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FREE VIBRATION OF UNIFORM AND NON-UNIFORM EULER BEAMS USING THE DIFFERENTIAL TRANSFORMATION METHOD

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Abstract. In this paper, the differential transformation method (DTM) is applied for free vibration analysis of beams with uniform and non-uniform cross sections. Natural frequencies and corresponding normalized mode shapes are calculated for different cases of cross section and boundary conditions. MATLAB code is designed to solve the differential equation of the beam using the differential transformation method. Comparison of the present results with the previous solutions proves the accuracy and versatility of the presented paper.

1. Introduction

In order to calculate fundamental natural frequencies and the corresponding mode shapes, different variational techniques such as Rayleigh_Ritz and Galerkin methods had been applied in the past. Besides these techniques, another numerical methods were also successfully applied to beam vibration analysis such as finite element method.

The DTM is a technique that uses Taylor series for the solution of differential equations in the form of a polynomial. The Taylor series method is computationally tedious for high order equations. The differential transform method leads to an iterative procedure for obtaining an analytic series solutions of functional equations. Since the beginning of 1986, Zhou and Pukhov [1] have developed a so-called differential transformation method (DTM) for electrical circuits problems. In recent years researchers had applied the method to various linear and nonlinear problems. Ayaz [2] applied The differential transform method to the differential algebraic equations , Moustafa El-Shahed [3] applied DTM to the non-linear oscillatory systems , Reza Attarnejad and Ahmad Shahba [4] applied DTM to the free vibration analysis of rotating non-prismatic beams, Rajasekaran S. [5] used the differential transformation (DT) to determine the natural frequency of beams and columns, Reza Attarnejad et al. [6] presented an application of differential transform in free vibration analysis of Timoshenko beams resting on two-parameter elastic foundation. Differential transform method is applied to the sine-Gordon equation by Jafar Biazar and Fatemeh Mohammadi [7], which arise in differential geometry,

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propagation of magnetic flux and stability of fluid motions. J. Biazar and M. Eslami [8] used DTM for Nonlinear Parabolic-hyperbolic Partial differential equations. DTM was applied to linear and nonlinear system of ordinary differential equations by Farshid Mirzaee [9]. Keivan Torabi et al. [10] applied DTM for longitudinal vibration analysis of beams with non-uniform cross section. The non-linear vibration analysis of beams is applied by Qiang Guo and Hongzhi Zhong [11] using a spline-based differential quadrate method.

In this paper, the vibration problems of uniform and non-uniform Euler-Bernoulli beams have been solved analytically using DTM for various end conditions.

2. Basic Idea of Differential Transformation Method

Following ref [2] we can obtain the idea of the DTM, The differential transformation of function $u(x)$ is defined as follows;

$$U(k)=\frac{1}{k!}\left(\frac{d^k u(x)}{dx^k}\right)_{x=0} \qquad (1)$$

In Eq. (1), $u(x)$ is the original function and $U(k)$ is the transformed function. Differential inverse transform of $U(k)$ is defined as follows;

$$u(x)=\sum_{k=0}^{\infty} x^k U(k) \qquad (2)$$

In fact, from (1) and (2), we obtain

$$u(x)=\sum_{x=0}^{\infty}\frac{x^k}{k!}\left(\frac{d^k u(x)}{dx^k}\right)_{x=0} \qquad (3)$$

Eq. (3) implies that the concept of differential transformation is derived from the Taylor series expansion. From the definitions (1) and (2), it is easy to obtain the following mathematical operations; [2, 3, 4, 6, 9]

1. If $f(x)=g(x)\pm h(x)$, then $F(k)=G(k)\pm H(k)$.
2. If $f(x)=cg(x)$, then $F(k)=cG(k)$, c is a constant.

3. If $f(x)=\dfrac{d^n g(x)}{dx^n}$, then $F(k)=\dfrac{(k+n)!}{k!}G(k+n)$.

4. If $f(x)=g(x)h(x)$, then $F(k)=\sum_{l=0}^{k}G(l)H(k-l)$.

5. If $f(x)=x^n$, then $F(k)=\delta(k-n)$, δ is the Kronecker delta.

7. If $f(x)=a(1-bx)^m\dfrac{d^n g(x)}{dx^n}$ then $F(k)=a\sum_{r=0}^{m}\dfrac{{}^mC_r(k+n-r)!}{(k-r)!}(-b)^r G(k+n-r)$, where a and b are constants

3. Formulation of the Problem

Free Vibration of a Non-uniform Beam

The governing differential equation for a non-uniform Euler beam shown in fig. 1 is given by:

$$\rho A(\overline{x})\frac{\partial^2 v(\overline{x},t)}{\partial t^2}+\frac{\partial^2}{\partial \overline{x}^2}\left(EI(\overline{x})\frac{\partial^2 v(\overline{x},t)}{\partial \overline{x}^2}\right)=0 \tag{4}$$

Where ρ is the density of the beam material, $A(\overline{x})$ is the cross sectional area of the beam, $v(\overline{x},t)$ is displacement of the beam, $I(\overline{x})$ is the inertia at distance x from the left end of the beam and E is young's modulus of the beam.

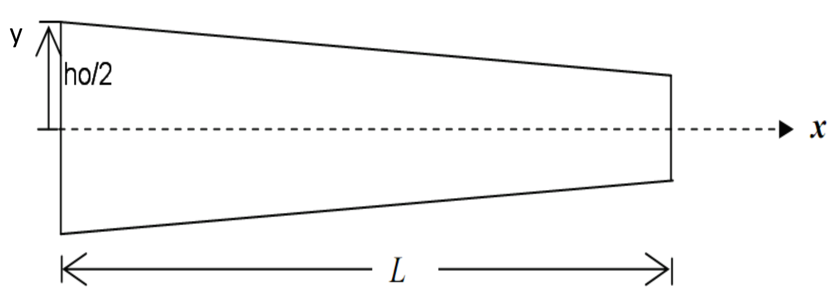

Figure 1 Beam with variable cross-section

Boundary conditions:

Case a. Simple –Simple Beam

$$v(0,t)=0\,; \quad \frac{\partial^2 v(0,t)}{\partial \overline{x}^2}=0\,; \quad v(L,t)=0\,; \quad \frac{\partial^2 v(L,t)}{\partial \overline{x}^2}=0 \tag{5}$$

Case b. Clamped- Clamped Beam

$$v(0,t)=0\,; \quad \frac{\partial v(0,t)}{\partial \overline{x}}=0\,; \quad v(L,t)=0\,; \quad \frac{\partial v(L,t)}{\partial \overline{x}}=0 \tag{6}$$

Case c. Clamped- Roller Beam

$$v(0,t)=0\,; \quad \frac{\partial v(0,t)}{\partial \overline{x}}=0\,; \quad v(L,t)=0\,; \quad \frac{\partial^2 v(L,t)}{\partial \overline{x}^2}=0 \tag{7}$$

Where L is the beam length and t is the time

4. Solution of the problem

Assume that, the displacement of the beam is given by:

$$v(\overline{x},t)=v(\overline{x})\exp(i\omega t) \tag{8}$$

Where ω is the natural frequency of the beam.

Equation (4) can be conveniently written as :

$$-\rho A(\overline{x})\omega^2 v(\overline{x}) + E\frac{d^4 v(\overline{x})}{d\overline{x}^4} + 2E\frac{dI(\overline{x})}{d\overline{x}}\frac{d^3 v(\overline{x})}{d\overline{x}^3} + E\frac{d^2 I(\overline{x})}{d\overline{x}^2}\frac{d^2 v(\overline{x})}{d\overline{x}^2}=0 \tag{9}$$

With boundary conditions as,

Case a. Simple –Simple Beam

$$v(0)=0\,;\quad \frac{d^2v(0)}{d\overline{x}^2}=0\,;\quad v(L)=0\,;\quad \frac{d^2v(L)}{d\overline{x}^2}=0 \tag{10}$$

Case b. Clamped- Clamped Beam

$$v(0)=0\,;\quad \frac{dv(0)}{d\overline{x}}=0\,;\quad v(L)=0\,;\quad \frac{dv(L)}{d\overline{x}}=0 \tag{11}$$

Case c. Clamped- Roller Beam

$$v(0)=0\,;\quad \frac{dv(0)}{d\overline{x}}=0\,;\quad v(L)=0\,;\quad \frac{d^2v(L)}{d\overline{x}^2}=0 \tag{12}$$

5. Dimensionless form

Equations (9) can be conveniently written in terms of dimensionless variables as :

$$S(x)\frac{d^4y(x)}{dx^4}+2\frac{dS(x)}{dx}\frac{d^3y(x)}{dx^3}+\frac{d^2S(x)}{dx^2}\frac{d^2y(x)}{dx^2}=\Omega^2(S(x))^{1/3}y(x) \tag{13}$$

where

$$\frac{EI(x)}{EI_0}=S(x)=(1-\beta x)^3\,,\qquad y=\frac{v}{L}\quad,\qquad x=\frac{\overline{x}}{L}\,,$$

$$\frac{A(x)}{A_0}=S^{1/3}(x)\,,\qquad S^{1/3}(x)=\frac{h(x)}{h_0}=(1-\beta x)\,,$$

$\Omega^2 = \omega^2 \dfrac{\rho A_0 L^4}{EI_0}$ (the non-dimensional frequency of the beam), h_0, A_0, I_0 are the beam height , the cross sectional area and the inertia at the point at the left edge of the beam, and β is a constant equals zero for uniform beam.

the non-dimensional boundary conditions are,

Case a. Simple –Simple Beam

$$y(0)=0\,; \quad \frac{d^2y(0)}{dx^2}=0\,; \quad y(1)=0\,; \quad \frac{d^2y(1)}{dx^2}=0 \tag{14}$$

Case b. Clamped- Clamped Beam

$$y(0)=0\,; \quad \frac{dy(0)}{dx}=0\,; \quad y(1)=0\,; \quad \frac{dy(1)}{dx}=0 \tag{15}$$

Case c. Clamped- Roller Beam

$$y(0)=0\,; \quad \frac{dy(0)}{dx}=0\,; \quad y(1)=0\,; \quad \frac{d^2y(1)}{dx^2}=0 \tag{16}$$

6. Applying the Differential Transform Method

In solving the problem, governing differential equations are converted to algebraic equations using DTM method which must be solved together with applied boundary conditions. Applying the differential transformation method (theorem (1-7)) to the non-dimensional governing equation (13) yield,

$$Y_{k+4}=\frac{1}{(k+1)(k+2)(k+3)(k+4)}\begin{Bmatrix}\Omega^2 Y_k-\beta^2\left[(k-1)(k)(k+1)(k+2)+6k(k+1)(k+2)+6(k+1)(k+2)\right]Y_{k+2}\\ +2\beta\left[k(k+1)(k+2)(k+3)+3(k+1)(k+2)(k+3)\right]Y_{k+3}\end{Bmatrix}$$

(17)

Then applying the Differential Transform Method to the non-dimensional boundary conditions equations (14)-(16) yield,

Case a. Simple –Simple Beam

The DT of Eqs. (14) is written as

$$y(0)=\sum_{k=0}^{\infty}Y[k]x^k=Y[0]x^0+Y[1]x^1+Y[2]x^2+Y[3]x^3+Y[4]x^4+Y[5]x^5+.....=0$$

leads to $Y[0]=0$ (18.a)

$$y''(0)=\sum_{k=0}^{\infty}(k+1)(k+2)Y[k+2]x^k=(1)(2)Y[2]x^0+(2)(3)Y[3]x^1+(3)(4)Y[2]x^2$$
$$+(4)(5)Y[3]x^3+(5)(6)Y[4]x^4+(6)(7)Y[5]x^5+.....=0$$

leads to $Y[2]=0$ (18.b)

$$y(1)=\sum_{k=0}^{\infty}Y[k]x^k=Y[0]x^0+Y[1]x^1+Y[2]x^2+Y[3]x^3+Y[4]x^4+Y[5]x^5+.....=0$$

leads to $\sum_{k=0}^{\infty}Y[k]=0$ (18.c)

Taking $Y[1]=c$ (18.d)

$$y''(1)=\sum_{k=0}^{\infty}(k+1)(k+2)Y[k+2]x^k=(1)(2)Y[2]x^0+(2)(3)Y[3]x^1+(3)(4)Y[2]x^2$$
$$+(4)(5)Y[3]x^3+(5)(6)Y[4]x^4+(6)(7)Y[5]x^5+.....=0$$

leads to $\sum_{k=0}^{\infty}k(k-1)Y[k]=0$ (18.e)

Taking $Y[3]=d$ (18.f)

Equations (18.c) and (18.e) may be written as

$$\begin{bmatrix} aa_1 & bb_1 \\ cc_1 & dd_1 \end{bmatrix} \begin{bmatrix} c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{19}$$

since c_1 and d_1 are not zero, for a non-trivial solution to exist the determinant of the matrix must be zero, i.e.

$$aa_1 \times dd_1 - cc_1 \times bb_1 = 0 \tag{20}$$

where aa_1, bb_1 are the coefficients of c_1 and d_1 in the equation (18.c) and cc_1, dd_1 are the coefficients of c_1 and d_1 in the equation (18.e). The root of Eq. (20) is the solution for case *a* of the problem.

Case b. Clamped- Clamped Beam

The DT of Eqs. (15) is written as

y(0)= $\sum_{k=0}^{\infty} Y[k]x^k = Y[0]x^0 + Y[1]x^1 + Y[2]x^2 + Y[3]x^3 + Y[4]x^4 + Y[5]x^5 + ... = 0$

leads to $Y[0] = 0$ (21.a)

$$y^{'}(0) = \sum_{k=0}^{\infty} (k+1)Y[k+1]x^k = (1)Y[1]x^0 + (2)Y[2]x^1 + (3)Y[3]x^2 + (4)Y[4]x^3$$

$$+(5)Y[5]x^4 + (6)Y[6]x^5 + ... = 0$$

leads to $Y[1] = 0$ (21.b)

y(1)= $\sum_{k=0}^{\infty} Y[k]x^k = Y[0]x^0 + Y[1]x^1 + Y[2]x^2 + Y[3]x^3 + Y[4]x^4 + Y[5]x^5 + ... = 0$

leads to $\sum_{k=0}^{\infty} Y[k] = 0$ (21.c)

Taking $Y[2] = c$ (21.d)

$$y'(1)=\sum_{k=0}^{\infty}(k+1)Y[k+1]x^{k}=(1)Y[1]x^{0}+(2)Y[2]x^{1}+(3)Y[3]x^{2}+(4)Y[4]x^{3}$$
$$+(5)Y[5]x^{4}+(6)Y[6]x^{5}+\dots=0$$

leads to $\sum_{k=0}^{\infty}kY[k]=0$ (21.e)

Taking $Y[3]=d$ (21.f)

Equations (21.c) and (21.e) may be written as

$$\begin{bmatrix} aa_2 & bb_2 \\ cc_2 & dd_2 \end{bmatrix}\begin{bmatrix} c_2 \\ d_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (22)$$

since c_2 and d_2 are not zero, for a non-trivial solution to exist the determinant of the matrix must be zero, i.e.

$$aa_2\times dd_2-cc_2\times bb_2=0 \quad (23)$$

where aa_2, bb_2 are the coefficients of c_2 and d_2 in the equation (21.c) and cc_2, dd_2 are the coefficients of c_2 and d_2 in the equation (21.e). The root of Eq. (23) is the solution for case *b* of the problem.

Case c. Clamped- Roller Beam

The DT of Eqs. (16) is written as

y(0)= $\sum_{k=0}^{\infty}Y[k]x^{k}=Y[0]x^{0}+Y[1]x^{1}+Y[2]x^{2}+Y[3]x^{3}+Y[4]x^{4}+Y[5]x^{5}+\dots=0$

leads to $Y[0]=0$ (24.a)

$$y'(0)=\sum_{k=0}^{\infty}(k+1)Y[k+1]x^{k}=(1)Y[1]x^{0}+(2)Y[2]x^{1}+(3)Y[3]x^{2}+(4)Y[4]x^{3}$$
$$+(5)Y[5]x^{4}+(6)Y[6]x^{5}+\dots=0$$

leads to $Y[1]=0$ (24.b)

y(1)= $\sum_{k=0}^{\infty} Y[k]x^k = Y[0]x^0 + Y[1]x^1 + Y[2]x^2 + Y[3]x^3 + Y[4]x^4 + Y[5]x^5 + ... = 0$

leads to $\sum_{k=0}^{\infty} Y[k] = 0$ (24.c)

Taking $Y[2] = c$ (24.d)

$$y''(1) = \sum_{k=0}^{\infty} (k+1)(k+2)Y[k+2]x^k = (1)(2)Y[2]x^0 + (2)(3)Y[3]x^1 + (3)(4)Y[2]x^2 + (4)(5)Y[3]x^3 + (5)(6)Y[4]x^4 + (6)(7)Y[5]x^5 + ... = 0$$

leads to $\sum_{k=0}^{\infty} k(k-1)Y[k] = 0$ (24.e)

Taking $Y[3] = d$ (24.f)

Equations (24.c) and (24.e) may be written as

$$\begin{bmatrix} aa_3 & bb_3 \\ cc_3 & dd_3 \end{bmatrix} \begin{bmatrix} c_3 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (25)$$

since c_3 and d_3 are not zero, for a non-trivial solution to exist the determinant of the matrix must be zero, i.e.

$$aa_3 \times dd_3 - cc_3 \times bb_3 = 0 \quad (26)$$

where aa_3, bb_3 are the coefficients of c_3 and d_3 in the equation (24.c) and cc_3, dd_3 are the coefficients of c_3 and d_3 in the equation (24.e). The root of Eq. (26) is the solution for case c of the problem.

7. Results and Discussion

For a beam with simply supported ends Ω^2 = 97.4091 leads to the first non-dimensional natural frequency as $\Omega = 9.8696$, which agrees with closed form value [5].

For a beam with clamped supported ends, Ω^2 =500.564 leads to the first non-dimensional natural frequency as $\Omega = 22.3733$, which agrees with closed form value [5]

For a beam with one end clamped and the other end free, $\Omega^2 = 237.721$ leads to the first non-dimensional natural frequency as $\Omega = 15.4182$, which agrees with closed form value [5].

Table 1 compares the accuracy of the first three non-dimensional frequencies of Simply supported uniform beams for different number of terms *N*. It is observed that the convergence speed increases with decreasing frequency order. i.e, the first frequency Ω_1 needs 25 terms to reach exact solution , while , third frequency Ω_3 needs 45 terms. Table 2 gives results of the first three non-dimensional frequencies of Euler beams with uniform cross section and different cases of boundary conditions. In table 2 exact values are also listed for direct comparison. It can be demonstrated that the differential transformation method is an efficient method in solving the vibrations of beams with good accuracy. Table 3 gives results of the first three non-dimensional frequencies of simply supported non-uniform Euler beams with different values of beta β. It can be demonstrated that the first three frequencies of the simply non-uniform beam decreases with increasing β due to decreasing the beam cross section. Tables 4 and 5 gives results of the first three non-dimensional frequencies of clamped-clamped and clamped-roller non-uniform Euler beams with different values of beta β which give different cross sections. It is also observed that the first three frequencies of the clamped-clamped and clamped-roller non-uniform beam decreases with decreasing of the cross section.

Table 1. The first three Non-dimensional frequencies of Simple-Simple uniform Euler Beams for different number of terms N

	Ω_1	Ω_2	Ω_3
N=10	9.8902098156	-	-
N=15	9.8696683979	37.2824413197	-
N=16	9.8696683979	37.2824413197	-
N=17	9.8696020437	40.064696792	58.0531514621
N=19	9.8696044699	39.4169139196	-
N=20	9.8696044699	39.4169139196	-
N=25	9.8696044011	39.4784501712	87.8912222720
N=30	9.8696044011	39.4784176959	88.8107229014
N=35	9.8696044011	39.4784176044	88.8264496362
N=40	9.8696044011	39.4784176044	88.8264396509
N=45	9.8696044011	39.4784176044	88.8264396098
N=46	9.8696044011	39.4784176044	88.8264396098

Table 2. Comparison of results for free vibration of uniform Euler beams.

The first three Non-dimensional frequencies of uniform Euler Beams:

Uniform Beam (β=0)	Ω_1			Ω_2			Ω_3		
	DTM	Exact]	Relative error %	DTM	Exact	Relative error %	DTM	Exact	Relative error %
Simple-Simple	9.8696	9.8696	0.00	39.4784	39.4784	0.00	88.8264	88.8264	0.00
Clamped-Clamped	22.3733	22.3733	0.00	61.6729	61.6728	0.00	120.9031	120.9030	0.00
Clamped-Roller	15.4182	15.4182	0.00	49.9649	49.9649	0.00	103.9982	103.9982	0.00

-The first three Non-dimensional frequencies of Non-uniform Euler Beams for different values of *β:*

Table 3. Simply supported Non-uniform Euler Beams

$\frac{EI(x)}{EI_0(x)}=(1-\beta x)^3$	Ω_1			Ω_2			Ω_3		
	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5
Simple-Simple	9.8696	8.5772	7.1215	39.4784	34.4062	28.9519	88.8107	77.3785	64.9802

Table 4. Clamped-Clamped Non- uniform Euler Beams

$\frac{EI(x)}{EI_0(x)}=(1-\beta x)^3$	Ω_1			Ω_2			Ω_3		
	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5
Clamped-Clamped	22.3733	19.4836	16.3356	61.6729	53.6971	44.9817	120.9031	105.2123	88.1593

Table 5. Clamped - Roller Non -uniform Euler Beams

$\frac{EI(x)}{EI_0(x)}=(1-\beta x)^3$	Ω_1			Ω_2			Ω_3		
	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5	β=0	β=0.25	β=0.5
Clamped-Roller	15.4182	13.9524	12.3001	49.9649	44.0199	37.5276	103.9982	91.2744	77.1247

Figures (2-4) show the first three mode shapes of uniform Euler beams with different boundary conditions. From figures (2-4) It can be seen that the first three mode shapes drawn using DTM is very agreement with the mode shaped drawn using exact methods. Figures (5-7) show, respectively, the first, the second and the third mode shapes of simply supported non-uniform Euler beams with different cross sections. Also figures (8-13) show, respectively, the first, the second and the third mode shapes of clamped-clamped and clamped-roller non-uniform Euler beams with different cross sections. From figures (5-13)). It can be seen that the value of beta (*β=0, 0.25 and 0.5)* has a significant effect on the mode shapes and the deflection values. Deflection of non-uniform beams increases as cross section decreases (β values increases). For Euler beam with uniform cross section (β=0).

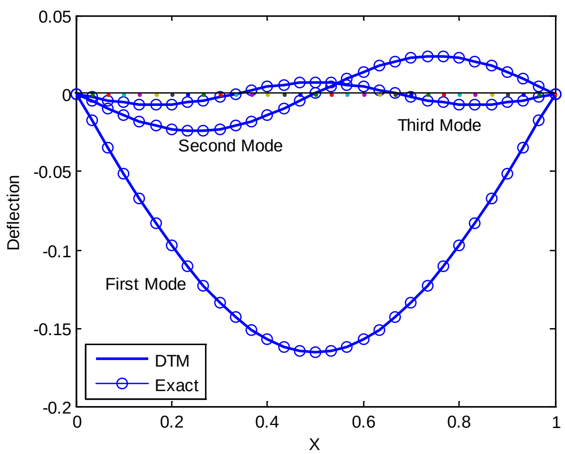

Fig. 2 The first three mode shapes of Simple –Simple uniform-beam

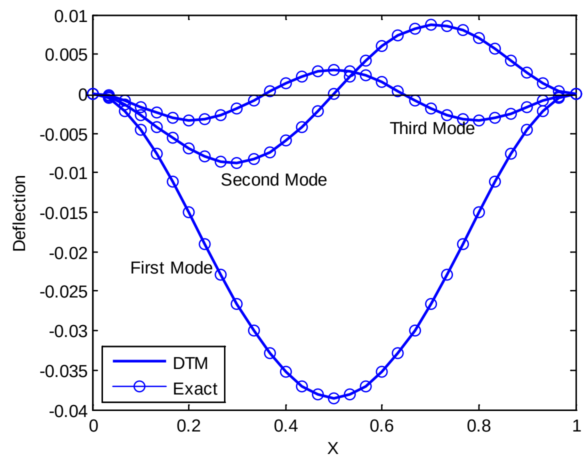

Fig. 3 The first three mode shapes of Clamped-Clamped uniform-beam

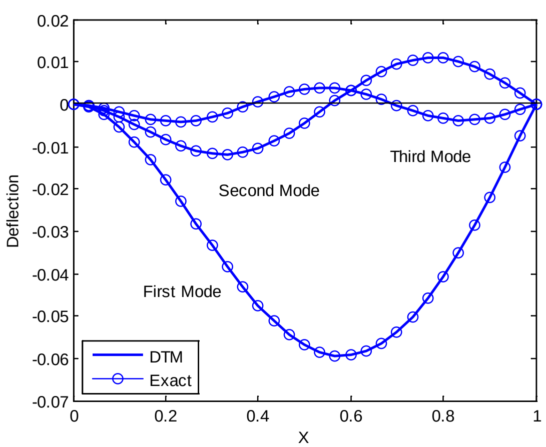

Fig. 4 The first three mode shapes of Clamped-Roller uniform beam

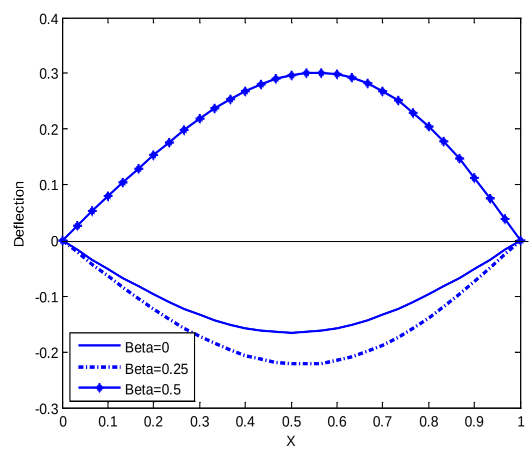

Fig. 5 The first mode shapes for three different values of Beta of Simple –Simple non-uniform beam

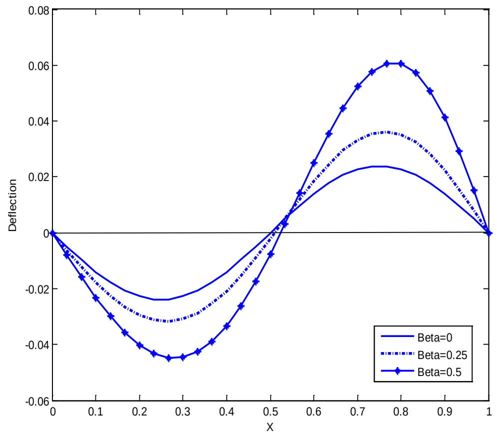

Fig. 6 The Second mode shapes for three different values of Beta of Simple –Simple non-uniform beam

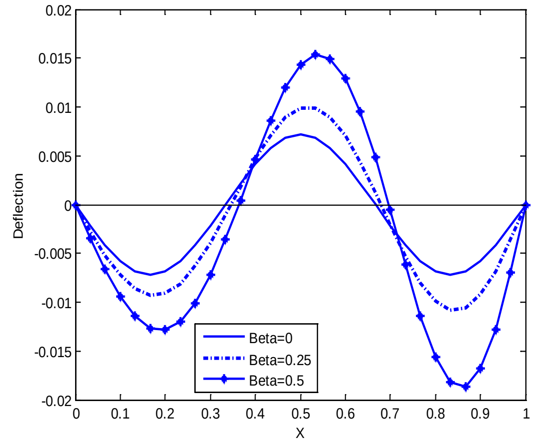

Fig. 7 The Third mode shapes for three different values of Beta of Simple –Simple non-uniform beam

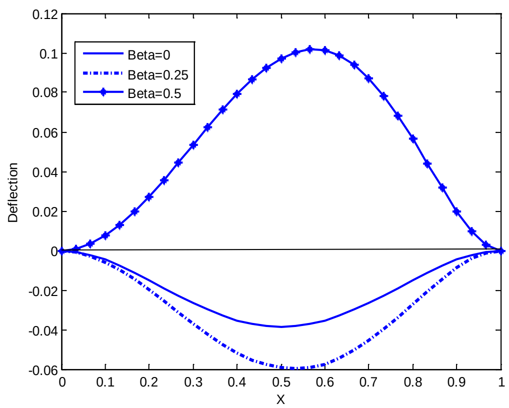

Fig. 8 The first mode shapes for three different values of Beta of Clamped-Clamped non-uniform beam

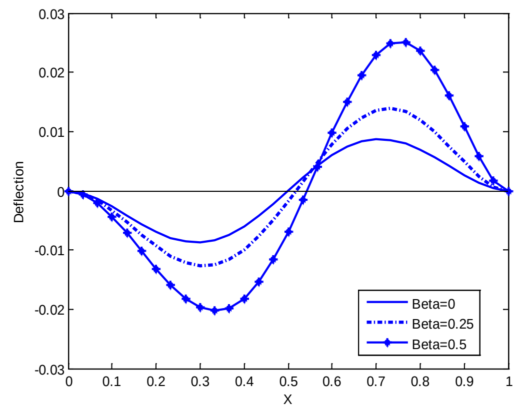

Fig. 9 The Second mode shapes for three different values of Beta of Clamped-Clamped non-uniform beam

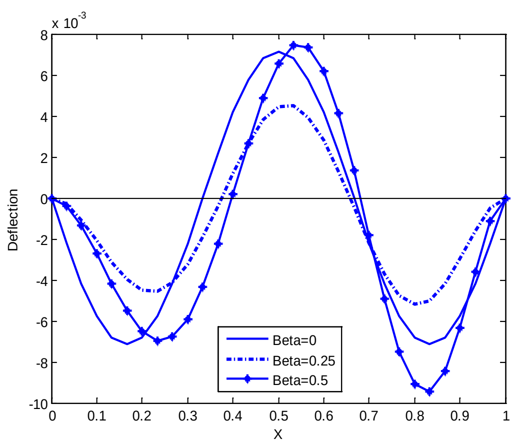

Fig. 10 The Third mode shapes for three different values of Beta of Clamped-Clamped non-uniform beam

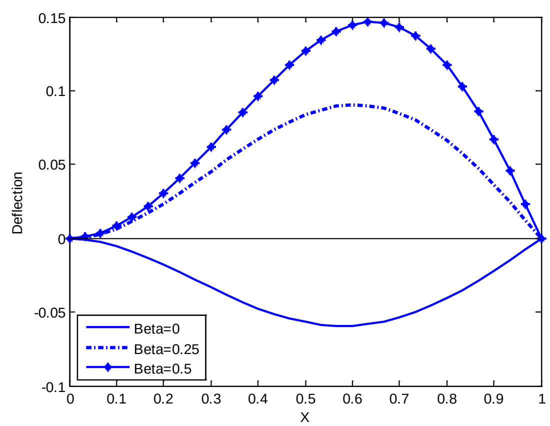

Fig. 11 The first mode shapes for three different values of Beta of Clamped- Roller non-uniform beam

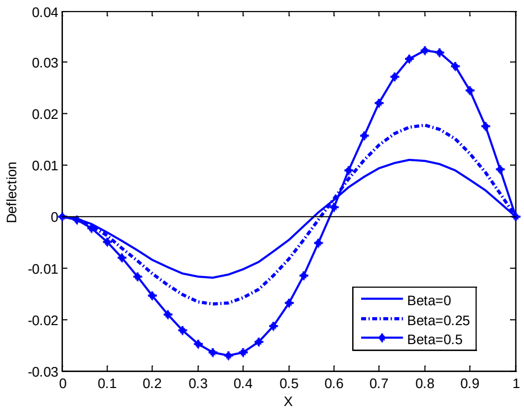

Fig. 12 The Second mode shapes for three different values of Beta of Clamped- Roller non-uniform beam

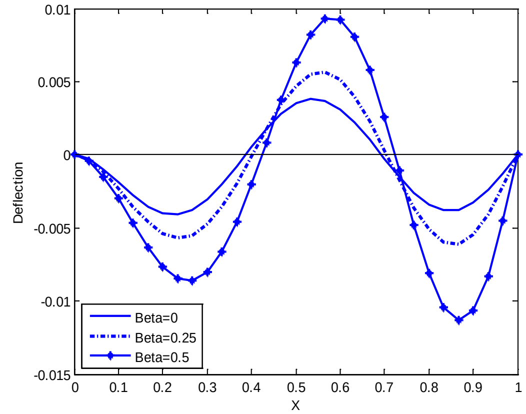

Fig. 13 The Third mode shapes for three different values of Beta of Clamped- Roller non-uniform beam

8. Conclusion

Based on the results presented, it can be demonstrated that the differential transformation method is an efficient method in solving the vibrations of beams with good accuracy using a few terms. The frequency of non-uniform beam decreases with decreasing of the cross section area and its inertia. Also it is observed that the deflection of the non-uniform Euler beam is increased as the cross section is decreased.

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