivation A_i, the objective function is given as follows:

$$\underset{A_i}{Minimize}\, TVC = \sum_{i=1}^{I} VC_i A_i \tag{7}$$

2.4. **Model Constraints.** The best combination of the decision variables A_i is found with respect of the defined limitations or constraints. In this paper the constraints of the system are as follows:

1 Total Available Area
The total area allocated for different crops in a particular season should be less than or equal to the corresponding cultivable command area:

$$\sum_{i=1}^{I} A_i \leq TA \tag{8}$$

2 Total Available Water
Sum of all Crop Irrigation Water Requirement (IWR) for all crops should be less than or equal to Total Available Water (TAW) in the given irrigation scheme in a year:

$$\sum_{i=1}^{I} IWR_i A_i \leq TAW \tag{9}$$

3 Upper and Lower bounds (Affinity)

The area of crop(s) in any season must be greater than or equal to the minimum area (A_i^{min}) and less than or equal to maximum area (A_i^{max}) . Maximum and Minimum area were fixed based on the existing and projected cropping area in the command area:

$$A_i^{min} \leq A_i \leq A_i^{max} \; for \; i = 1, ..., I \tag{10}$$

2.5. **Model Input.** Table 2 shows the input parameters for the formulated model as collected from Nkoanrua region and literature. k_i^a and λ_i^b are Yield response factor and Crop

Table 2. Some critical data for the annual crops under Nduruma Catchment

		Crop					
Growth	Parameter	Maize	Carrots	Cucumber	Tomato	Melon	Cabbage
Initial Stage	k^a	0.4	0.45	0.45	0.45	0.45	0.45
	λ^b	0.35	0.39	0.39	0.39	0.39	0.39
Devt. Stage	k^a	0.80	0.75	0.70	0.75	0.75	0.75
	λ^b	0.75	0.69	0.64	0.69	0.69	0.69
Mid Stage	k^a	1.15	1.05	0.90	1.15	1.00	1.05
	λ^b	1.20	1.06	0.87	1.02	0.99	1.06
Late Stage	k^a	0.70	0.90	0.75	0.80	0.75	0.90
	λ^b	0.64	0.87	0.69	0.75	0.69	0.87
	P^e (TZS/Kg)	540	200	634	600	300	165
	VC^e (TZS/Ha×10^5)	6.5	4.9	6.1	15.1	6.0	4.5
	Ym^e (Kg)	3000	13000	15000	22000	24000	25000
	IWR (m^3/Ha)	4005	4300	2400	4500	3000	4000

sensitivity index to water stress respectively. [a] Enza Zaden; [b] Estimated by using [8] model and [e]Nkoanrua Region.

3. Multi-Objective Optimization Problem (MOOP)

Consider the general form of a MOOP:

$$\begin{aligned} max/min \; f_m(\mathbf{x}), \; m = 1, 2, ..., M; \; M \geq 2; \\ \textit{Subject to: } g_j(\mathbf{x}) \geq 0, \; j = 1, 2, ..., J; \\ h_k(\mathbf{x}) = 0, \; k = 1, 2, ..., K; \\ x_i^L \leq x_i \leq x_i^U, \; i = 1, 2, ..., n. \end{aligned} \tag{11}$$

A solution $\mathbf{x} \in \mathbb{R}^n$ is a vector of n decision variables representing the quantities for which values are to be chosen in the optimization problem: $\mathbf{x} = [x_1, x_2, ..., x_n]^T$. The image of the *decision variable space* $\mathbb{R}^n$ under f is the *objective space* which consists of the points $\mathbf{z} = [z_1,\ z_2, ...,\ z_M]^T$.

Definition 3.1. *Let $v,\ w \in \mathbb{R}^M$. The vector v is less than w ($v <_p w$) if $v_m < w_m$ $\forall m \in \{1, ..., M\}$. The relation $\leq_p$ is defined in an analogous way.*

In Multi-Objective Optimization there is no single solution (as in single objective) that simultaneously optimizes each objective function. Instead, we usually look for set of "trade-offs". The set of trade-offs is called *Pareto set* (after Vilfredo Pareto) and is defined as follows:

Definition 3.2. [Pareto Dominance relation, Pareto Optimal]

(a) A point $x^* \in \mathbb{R}^n$ Pareto dominates a point $x \in \mathbb{R}^n$, denoted by $x^* \prec_p x$, if $f(x^*) \leq_p f(x)$ and $f(x^*) \neq f(x)$.
(b) A point $x^* \in \mathbb{R}^n$ is called (globally) Pareto optimal *if there exists no point x dominating $x^* \in \mathbb{R}^n$.*
(c) The non-dominated set of the entire feasible search space is the globally Pareto-optimal set, *its image is called* Pareto front.

A more detailed introduction to multi-objective optimization can be found in [16] and [12].

4. Subdivision Algorithm

The subdivision algorithm was used to solve the formulated problem. The algorithm has been implemented in the software package Global Analysis of Invariant Objects (GAIO). To be able to use GAIO, the optimization model and the constraints have to be implemented in MATLAB [6, 21]. GAIO has an advantage over other methods when the image space dimension increases, Pareto front is disconnected and derivatives are hard to compute or not available. The Pareto set is approximated by a nested sequence of increasingly refined box coverings. Each iteration of the algorithm consists of a *subdivision step* and a *selection step*. In the subdivision step, the current box covering is subdivided into a finer covering that covers the same set. In the selection step, all boxes of the new box covering that are completely dominated by other boxes are removed from the collection. In this way each box covering is a more precise covering of the Pareto set (Algorithm 1).

4.1. Hausdorff Distance.

Hausdorff distance (d_H) is the farthest distance any point of B is from the set A, or the farthest any point of A is from B, whichever is greater.

Definition 4.1. *Given a metric space X, the set of closed sets of X supports a metric, the Hausdorff metric. If A is a set in X and $r > 0$, we define the $r-$thickening, or $r-$neighbourhood, of A to be the set $A^{(r)}$ defined by*

$$A^{(r)} = \bigcup_{x \in A} B_x(r) \tag{12}$$

where $B_x(r)$ is the (open) ball of radius r about x. if $A, B \subset X$ are closed sets, define their $Hausdorff$ distance $d_H(A, B)$ to be the number

$$d_H(A,B) = \inf\left\{r > 0 \mid B \subset A^{(r)} \text{ and } A \subset B^{(r)}\right\} \tag{13}$$

Recall that the infimum of an empty set is regarded to be $+\infty$. A equivalent definition is as follows. Given a point $p \in X$ and a closed set $A \subset X$, define

$$d(p, A) = \inf_{y \in A} \operatorname{dist}(p, y) \tag{14}$$

then the Hausdorff distance is

$$d_H = \max\left\{\sup_{x \in A} d(x, B),\ \sup_{x \in B} d(y, A)\right\} \tag{15}$$

Theorem 4.2. *If (X, d) is a bounded metric space, the set of closed sets of X is itself a metric space with the Hausdorff metric*

Algorithm 1 Subdivision (Derivative Free)

Required: Box constraints $x^{min},\ x^{max} \in \mathbb{R}^n$, number of subdivision steps N_{sub}

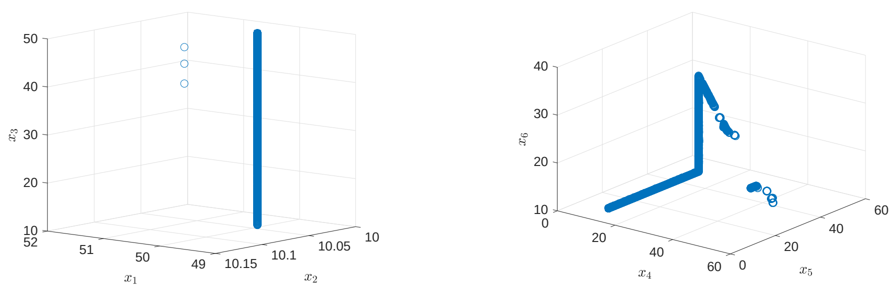
1: Create initial box collection β_0 defined by the constraints $x^{min},\ x^{max}$, for example $\beta_0 = B = [x^{min},\ x^{max}] \times ... \times [x^{min},\ x^{max}]$
2: **for** $i = 1, ..., N_{sub}$ **do**
3: Subdivision step: Construct from β_{i-1} a new collection $\hat{\beta}_i$ of set such that: $\bigcup_{B \in \hat{\beta}_i} B = \bigcup_{B \in \beta_{i-1}} B$ and $\operatorname{diam}(\hat{\beta}_i) = \theta_i \operatorname{diam}(\beta_{i-1})$, where $0 < \theta_{min} \leq \theta i < \theta_{max} \leq 1$
4: Insert S sampling points $x_1, ..., x_S$ into each box and evaluate the objective function $f(x_s), s = 1, ..., S$
5: Selection step: Eliminate all boxes that contain only dominated points: $\beta_i = \left\{B \in \hat{\beta}_i \mid \textbf{There exists no } x^* \in \bigcup_{\hat{B} \in \hat{\beta}_i \setminus B} \hat{B} \textbf{ such that } f(x^*) \leq_p f(x) \forall x \in B\right\}$
6: **end for**

5. Results

Nkoanrua irrigation scheme is operated traditionally and designed to supply water to irrigate 200 Hectare, with average discharge of the $3000000 m^3$ annually. The model analyses the farm benefit, crop water requirement, available land, production costs, crop prices, market considerations and minimum food requirement. Once the optimal irrigation areas are obtained, a sensitivity analysis was performed to test the effectiveness of the model, specifically the efficiency of the model was tested by changing the model parameters (Net Benefit and Production Costs) in the following scenarios: (1) Decreasing and increasing in crop market price and (2) Increasing and decreasing in crop production costs. Both price and cost have an effect on the farm benefit and production costs which are the coefficients of the objective functions.

5.1. **Pareto Set and Pareto Front.** 54 subdivision steps, Monte Carlo sampling method and affinity constraints as initial box were used for model computation. After 9 minutes and 33.20 seconds, a total number of 2227000 functions were evaluated with 2973 non dominated boxes, the model was implemented in a Debian GNU/Linux 8 (jessie) operating system desktop computer with 8 GB memory, and Intel(R) Core(TM)2 Duo CPU 3.16 GHz each.

Fig.1a and Fig.1b show the entire approximated Pareto set of the problem, the Pareto set represents the possible optimal area ($x_i,\ i = 1, 2, ..., 6$). The farmer may choose crop pattern from the Pareto set depending on his/her preference. In multi-objective optimization problem, the preference attitudes of the decision maker play a vital role that specifies the sense of optimality or desirability. From the figures, the amount of land required for maize (x_1) and carrots (x_2) is based on the minimum limitations requirements, therefore the decision maker has no choice based on his/her preference.



(A) (B)

FIGURE 1. Pareto set

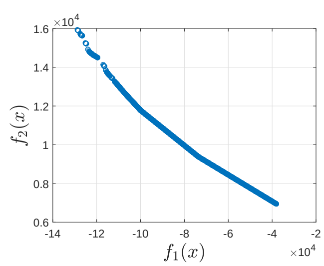


FIGURE 2. Pareto Front

Fig.2, shows the image of the Pareto set (Pareto front) under the objective functions, it is the *trade-off*" curve that explain the optimal compromise point between Total Benefits ($f_1(x)$) derived from the farm against the Total Variable Costs ($f_2(x)$) incurred during crops production.

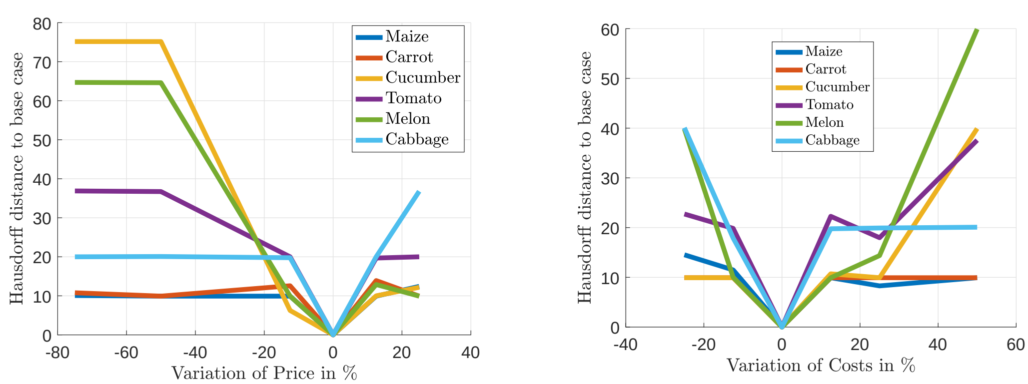
5.2. **Sensitivity Analysis.** In view of mathematics, the Hausdorff distance (d_H) was used to analyse the distance between Pareto set of the base model and the Pareto sets of the new models after parameter variation due to market price and production cost change. Fig.3a shows that, the model is not sensitive to the price of maize and carrots (less profitable crops), the change in prices does not produce significant Hausdorrf distance ($d_H \leq 20$). Meanwhile, the Hausdorff distance suggested that, the market price of cucumber, melon, tomato and cabbage are sensitive parameters to the model, the new model suggested different Pareto set ($d_H \geq 20$). Fig.3b shows that, the variation of maize and carrots production costs have no impact to the solutions of the base model, the variations produce insignificant Hausdorff distances ($d_H \leq 20$). Nevertheless, variations of cucumber, tomato, melon, and cabbage production costs have an impact to the optimal solutions of the basic model because, the calculated Hausdorff distance is lager enough ($d_H \geq 20$).

The table 3 shows the summary of the information expressed in Fig.3a and Fig.3b. The parameters are given in reference to the index of the crops, $i = 1, 2, 3, 4, 5, 6.$, where, $1 = maize,\ 2 = carrots,\ 3 = cucumber,\ 4 = tomato,\ 5 = melon,\ 6 = cabbage.$

TABLE 3. Summary of sensitivity analysis of the model parameters

Parameter	**Sensitivity**
$P_1,\ PC_1$	No, No
$P_2,\ PC_2$	No, No
$P_3,\ PC_3$	Yes, Yes
$P_4,\ PC_4$	Yes, Yes
$P_5,\ PC_5$	Yes, Yes
$P_6,\ PC_6$	Yes, Yes

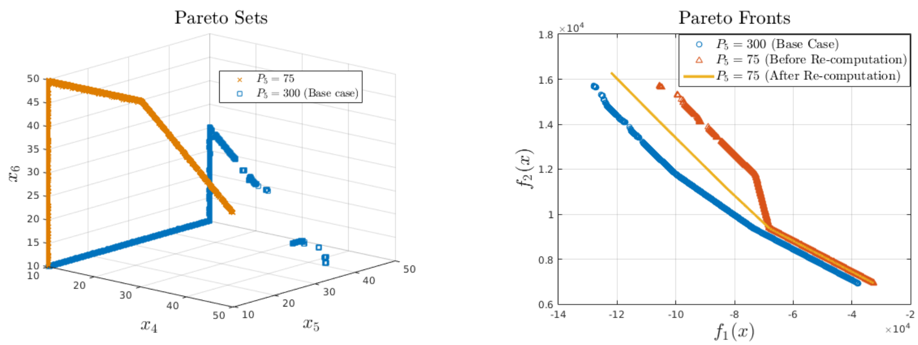
P_i and PC_i are Crop i Market Price and Crop i Production Costs respectively.



(A) (B)

FIGURE 3. Hausdorff Distance (d_H)

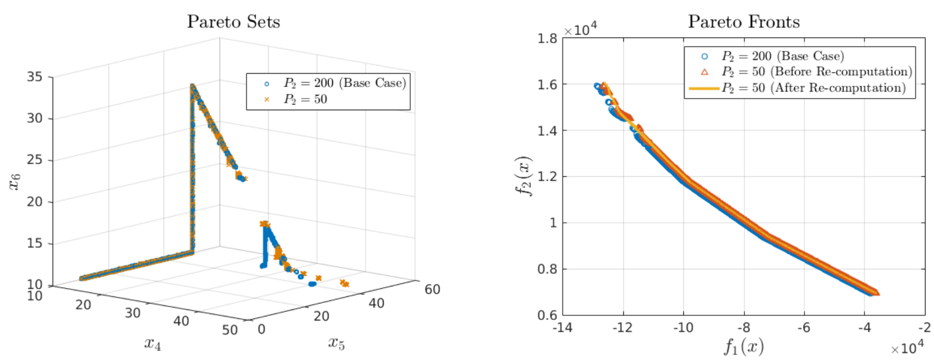
The realization of the impact of parameters variation to the model solutions as described by Hausdorff distance were expressed by comparing the Pareto set and Pareto front of the basic model and the new models after Price and Production costs variation. Fig.4a shows the Pareto set of the basic model and the new Pareto set after price of melon was reduced by 75%. The reduction of melon price caused the Pareto set to shift in the direction of x_6 by 20 units above (From 30 to 50 Hectares as the maximum possible land allocation)



(A) (B)

FIGURE 4. Pareto set and Pareto front of the base case and after price of melon reduced by 75%

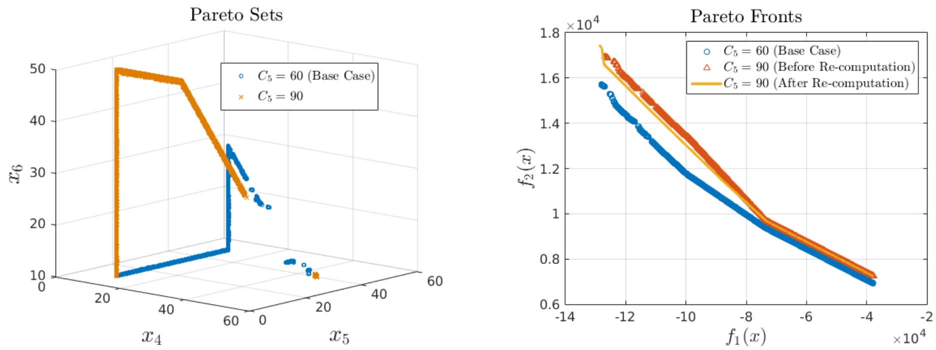
Fig.5a presents the comparison between the Pareto sets of the basic model and new model after reduction of price of carrots by 75%, as in Hausdorff distance scenario, change in carrots market price does not have any impact to the solution of the problem. The Pareto set and Pareto front (Fig. 5b) of the new model after Re-computation remains the same as of the basic model.



(A) (B)

FIGURE 5. Pareto set and Pareto front of the base case and after carrots price reduced by 75%

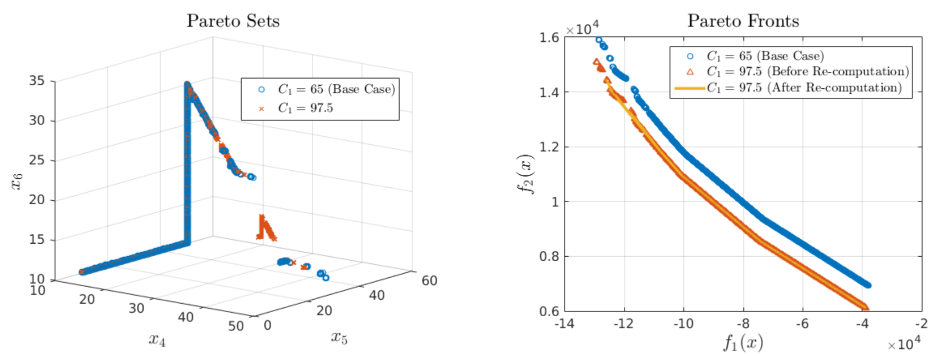
Similar to price changes, the sensitivity with respect to crop production costs has been investigated: For example, Increasing of melon production costs by 50%, the new model suggested a significant change of the optimal solution. Fig.6a shows the Pareto sets of the base case and new model due to melon production cost change. There is a slight different to the Pareto front as shown in Fig.6b. The new model propose the maximum land allocation for cabbage to be 50 Hectares from the basic model with 30 Hectares.



(A) (B)

FIGURE 6. Pareto set and Pareto front of the base case and after costs of melon increased by 50%

Lastly, the impact of maize production costs parameter to the Pareto set of the basic model was analysed, the production costs was changed by +50%, Fig.7a shows the Pareto sets of the basic model and new model after maize production costs change looks similar. The Pareto front before and after Re-computation also are the same. Therefore, as in Hausdorff distance scenarios, the change in maize production costs has no impact to basic model solutions. The basic model solutions can be used as the optimal solutions to the new model when the costs of maize change.



(A) (B)

FIGURE 7. Pareto set and Pareto front of the base case and after costs of maize increased by 50%

6. Conclusion

This work aimed to formulate and analyse a mathematical model for optimizing water use in agriculture by determining the best crop pattern which maximize the value of farm benefit and minimize the value of production cost according to regional limitations and water availability by using multi-criteria decision support. The model well-found the optimal distribution of land, the water requirements, total variables cost, and the total benefit. The model provides several crop patterns (Pareto optimal set) from which the decision maker has opportunity to choose the best pattern as per preference. Through sensitivity analysis the model suggested important parameters which require much attention by decision maker on the course of choosing the pattern from the Pareto optimal set. Sensitivity analysis results show that, price and production costs for cucumber, melon, tomato and cabbage had a large impact on the optimal solution. The results support the study by [3], selection of profitable crops, farmers must pay much consideration on the crop market price and production cost.

Acknowledgement: The authors thank German Academic Exchange Service (DAAD) scholarship that was necessary to cover all related costs towards this work. We also appreciate the material and moral support from Prof. Dr. Michael Dellnitz and Dr. Sebastian Peitz of the Applied Mathematics Research Group at the University of Paderborn, Germany.

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