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COMPARISON OF ADOMIAN AND TAYLOR POLYNOMIAL SOLUTIONS OF SOME NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS OF VARIABLE COEFFICIENTS

A. K. JIMOH[1,*], A. M. AYINDE[2]

[1]*Department of Mathematics and Statistics, Faculty of Pure and Applied Sciences, Kwara State University, Malete, Nigeria*

[2]*Department of Mathematics, School of Pure and Applied Sciences, Modibbo Adama University of Technology, Yola, Nigeria*

*Correspondence: abdulazeez.jimoh@kwasu.edu.ng

Abstract. In this paper, a comparison of the Adomian and Taylor polynomial solutions of some nonlinear Ordinary Differential Equations of constant coefficients to those equations with variable coefficients is presented. The total derivatives of the nonlinear functions involved in the problem considered were derived in order to obtain the Adomian polynomials for the problems. For Taylor polynomials, the nonlinear functions involved were iteratively differentiated. Numerical experiments show that Adomian Decomposition Method can be extended as alternative way for finding numerical solutions to ordinary differential equations of variable coefficients. Furthermore, the methods are easy with no assumption and they produce accurate results when compared with other methods in literature. Moreover, the Taylor polynomial solution gives more accurate results compared to the results by Adomian polynomial solution in terms of the absolute errors.

1.0 Introduction

Differential equations generally and nonlinear differential equations in particular often do not have exact or closed form solutions, Batiha *et al* (2008). Nonlinear differential equations abound in many branches of applied mathematics such as psychology, chaos, potential theory, growth rate in biology and elasticity, Jimoh and Taiwo (2015). The Adomian Decomposition Method (ADM) proposed by G. Adomian in the 1980's has been a subject of discussion and investigation by researchers, Almazmumy *et al* (2012) and Luo (2005). The (ADM) has been used by several mathematicians to solve nonlinear problems due to its simplicity and its ability to circumvent the problems of linearization and perturbation, Zhang *et al* (2006). The Adomian Decomposition Method involves separating the problem under consideration into

Key words and phrases. Adomian polynomials, Taylor polynomials, Nonlinear Ordinary Differential Equations, total derivatives, iterative differentiation, equations with variable coefficients.

linear and nonlinear components, Hendi *et al* (2012). The nonlinear portion of the problem is decomposed into a series of polynomials. These are then called Adomian polynomials.

Adomian decomposition method is a semi-analytical method for solving ordinary and partial nonlinear differential equations, Wazwaz (2000). The method, developed from 1970s to 1990s by George Adomian is also applied to solve both linear and nonlinear Boundary Value Problems (BVPs) and integral equations. The numerical result is obtained with minimum amount of computation or mathematics, Hosseini (2006). Adomian technique is based on a decomposition of a solution of nonlinear functional equation in a series of functions. Each term of the series is obtained from polynomial generated by a power series expansion of an analytic function, Hosseini and Nasabzadeh (2007). Some of the advantages of the Adomian decomposition are that; it can be applied directly for all types of functional equations both linear and nonlinear and it has ability of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution, Hosseini and Nasabzadeh (2006). Adomian Decomposition Method (ADM) provides an analytical approximate solution for nonlinear functional equations in terms of a rapidly converging series, without linearization, perturbation or discretization, Hasan and Zhu (2009).

Taylor series method is one of the singlestep methods which have different increment functions. From the application point of view, the Taylor series method has a major disadvantage. The method requires evaluation of partial derivatives of higher orders manually, Hoffman (2001). However, with the advent of computers, this challenge is being surmounted. The Taylor series can be written as the finite Taylor series, also known as the Taylor formula or Taylor polynomial. The Taylor series is a polynomial of infinite order, Vygodsky (1975). Thus,

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f''(x_0)(x - x_0)^2 + ...$$

It is, of course, impractical to evaluate an infinite number of terms, Hoffman (2001). The Taylor polynomial of degree n is defined by

$$f(x) = P_n(x) + R_{n+1}(x)$$

where the Taylor polynomial $P_n(x)$, and the remainder term $R_{n+1}(x)$ are given by

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f''(x_0)(x - x_0)^2 + ... + \frac{1}{n!}f^{(n)}(x_0)(x - x_0)^n$$

$$R_{n+1}(x) = \frac{1}{(n+1)!}f^{(n+1)}(\xi)(x - x_0)^{(n+1)},\ x_0 \leq \xi \leq x$$

The Taylor polynomial is a truncated Taylor series, with an explicit remainder, or error term, Mileties and Molnarka (2004). The Taylor polynomial cannot be used as an approximating function for discrete data because the derivatives required in the coefficients cannot be determined. It does have great significance, however, for polynomial approximation, because it has an explicit error term, Mileties and Molnarka (2005).

2.0 Adomian Polynomials

Consider a functional equation

$$u = f + L(u) + N(u) \tag{1}$$

where L and N are respectively linear and nonlinear operators and f is a known function. By Adomian decomposition method, the solution $u(x,t)$ of (1) is decomposed in the form of an infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{2}$$

Furthermore, the nonlinear function $N(u)$ assumes the following representation:

$$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, ..., u_n) \tag{3}$$

where, $A_n's$ are nth order Adomian polynomials. In the linear case, $N(u) = u$ then A_n simply reduces to u_n.

Cherruault and Adomian (1993) gave a method for determining these polynomials by parametrizing $u(x,t)$ as

$$u\alpha(x,t) = \sum_{n=0}^{\infty} u_n(x,t)\alpha^n \tag{4}$$

and assuming $N(u\alpha)$ to be analytic in α, which decomposes as

$$N(u\alpha) = \sum_{n=0}^{\infty} A_n(u_0, u_1, ..., u_n)\alpha^n \tag{5}$$

Hence, the Adomian polynomials A_n are given by

$$A_n(u_0, u_1, ..., u_n) = \frac{1}{n!}\frac{\partial^n N(u\alpha)}{\partial \alpha^n}|_{\alpha=0} \forall n \in N_0 \tag{6}$$

where, $N_m = [n \in Nu0 : n \geq m]$ and N denotes the set of positive integers.

Rach (1984), suggested the following formulae for determining Adomian polynomials, Rach (2008):

$$A_0(u_0) = N(u_0) \tag{7}$$

$$A_n(u_0, u_1, ..., u_n) = \sum_{k=1}^{n} c(k,n) N^k(u_n) \forall n \in N \tag{8}$$

The first-few Adomian polynomials are listed by Zhu *et al* (2005) as follows:

$$\left.\begin{array}{c} A_0 = f(t, y_0) \\ A_1 = y_1 f'(t, y_0) \\ A_2 = y_2 f'(t, y_0) + \frac{1}{2} {y_1}^2 f''(t, y_0) \\ A_3 = y_3 f'(t, y_0) + y_1 y_2 f''(t, y_0) + \frac{1}{6} {y_1}^3 f'''(t, y_0) \end{array}\right\} \tag{9}$$

where primes denote the partial derivatives with respect to y.

3 Adomian Polynomial Solutions of Ordinary Differential Equations

The generalized first order nonlinear equation considered is given by

$$y' = f(x, y) \tag{10}$$

together with the initial condition

$$y(x_a) = y_a \tag{11}$$

The main goal of this article is to extend the Adomian decomposition method, with modification in order to obtain a polynomial solution of (10) and (11). Adomian Decomposition Method (ADM) solves nonlinear operator equations for any analytic nonlinearity, providing an easily computable, readily verifiable and rapidly convergent sequence of analytic approximate solutions. Since it was first presented in the 1980's, Adomian decomposition method has led to several modifications on the method made by various researchers in an attempt to improve the accuracy or expand the application of the original method, Duan (2011). The choice of decomposition is non-unique and provides a valuable advantage to the analyst, permitting the freedom to design modified recursion schemes for ease of computation in realistic systems, Duan (2011). In order to obtain the Adomian polynomial solution of (10) and (11), the nonlinear variable coefficient equation (10) is written in its operator form as

$$Ly + Ry + Ny = F \tag{12}$$

where F is a known function and y is the unknown function to be determined, L is the linear operator to be inverted, R is the linear remainder operator and N is the nonlinear operator, which is assumed to be analytic. The choice for L and its pair L^{-1} (inverse of L) are determined by the equation being considered, hence the choice is non-unique. Here, we L is chosen to be

$$L = \frac{d}{dx}(.)$$

and thus its inverse L^{-1} follows as the one-fold definite integration operator from x_0 to x. Thus,

$$L^{-1}Ly = y - \psi$$

assumes the initial value as

$$\psi = y_a$$

Remark: For nth-order differential equation, the choice of L is given by

$$L = \frac{d^n}{dx^n}(.)$$

and its inverse, L^{-1} is the n-fold definite integration operator from x_0 to x. Thus, ψ absorbs the initial value as

$$\psi = \sum_{k=0}^{n-1} \alpha_k \frac{(x-x_0)^k}{k!}$$

Applying the inverse linear operator L^{-1} to both sides of equation (12), leads to

$$y = \beta(x) - L^{-1}[Ry + Ny] \tag{13}$$

where $\beta(x) = \psi + L^{-1}F$

The unknown function, y is expressed in a series of the form

$$y = \sum_{k=0}^{\infty} y_k \tag{14}$$

and the nonlinear term Ny is decomposed into a series

$$Ny = \sum_{k=0}^{\infty} A_k \tag{15}$$

where the $A_k' s$ which depend on $y_0, y_1, ..., y_k$, are called the Adomian polynomials, and are obtained for the nonlinearity $Ny = f(y)$ by

$$A_k = \frac{1}{k!}\frac{\partial^k}{\partial \lambda^k}[f(\sum y_n \lambda^n)]_{\lambda=0}, \; k = 0, 1, 2, ... \tag{16}$$

where λ is a formal parameter.

However, in this work, the formulae (9) are modified to obtain expressions for the first-few Adomian polynomials A_0 through A_4, inclusively, as

$$\left.\begin{array}{c} A_0 = f(t_0, y_0) \\ A_1 = y_1 f'(t_0, y_0) \\ A_2 = y_2 f'(t_0, y_0) + \frac{1}{2}{y_1}^2 f''(t_0, y_0) \\ A_3 = y_3 f'(t_0, y_0) + y_1 y_2 f''(t_0, y_0) + \frac{1}{6}{y_1}^3 f'''(t_0, y_0) \\ A_4 = y_4 f'(t_0, y_0) + \frac{1}{2}{y_2}^2 f''(t_0, y_0) + \frac{1}{6} y_1 y_3 f'''(t_0, y_0) + \frac{1}{24}{y_1}^4 f^{(4)}(t_0, y_0) \end{array}\right\} \tag{17}$$

where primes denote total derivatives of $f(t, y)$ at (t_0, y_0).

Using the $A_k' s$ in (13) - (15), the recursive formula for y_{n+1} is obtained as

$$y_{n+1} = \int_0^x A_n[t, y_0(t), y_1(t), ..., y_n(t)]dt, \; n = 0, 1, 2, ... \tag{18}$$

4 Taylor Polynomial Solutions of Ordinary Differential Equations

Barrio (2005), used a generalized Taylor series method for solving nonlinear ordinary differential equations of constant coefficients to find a general expansion for a given function $f(t)$. For Taylor polynomial solutions of first order ordinary differential equations, the initial value problem given by an implicit differential equation (10) and initial value given by equation (11) is considered.

Thus, if the solution $g(x)$ of (10) and (11) is n times differentiable, the n-th degree Taylor polynomial of $g(x)$ is then sought for. A typical situation is an initial value problem (10) and (11) for a complex function g with analytic right hand side f. In this case, the solution g is analytic in a neighbourhood of the initial point x_0. The Taylor coefficients are given by

$$T_n(g, x, x_0) = g_0 + \sum_{k=1}^{n} \frac{g^{(k)}(x_0)}{k!}(x - x_0)^k \tag{19}$$

with the derivative list as

$$g'(x) = f(x, y)|_{y=g(x)} \equiv f_{(x,y)}$$

$$g''(x) = \frac{\partial f}{\partial x}(x, y) + \frac{\partial f}{\partial y}(x, y)g'(x)$$

$$= \frac{\partial f}{\partial x}(x, y) + \frac{\partial f}{\partial y}(x, y)f(x, y)|_{y=g(x)} \equiv f_2(x, y)$$

. .

. .

. .

$$g^{(k)} = \frac{\partial f_{k-1}}{\partial x}(x, y) + \frac{\partial f_{k-1}}{\partial y}(x, y)f(x, y)|_{y=g(x)} \equiv f_k(x, y) \tag{20}$$

In the case of an implicit second order differential equation

$$y'' = f(x, y, y') \tag{21}$$

with initial conditions

$$y(x_a) = y_a \tag{22}$$

and

$$y'(x_a) = y_b \tag{23}$$

the n-th degree Taylor polynomial solution has the form

$$T_n(g, x, x_0) = g_0 + g_1(x - x_0) + \sum_{k=2}^{n} \frac{g^{(k)}(x_0)}{k!}(x - x_0)^k \tag{24}$$

and the same method as before after calculation of $g^{(k)}(x_0)$ is applied for $k \geq 2$.
If the right hand side of (21) is a function of the three variables x, y and u, then the derivatives of $g(x)$ are obtained iteratively as

$$g''(x) = f(x, g(x), g'(x))$$

and by the chain rule

$$g'''(x) = \frac{\partial f}{\partial x}(x,y,u) + \frac{\partial f}{\partial y}(x,y,u)g'(x) + \frac{\partial f}{\partial u}(x,y,u)g''(x)$$

$$= \frac{\partial f}{\partial x}(x,y,u) + \frac{\partial f}{\partial y}(x,y,u)u + \frac{\partial f}{\partial u}(x,y,u)f(x,y,u)|_{u=g'(x)}$$

for $y = g(x)$ and iteratively

$$g^{(k)}(x) = \frac{\partial f_{k-1}}{\partial x}(x,y,u) + \frac{\partial f_{k-1}}{\partial y}(x,y,u)g'(x) + \frac{\partial f_{k-1}}{\partial u}f(x,y,u)|_{u=g'(x)} \equiv F_k(x,y,u)$$

and $g^{(k)}(x)$ is evaluated at the point $x = x_0$ to get $g^{(k)}(x_0)$ by taking the limit $u \longrightarrow y_1$, $y \longrightarrow y_0$ and $x \longrightarrow x_0$ which yields the result.
After obtaining the polynomials $g^{(k)}(x_0)$ for $k = 1,\ 2,\ 3, ..., n$, they are then substituted into equation (19) for the Taylor polynomial solution of equations (10) and (11).

5 Evaluation of the Error

In this paper, error is defined as

$$\text{Error} = \max_{a \le x \le b} |ExactValue - ApproximateValue|$$

In case the exact solution is not available, the results are compared with those in literature.

6 Illustrative Examples

The Adomian and Taylor polynomial solutions obtained for some nonlinear first order ordinary differential equations of variable coefficients are presented in this section. The results obtained are tabulated for comparison.

Example 1: Consider the first order nonlinear differential equation

$$y' = xy^2 + 1 \tag{25}$$

with the initial condition

$$y(0) = 1$$

(Griffths and Higham, 2010)
Example 2: Consider the first order nonlinear differential equation

$$y' = xy^2 \tag{26}$$

with the initial condition

$$y(0.1) = 1$$

The exact solution is

$$y(x) = \frac{2}{2.01 - x^2}$$

Solution by the Adomian Decomposition method:

$$y(x) = 0.98973 + 0.169367x - 1.64583x^2 + 13.4375x^3 - 41.6667x^4 + 52.0833x^5$$

(Griffths and Higham, 2010)

Example 3: Consider the first order nonlinear ordinary differential equation

$$y' = x^2 + y^2 \tag{27}$$

with the initial condition

$$y(0) = 1$$

Solution by the Adomian Decomposition method:

$$y(x) = 1 + x + x^2 + \frac{4}{3}x^3 + \frac{5}{3}x^4 + \frac{16}{15}x^5$$

(Jain *et, al*, 2012)

Example 4: Consider the first order differential equation

$$y' = \frac{y}{x} - \frac{5}{2}x^2y^3 \tag{28}$$

with the initial condition

$$y(1) = \frac{1}{\sqrt{2}}$$

Solution by the Adomian Decomposition method:

$$y(x) = -1.34642 + 7.33484x - 11.4929x^2 + 10.6446x^3 - 5.4401x^4 + 1.30000x^5$$

(Griffths and Higham, 2010) **Tables of Results**

TABLE 1

Numerical Results for Example 1

x	Literature	Adomian Poly.	Taylor's Poly.
0.0	1.00000	1.00000	1.00000
0.1	1.10572	1.10593	1.10569
0.2	1.22600	1.22832	1.22579
0.3	1.36856	1.37617	1.36543
0.4	1.54210	1.56171	1.53077
0.5	1.76596	1.80078	1.72917
0.6	2.06740	2.11336	1.96936
0.7	2.50470	2.52396	2.26170
0.8	3.20741	3.06208	2.61834
0.9	4.53927	3.76270	3.05344
1.0	8.00782	4.66667	3.58333

Literature: Taylor Series Method

TABLE 2

Numerical Results for Example 2

x	Exact Solution	Adomian Poly.	Error	Taylor's Poly.	Error
0.10	1.00000	1.00000	0.0000E-5	1.00000	0.0000E-5
0.11	1.00105	1.00106	1.0000E-5	1.00105	0.0000E-5
0.12	1.00220	1.00223	3.0000E-5	1.00221	1.0000E-5
0.13	1.00346	1.00348	2.0000E-5	1.00346	0.0000E-5
0.14	1.00482	1.00485	3.0000E-5	1.00482	0.0000E-5
0.15	1.00629	1.00631	2.0000E-5	1.00629	0.0000E-5
0.16	1.00786	1.00789	3.0000E-5	1.00786	0.0000E-5
0.17	1.00954	1.00956	2.0000E-5	1.00954	0.0000E-5
0.18	1.01133	1.01136	3.0000E-5	1.01133	0.0000E-5
0.19	1.01322	1.01324	2.0000E-5	1.01322	0.0000E-5
0.20	1.01523	1.01527	4.0000E-5	1.01523	0.0000E-5

TABLE 3

Numerical Results for Example 3

x	Lit.	Adomian Poly.	Taylor's Poly.
0.0	1.000000	1.00000	1.00000
0.1	-	1.11151	1.11146
0.2	1.25253	1.25367	1.25292
0.3	-	1.44209	1.43837
0.4	1.69318	1.69892	1.68749
0.5	-	2.05417	2.02708
0.6	2.62293	2.54694	2.49251
0.7	-	3.22677	3.12913
0.8	5.49363	4.15486	3.99375
0.9	-	5.40536	5.15604
1.0	34.3075	7.06667	6.70000

Lit. Taylor series method of order four

TABLE 4

Numerical Results for Example 4

x	Lit.	Adomian Poly.	Taylor's Poly.
1.0	1.00000	1.00000	1.00000
1.1	-	1.11227	1.11145
1.2	1.25478	1.25371	1.25289
1.3	-	1.44140	1.43833
1.4	1.69912	1.69808	1.68744
1.5	-	2.05371	2.02702
1.6	2.57034	2.54703	2.49243
1.7	-	3.22713	3.12903
1.8	4.27532	4.15499	3.99363
1.9	-	5.40508	5.15590
2.0	7.20134	7.06686	6.69983

Lit. Modified Euler Method

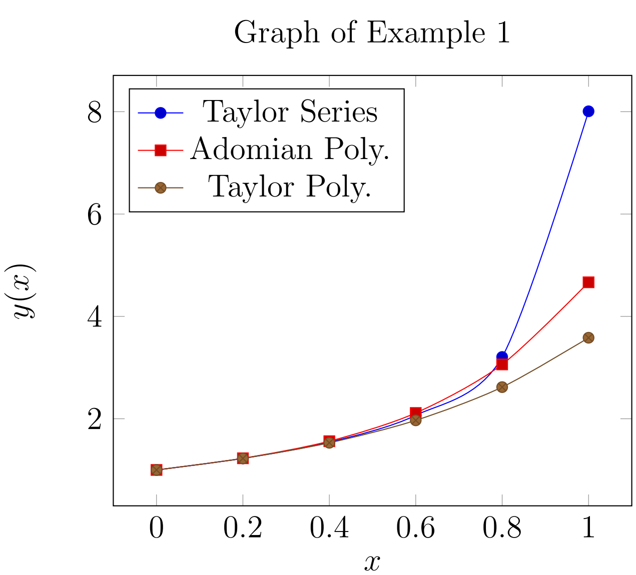

Figure 1: The behaviour of the Taylor Series Method compared with the Adomian and Taylor polynomial solutions.

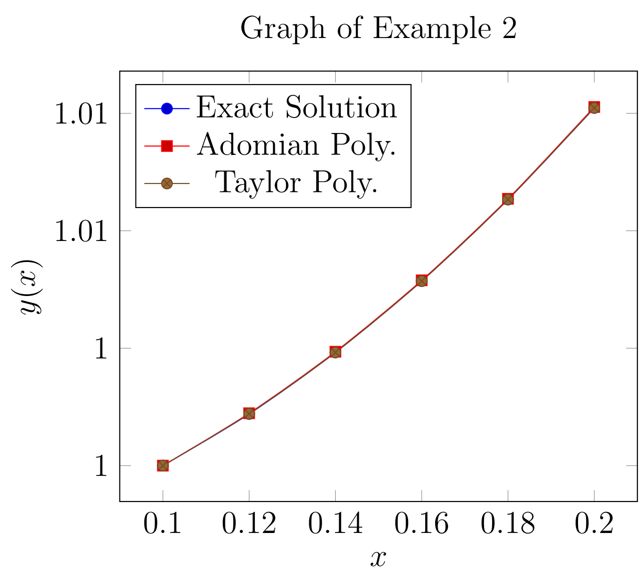

Figure 2: The behaviour of the exact solution compared with the Adomian and Taylor polynomial solutions.

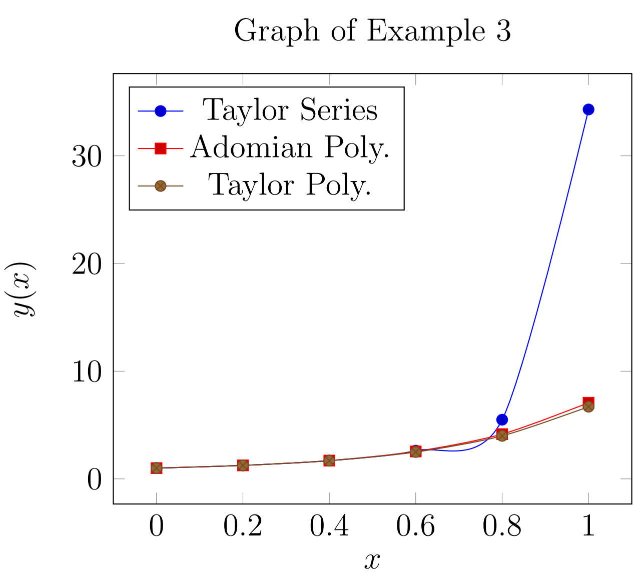

Figure 3: The behaviour of the Taylor Series Method compared with the Adomian and Taylor polynomial solutions.

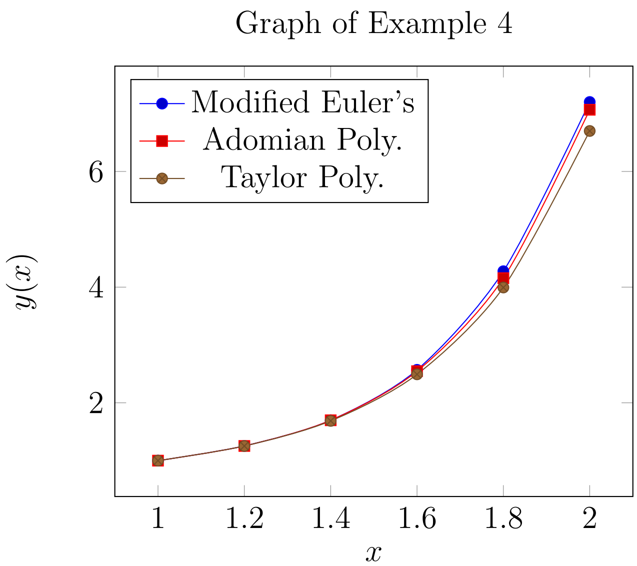

Figure 4: The behaviour of the Modified Euler's Method compared with the Adomian and Taylor polynomial solutions.

7 Discussion of Results

The Adomian and Taylor polynomial solutions of some nonlinear ordinary differential equations of variable coefficients were obtained. The results were compared with the exact solutions (where available) and some existing results in literature. The absolute errors show that the results by the two polynomial solutions are in excellent agreement with the exact solutions. Moreover, the Taylor polynomial solution gives a better approximation than the Adomian polynomial solution as can be seen in the tables 1 - 4 presented. It is also noted that the Taylor polynomial solution performs better than the Taylor series method as the results by the Taylor series method diverge as the value of x moves away from the initial point.

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