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SOME DISTANCE BASED TOPOLOGICAL INDICES OF STRONG DOUBLE GRAPHS

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Abstract. In this paper, we investigated some famous distance based topological indices of strong double graphs. Some bounds are also defined on certain distance based topological indices of some families of graphs of strong double graphs as corollaries of main results.

1. Introduction

Let *G(V,E)* be a graph, where V and E represent the vertex and edge sets, respectively. The number of elements in vertex set is called the *order* of the *graph* G and the number of elements in edge set is called the *size* of the graph G. In a graph G, the *distance* between two vertices u and v is the length of the shortest path connecting them. It is denoted as $d_G(u,v)$.The maximum distance between the vertex u and any other vertex of the graph G is called the *eccentricity* of the vertex u , $ecc_G(u)$, in the graph G. The *degree* of a vertex $u \in V(G)$ is the number of adjacent vertices to the vertex u, which is denoted as $d_G(u)$. P_n, S_n, and K_n notations represent the path, star and complete graphs with order n respectively.

Key words and phrases. degree, distance, topological indices, strong double graph.

Munarini [10] introduced the *double graph D[G]* of a simple graph. The double graph of a simple G can be constructed by taking two distinct copies of the graph G and joining each vertex in one copy with the neighbors of the corresponding vertex in another copy. The *k-fold* graph [2] $D^t[G]$ can be obtained in a similar way, by talking t distinct copies of graph G and joining each vertex in one of the copy with the neighbors of the corresponding vertices in other copies.

The *strong double graph* [2] of a graph G, *SD(G),* is the graph obtained by taking two copies of the graph G and joining each vertex u_i in one copy with the closed neighborhood $N[u_i] = N(u_i) \bigcup \{u_i\}$ of the corresponding vertex in another copy.

Topological index is a numerical quantity associated with the molecular graph G obtained from the corresponding molecular graph structure. This quantity is invariant under graph isomorphism. In chemical graph theory, topological indices are used to describe some physical and chemical properties of different compounds. Hundreds of distance and degree based topological indices have been introduced, but few succeed to attain the attraction of chemists.

The *Wiener index* is a distance-based graph invariant defined as [4]

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)$$

In 1993, *Randiċ* proposed the *hyper-Wiener index* [13] as a kind of extension of the Wiener index. It is defined as

$$WW(G) = \frac{1}{2} \sum_{\{u,v\} \subset V(G)} [d_G(u,v) + d_G(u,v)^2]$$

The *Harary index* of G is defined as the sum of reciprocals of distances between all unordered pairs of vertices of a connected graph [5].

$$H(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{d(u,v)}$$

[1] and [20] modified the Harary index and defined the *additively* and *multiplicatively Harary index* as

$$H_A(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{d(u)+d(v)}{d(u,v)}$$
$$H_M(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{d(u)d(v)}{d(u,v)}$$

The *degree distance index* of a graph G was proposed by *Dobrynin* and *Kochetova* [4] as

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} [d_G(u) + d_G(v)] d_G(u,v)$$

I. Gutman introduced the *Gutman index* [7]

$$Gut(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u) d_G(v) d_G(u,v)$$

Sharma [14] introduced the *eccentric connectivity index* of a graph G which is defined as

$$\zeta^c(G) = \sum_{u \in V(G)} d(u)\, ecc(u)$$

and the *total eccentric connectivity index* is defined as

$$\zeta(G) = \sum_{u \in V(G)} ecc(u)$$

Jamil [9] found some distance based topological indices of double graphs. In this paper, we compute some topological indices of strong double graphs and find sharp bounds on strong double graphs of different families of graphs.

2. Main Results

The composition of two graphs G and K with disjoint vertex sets is the graph $G[K]$ with vertex set $V(G) \times V(K)$ such that two vertices (u_1, u_2) and (v_1, v_2) are adjacent if and only if $u_1 = v_1$ and $u_2v_2 \in E(K)$ or $u_1v_1 \in E(G)$. One can see that a strong double graph can be obtained by composing the graph G to the complete graph K_2, i.e. $SD(G) = G[K_2]$. If a graph G has order n and size m then its strong double graph $SD[G]$ is of order $2n$ and size $5m$. Fig. 1 shows a graph and its strong double graph. For further results and properties, refer to [3,10]. In this section, we compute some topological indices of strong double graphs and also mention some bounds on strong double graphs of some certain families of graphs.

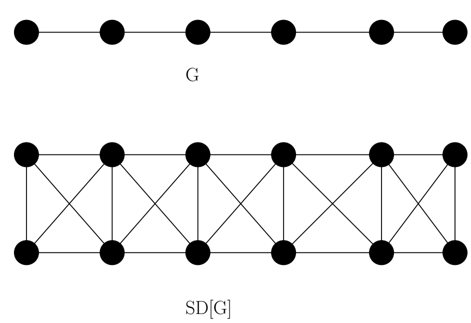

Figure 1. A graph G and its strong double graph SD[G].

Let $G'(V', E')$ be the distinct copy of the graph $G(V,E)$. Then the strong double graph $SD[G]$ have the vertex set $V(SD[G]) = V(G) \bigcup V(G')$, where $V(G) = \{x_1, x_2, \cdots, x_n\}$, $V(G') = \{y_1, y_2, \cdots, y_n\}$ and y_j are the corresponding vertex of x_j in $V(G')$.

Lemma 2.1. *Let G be a simple graph and $SD[G]$ be its strong double graph. Then*

$$d_{SD[G]}(x_i, y_j) = d_G(x_i, x_j) \quad ; i, j = 1, \ldots, n, i \neq j$$

Proof.

Let $x_i \in V(G)$ and $y_j \in V(G')$. Suppose $l = d_{SD[G]}(x_i, y_j) < d_G(x_i, x_j) = p$ and a shortest path in $SD[G]$ is $x_i v_1 v_2 \cdots v_{l-1} y_j$. If $l = 1$ the property is true. Let $l \geq 2$. It follows that there exists some $v_k \in V(G')$. Since v_{k-1} and v_{k+1} are adjacent to v_k, by construction v_{k-1} and v_{k+1} are adjacent to x_k(corresponding vertex of v_k in $V(G)$). We have obtained a path $x_i v_1 v_2 \ldots x_k \ldots v_{l-1} y_j$ in $SD[G]$, which implies the existence of a path $x_i x_1 x_2 \ldots x_k \ldots x_{l-1} x_j$ in G of length l, a contradiction. If $l = d_{SD[G]}(x_i, y_j) > d_G(x_i, x_j) = p$, we get a similar contradiction. Consequently, $d_{SD[G]}(x_i, y_j) = d_G(x_i, x_j)$.

Lemma 2.2. *Let G be a simple graph and $SD[G]$ be its strong double graph. Then*

$$d_{SD[G]}(x_i, x_j) = d_G(x_i, x_j) \quad ; i, j = 1, \ldots, n.$$

Proof.

Clearly, $G \subset SD[G]$. Let $\{x_i, x_j\} \subseteq V(G) \subset V(D[G])$ then $d_{SD[G]}(x_i, x_j) \leq d_G(x_i, x_j)$. Suppose $l = d_{SD[G]}(x_i, x_j) < d_G(x_i, x_j) = p$ and a shortest path in $SD[G]$ from x_i to x_j is $x_i v_1 v_2 \cdots v_{l-1} x_j$. If $l = 1$ then the property is obvious. Suppose $l \geq 2$. Since $l < p$, there exists some $v_k \in V(G')$. As v_{k-1} and v_{k+1} are adjacent to v_k, by definition of the double graph v_{k-1} and v_{k+1} are adjacent to x_k(corresponding vertex of v_k in $V(G)$). Now we have obtained a path $x_i v_1 v_2 \ldots x_k \cdots v_{l-1} x_j$. In this way we can find a path $x_i x_1 x_2 \ldots x_k \cdots x_{l-1} x_j$ in G of length l, which is a contradiction. It follows that $d_{SD[G]}(x_i, x_j) = d_G(x_i, x_j)$. Similarly, $d_{SD[G]}(y_i, y_j) = d_G(y_i, y_j)$.

Lemma 2.3. *For a simple graph G and its strong double graph $SD[G]$, we have*

$$ecc_{SD[G]}(x_i) = ecc_{SD[G]}(y_i) = ecc_G(x_i) \quad ; i = 1, \ldots, n$$

Lemma 2.4. *For the strong double graph $SD[G]$*

$$d_{SD[G]}(x_i) = d_{SD[G]}(y_i) = 2d_G(x_i) + 1 \;\; ; i = 1, \ldots, n.$$

Theorem 2.5. *Let G be a simple graph with n vertices. Then the Wiener index of $SD[G]$ is given by*

$$W(SD[G]) = 4W(G) + n.$$

Proof.

The Wiener index of $SD[G]$ is

$$W(SD[G]) = \sum_{1 \leq i < j \leq n} d_{SD[G]}(v_i, v_j)$$

$$= \sum_{1 \leq i < j \leq n} d_{SD[G]}(x_i, x_j) + \sum_{1 \leq i < j \leq n} d_{SD[G]}(y_i, y_j)$$

$$+\sum_{i,j=1,\ldots,n} d_{SD[G]}\left(x_i, y_j\right)$$

By lemmas 2.1 and 2.2 , we deduce

$$\begin{aligned} W\left(D\left[G\right]\right) &= \sum_{1\leq i<j\leq n} d_G\left(x_i, x_j\right) + \sum_{1\leq i<j\leq n} d_G\left(x_i, x_j\right) \\ &+ \sum_{\substack{i,j=1,\ldots,n \\ i\neq j}} d_G\left(x_i, x_j\right) + n \\ &= W(G) + W(G) + 2W(G) + n \\ &= 4W\left(G\right) + n. \end{aligned}$$

A well known property of the Wiener index of trees implies the following corollary.

Corollary 2.6. *Suppose T_n is a tree with n vertices. Then*

$$W\left(SD\left[S_n\right]\right) \leq W\left(SD\left[T_n\right]\right) \leq W\left(SD\left[P_n\right]\right).$$

Theorem 2.7. *Let G be a simple graph with n vertices. Then the hyper-Wiener index of $SD[G]$ is given by*

$$WW\left(SD[G]\right) = 4WW(G) + n.$$

Proof.
The hyper-Wiener index of $SD[G]$ is

$$WW(SD[G]) = \frac{1}{2}\sum_{1\leq i<j\leq n}(d_{SD[G]}(v_i, v_j) + (d_{SD[G]}(v_i, v_j)^2)$$

$$= \frac{1}{2}\sum_{1\leq i<j\leq n}(d_{SD[G]}\left(x_i, x_j\right) + (d_{SD[G]}\left(x_i, x_j\right))^2)$$

$$+\frac{1}{2}\sum_{1\leq i<j\leq n}(d_{SD[G]}\left(y_i, y_j\right) + (d_{SD[G]}\left(y_i, y_j\right))^2)$$

$$+\frac{1}{2}\sum_{1\leq i<j\leq n}(d_{SD[G]}\left(x_i, y_j\right)(d_{SD[G]}\left(x_i, y_j\right))^2)$$

By lemmas 2.1-2.2 , we deduce

$$\begin{aligned} WW\left(SD\left[G\right]\right) &= \frac{1}{2}\sum_{1\leq i<j\leq n}\left(d_G\left(x_i,x_j\right)+\left(d_G\left(x_i,x_j\right)\right)^2\right) \\ &+\frac{1}{2}\sum_{1\leq i<j\leq n}\left(d_G\left(x_i,x_j\right)+\left(d_G\left(x_i,x_j\right)\right)^2\right) \\ &+\frac{1}{2}\sum_{\substack{i,j=1,\ldots,n \\ i\neq j}}\left(d_G\left(x_i,x_j\right)+\left(d_G\left(x_i,x_j\right)\right)^2\right)+n \\ &= WW(G)+WW(G)+2WW(G)+n \\ &= 4WW\left(G\right)+n. \end{aligned}$$

From [8], we have the following corollary.

Corollary 2.8. *If T is an n vertex tree, then for all values of n, $n\geq 1$*

$$WW(SD[S_n])\leq WW(SD[T])\leq WW(SD[P_n])$$

Theorem 2.9. *Let G be a simple graph with n vertices. Then the Harary index of $SD[G]$ is given by*

$$H\left(SD[G]\right)=4H(G)+n.$$

Proof. The Harary index of $SD[G]$ is

$$H(SD[G])=\sum_{1\leq i<j\leq n}\frac{1}{d_{SD[G]}(v_i,v_j)}$$

$$=\sum_{1\leq i<j\leq n}\frac{1}{d_{SD[G]}\left(x_i,x_j\right)}+\sum_{1\leq i<j\leq n}\frac{1}{d_{SD[G]}\left(y_i,y_j\right)}$$

$$+\sum_{i,j=1,\ldots,n}\frac{1}{d_{SD[G]}\left(x_i,y_j\right)}$$

By Lemmas 2.1, 2.2 we have

$$\begin{aligned} H\left(SD\left[G\right]\right) &= \sum_{1\leq i<j\leq n}\frac{1}{d_G\left(x_i,x_j\right)}+\sum_{1\leq i<j\leq n}\frac{1}{d_G\left(x_i,x_j\right)} \\ &+\sum_{\substack{i,j=1,\ldots,n \\ i\neq j}}\frac{1}{d_G\left(x_i,x_j\right)}+n \\ &= H(G)+H(G)+2H(G)+n \\ &= 4H\left(G\right)+n. \end{aligned}$$

From [19], we have the following result

Corollary 2.10. *Let G be a connected graph of order n. Then*

$$H(SD[G]) \leq H(SD[K_n])$$

and equality holds if and only if $G \cong K_n$.

Corollary 2.11. *Let T_n be a tree with n vertices. Then*

$$H\left(SD\left[P_n\right]\right) \leq H\left(SD\left[T_n\right]\right) \leq H\left(SD\left[S_n\right]\right).$$

Theorem 2.12. *Let G be a simple graph with m edges. Then the additively weighted Harary index of $SD[G]$ is given by*

$$H_A\left(SD\left[G\right]\right) = 8H_A\left(G\right) + 8H\left(G\right) + 8m + 2n.$$

Proof. The additively Harary index of $SD[G]$ is

$$H_A(SD[G]) = \sum_{1 \leq i < j \leq n} \frac{d_{SD[G]}(v_i) + d_{SD[G]}(v_j)}{d_{SD[G]}(v_i, v_j)}$$

$$= \sum_{1 \leq i < j \leq n} \frac{d_{SD[G]}\left(x_i\right) + d_{SD[G]}\left(x_j\right)}{d_{SD[G]}\left(x_i, x_j\right)} + \sum_{1 \leq i < j \leq n} \frac{d_{SD[G]}\left(y_i\right) + d_{SD[G]}\left(y_j\right)}{d_{SD[G]}\left(y_i, y_j\right)}$$

$$+ \sum_{i,j=1,\ldots,n} \frac{d_{SD[G]}\left(x_i\right) + d_{SD[G]}\left(y_j\right)}{d_{SD[G]}\left(x_i, y_j\right)}$$

By lemmas 2.1, 2.2, 2.4 the last expression is equal to

$$\sum_{1 \leq i < j \leq n} \frac{2d_G\left(x_i\right) + 2d_G\left(x_j\right) + 2}{d_G\left(x_i, x_j\right)} + \sum_{1 \leq i < j \leq n} \frac{2d_G\left(x_i\right) + 2d_G\left(x_j\right) + 2}{d_G\left(x_i, x_j\right)}$$

$$+ \sum_{i,j=1,\ldots,n} \frac{2d_G\left(x_i\right) + 2d_G\left(x_j\right) + 2}{d_G\left(x_i, x_j\right)}$$

$$= 2H_A\left(G\right) + 2H(G) + 2H_A\left(G\right) + 2H(G) + 4H_A\left(G\right) + 4H(G)$$

$$+ 4 \sum_{x_i \in V(G)} d_G\left(x_i\right) + \sum i \in \{1, 2, 3, \ldots, n\} \frac{2}{d_G(x_i, y_i)}$$

$$= 8H_A\left(G\right) + 8H(G) + 8m + 2n.$$

Corollary 2.13. *Suppose T_n and U_n be tree and unicyclic graphs, respectively, with n vertices. Then*

$$H_A\left(SD[T_n]\right) = 8H_A\left(T_n\right) + 8H\left(T_n\right) + 10n - 8.$$

$$H_A\left(SD[U_n]\right) = 8H_A\left(U_n\right) + 8H\left(U_n\right) + 10n.$$

$$H_A\left(SD[B_n]\right) = 8H_A\left(B_n\right) + 8H\left(B_n\right) + 10n + 8.$$

Corollary 2.14. *Suppose T_n is a tree with n vertices. Then*

$$H_A\left(SD\left[P_n\right]\right) \leq H_A\left(SD\left[T_n\right]\right) \leq H_A\left(SD\left[S_n\right]\right).$$

Theorem 2.15. *Let G be a simple graph. The multiplicative weighted Harary index of $SD[G]$ is given by*

$$H_M\left(SD\left[G\right]\right) == 16H_M(G) + 8H_A(G) + 4H(G) + 4M_1(G) + 8m + n$$

Proof. The multiplicative Harary index of $SD[G]$ is

$$H_M(SD[G]) = \sum_{1\leq i<j\leq n} \frac{d_{SD[G]}(v_i)d_{SD[G]}(v_j)}{d_{SD[G]}(v_i, v_j)}$$

$$= \sum_{1\leq i<j\leq n} \frac{d_{SD[G]}\left(x_i\right)d_{SD[G]}\left(x_j\right)}{d_{SD[G]}\left(x_i, x_j\right)} + \sum_{1\leq i<j\leq n} \frac{d_{SD[G]}\left(y_i\right)d_{SD[G]}\left(y_j\right)}{d_{SD[G]}\left(y_i, y_j\right)}$$

$$+ \sum_{i,j=1,\ldots,n} \frac{d_{SD[G]}\left(x_i\right)d_{SD[G]}\left(y_j\right)}{d_{SD[G]}\left(x_i, y_j\right)}$$

By lemmas 2.1, 2.2 and 2.4 this expression equals to

$$\sum_{1\leq i<j\leq n} \frac{(2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)}{d_G\left(x_i, x_j\right)} + \sum_{1\leq i<j\leq n} \frac{(2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)}{d_G\left(x_i, x_j\right)} +$$

$$\sum_{\substack{i,j=1,\ldots,n\\ i\neq j}} \frac{(2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)}{d_G\left(x_i, x_j\right)} + \sum_{x_i\in V(G)} (2d_G\left(x_i\right)+1)(2d_G\left(x_i\right)+1)$$

$$= 4H_M\left(G\right) + 2H_A\left(G\right) + H\left(G\right) + 4H_M\left(G\right) + 2H_A\left(G\right) + H\left(G\right)$$

$$+ 8H_M\left(G\right) + 4H_A\left(G\right) + 2H\left(G\right)$$

$$+ \sum_{x_i\in V(G)} \left(2d_G(x_i)+1\right)^2$$

$$= 16H_M(G) + 8H_A(G) + 4H(G) + \sum_{x_i\in V(G)} \left(4d_G(x_i)^2 + 4d_G(x_i) + 1^2\right.$$

$$= 16H_M(G) + 8H_A(G) + 4H(G) + 4M_1(G) + 8m + n.$$

Corollary 2.16. *Suppose P_n, S_n, C_n and K_n be the path, star cyclic and complete graphs with n vertices. Then*

$$H_M\left(SD\left[P_n\right]\right) = 16H_M\left(P_n\right) + 8H_A\left(P_n\right) + 4H\left(P_n\right) + 25n - 32$$

$$H_M\left(SD\left[S_n\right]\right) = 16H_M\left(S_n\right) + 8H_A\left(S_n\right) + 4H\left(S_n\right) + 10n - 10$$

$$H_M\left(SD\left[C_n\right]\right) = 16H_M\left(C_n\right) + 8H_A\left(C_n\right) + 4H\left(C_n\right) + 25n$$

$$H_M\left(SD\left[K_n\right]\right) = 16H_M\left(K_n\right) + 8H_A\left(K_n\right) + 4H\left(K_n\right) + n(2n-1)^2.$$

Theorem 2.17. *Let G be a simple graph with m edges. Then the degree distance index of $SD[G]$ is given by*

$$DD\left(SD\left[G\right]\right) = 8DD\left(G\right) + 8W\left(G\right).$$

Proof. The degree distance index of $SD[G]$ is

$$DD(SD[G]) = \sum_{1\leq i<j\leq n} [d_{SD[G]}(v_i) + d_{SD[G]}(v_j)]d_{SD[G]}(v_i, v_j)$$

$$= \sum_{1\leq i<j\leq n} [d_{SD[G]}(x_i) + d_{SD[G]}(x_j)]d_{SD[G]}(x_i, x_j)$$

$$+ \sum_{1\leq i<j\leq n} [d_{SD[G]}(y_i) + d_{SD[G]}(y_j)]d_{SD[G]}(y_i, y_j)$$

$$+ \sum_{i,j=1,\ldots,n} [d_{SD[G]}(x_i) + d_{SD[G]}(y_j)]d_{SD[G]}(x_i, y_j)$$

By lemmas 2.1, 2.2, 2.4 the last expression equals to

$$\sum_{1\leq i<j\leq n} [2d_G(x_i) + 2d_G(x_j) + 2]d_G(x_i, x_j)$$
$$+ \sum_{1\leq i<j\leq n} [2d_G(x_i) + 2d_G(x_j) + 2]d_G(x_i, x_j)$$
$$+ \sum_{i,j=1,\ldots,n} [2d_G(x_i) + 2d_G(x_j) + 2]d_G(x_i, x_j)$$
$$= 2DD(G) + 2W(G) + 2DD(G) + 2W(G) + 4DD(G) + 4W(G)$$
$$= 8DD(G) + 8W(G).$$

Theorem 2.18. *Let G be a simple graph. The Gutman index of $SD[G]$ is given by*

$$Gut(SD[G]) = 16Gut(G) + 8DD(G) + 4W(G) + \sum_{x_i\in V(G)} (2d_G(x_i) + 1)^2.$$

Proof. The multiplicative Harary index of $SD[G]$ is

$$Gut(SD[G]) = \sum_{1\leq i<j\leq n} d_{SD[G]}(v_i)d_{SD[G]}(v_j)d_{SD[G]}(v_i, v_j)$$

$$= \sum_{1\leq i<j\leq n} d_{SD[G]}(x_i)\, d_{SD[G]}(x_j)d_{SD[G]}(x_i, x_j)$$

$$+ \sum_{1\leq i<j\leq n} d_{SD[G]}(y_i)\, d_{SD[G]}(y_j)d_{SD[G]}(y_i, y_j)$$

$$+ \sum_{i,j=1,\ldots,n} d_{SD[G]}(x_i)\, d_{SD[G]}(y_j)d_{SD[G]}(x_i, y_j)$$

By lemmas 2.1, 2.2 and 2.4 this expression equals to

$$\sum_{1\leq i<j\leq n} (2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)d_G\left(x_i,x_j\right)$$
$$+\sum_{1\leq i<j\leq n} (2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)d_G\left(x_i,x_j\right)+$$
$$\sum_{\substack{i,j=1,\dots,n\\ i\neq j}} (2d_G\left(x_i\right)+1)(2d_G\left(x_j\right)+1)d_G\left(x_i,x_j\right)$$
$$+\sum_{x_i\in V(G)} (2d_G\left(x_i\right)+1)(2d_G\left(x_i\right)+1)$$
$$=4Gut\left(G\right)+2DD\left(G\right)+W\left(G\right)+4Gut\left(G\right)+2DD\left(G\right)+W\left(G\right)$$
$$+8Gut\left(G\right)+4DD\left(G\right)+2W\left(G\right)$$
$$+\sum_{x_i\in V(G)} \left(2d_G(x_i)+1\right)^2$$
$$=16Gut(G)+8DD(G)+4W(G)+\sum_{x_i\in V(G)} \left(2d_G(x_i)+1\right)^2.$$

Theorem 2.19. *Suppose G is a graph of order n. The eccentric connectivity index of $SD[G]$ is given by*

$$\zeta^c\left(SD\left[G\right]\right)=4\zeta^c\left(G\right)+2\zeta\left(G\right).$$

Proof.

$$\zeta^c\left(SD\left[G\right]\right)=\sum_{i=1}^{n} d_{SD[G]}(x_i)ecc_{SD[G]}(x_i)+\sum_{i=1}^{n} d_{SD[G]}(y_i)ecc_{SD[G]}(y_i).$$

By Lemmas 2.3 and Lemma 2.4 we have

$$\zeta^c\left(SD\left[G\right]\right)=\sum_{i=1}^{n}(2d_G(x_i)+1)ecc_G(x_i)+\sum_{i=n}^{n}(2d_G(x_i)+1)ecc_G(x_i)$$

$$=2\sum_{i=1}^{n} d_G(x_i)ecc_G(x_i)+\sum_{i=1}^{n} ecc_G(x_i)+2\sum_{i=1}^{n} d_G(x_i)ecc_G(x_i)+\sum_{i=1}^{n} ecc_G(x_i)$$
$$=4\zeta^c\left(G\right)+2\zeta\left(G\right)$$

The following theorem is clear from Lemma 2.3.

Theorem 2.20. *Let G be a simple graph of order n. The total eccentricity index of $SD[G]$ is given by*

$$\zeta\left(SD\left[G\right]\right)=2\zeta\left(G\right).$$

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