Multi-Objective Optimization Model Formulation for Solid Waste Management in Dar es Salaam, Tanzania

Halidi Ally Lyeme1,3, Allen Mushí2, Yaw Nkansah-Gyekye1

Abstract: Solid waste management is a challenging problem in developing nations. The health and environmental negative implications associated with solid waste management are very serious particularly in the developing nations where a large percent of waste is dumped into open areas. These implications are essentially on climate change and global warming due to environmental problems. In this paper, a multi-objective optimization model is developed to address the conflicting objectives of cost minimization, minimization of final waste disposal to the landfill, and environmental impact minimization. The model follows a mixed-integer programming formulation and tested by data from selected wards in Dar es Salaam city. The output is the best location of recycling plants, separating plants, composting plants, incinerating plants, landfill and waste flow allocation between them. The solution shows a high reduction of the amount of waste to the landfill and greenhouse gas emissions by 76% and 55.2% respectively compared to the current system.

1. Introduction
Societies in the world, especially in urban areas, are facing extended complications in managing their municipal solid waste (MSW) successfully and cost-effectively. The growing of waste portions, declining of landfill space, increasing public environmental consciousness, stern technical requisites on management options, as well as waste avoidance protocols and waste diversion goals have forced us to have different insight in MSW management. Observing that claims for appropriate MSW management has risen over time, the standpoint of waste management has constantly altered from open disposal to controlled dumpsite to integrated solid waste management (ISWM) system, which needs a mixture of strategies and agenda to manage the waste flow [1].

The disposal and management of solid waste are worldwide threats, particularly in developing nations due to their detrimental effects and the environmental and high consumption of public funds with little output service [2]. It is estimated that municipal authority in developing nations allocate up to 50% of their budget in each year to the management of solid waste, while service covers less than 50% of the population in the regions [3]. The viable means for solid waste management (SWM) in our time has become a global challenge. Solid waste generation is a natural incident but on the other hand, it has grown
rapidly due to increase in the worldwide population levels over the last five decades [4].

The rapid urbanization, booming economies, and rise in human living standards are among the factors which contributed to high generation rate of solid waste [5]. Approximately about 95% of the generated solid waste is landfilled in open areas in developing countries, which causes problems to the environment. These landfills are the sources of Green House Gases (GHGs) emissions particularly methane and carbon dioxide emissions. These landfill gasses account for about 4% of total global GHG emissions which are causing climate change and global warming [6].

Solid waste management involves activities related to generation, collection and depository, transfer and transportation, treatment and dumping of solid wastes. The SWM requires an appropriate framework, upkeep and improvement for all operations. Consequently, this has become more costly and very complicated caused by the progressive and unexpected advancement of the town areas. The problems in providing the desired service in the town areas are frequently attributed to the poor economic condition to manage municipal corporate body [7]. Different alternatives are still accessible to manage the municipal solid waste like a waste to energy technology, use of a land application for waste composting, vermin-composting, digestion, and landfilling [8]. In the existing plot, most of the generated wastes are throw away in open dumps in developing nations and in landfills in developed nations [4].

Decision-makers and the professional must analyze the key technical, legal, economic, environmental, social and political issues connected to ISWM systems to establish a persuasive program for waste management. As the complication of SWM options increased, the choice of the most excellent waste management systems grow into a very complicated task. Thus, systems analysis and mathematical modeling methodology were introduced to solid waste management in order to assist the decision-maker [9]. With such approaches, each nation can make the exclusive system to take care of the diverse ingredients of the waste flow in an environmentally and economical sound manner.

The mathematical modeling technique introduced in this article was fostered by a municipal solid waste management problem that contains the location of various technologies. The benefits of multi-objective techniques to a single objective function were documented in SWM system literature. Some of the benefits referred to are the improvements of the extent to the decision-maker by considering the various angle of the problems, and the added resilience related to the models which are based only on economic [10]. The review of the SWM system literature reveals the limited extension of environmental factors as constraints in some models should exceed that of adding new environmental aspects. Therefore, the addition of more effective environmental objective in models, which includes GHG emissions $CO_2$ and $CH_4$ from both technologies and transportation with enhanced multi-objective techniques is required.
Multi-objective models have been widely applied in the SWM problem with environmental considerations in the research literature. For example, [11] has developed a multi-objective model for SWM with two conflicting objectives. The total cost minimization and the environmental impact minimization which is measured by pollution are two objectives respectively.

The research done by [12] in the city of Duisburg (Germany) presented a multi-objective optimization model for solid waste flows, which aimed to assist the decision-maker on the optimum flows of solid waste transported to different facilities such as organic material treatment, refusal derived fuel (RDF), incinerator plants and sanitary landfill. Four objectives were considered which related to unrecycled waste, economic costs, sanitary landfill disposal and environmental impact (incinerator emissions). In addition, [13] proposed a multi-objective optimization model which integrates economic and environmental factors for the SWM system.

The study conducted by [14] formulated a model based on a multi-objective integer programming technique. The objective of their model is to recommend the optimum solution for transportation, processing and final disposal to different waste facilities with minimal cost and minimum environmental risk.

Furthermore, [9] have developed a multi-objective mixed integer programming model for interpreting the hidden adverse among the environmental and economic aspirations. The model was assessed in Kaohsiung city in Taiwan for the sustainable SWM programs. [15] developed a multi-objective model for solid waste collection system, whereby decision-making approach was considered with economic, environmental, technical and social aspects.

The models above are seen as a good representation of sustainable waste management systems that includes environmental objective with GHGs emission minimization. Moreover, the models are well addressed by presenting decision variables for the selection of technology type. However, the environmental impact cannot only be considered in incinerator emissions as presented by [12] but should be considered in other technologies such as recycling, composting and landfill as the sources for the emission of GHGs, where it will contribute towards new trade-off solutions. This paper adds environmental factors in the objective function which includes GHG emissions $CH_4$ and $CO_2$ from technologies. The rest of the paper is organized as follows: following the proposed MSW management model, then we present a mathematical programming formulation, followed by a summary of results and lastly a conclusion with areas of further research.

2. The Solid Waste Management System Model

A sample SWM is assumed as follows: there are $I$ potential sources of solid waste, which can be removed directly to $K$ potential separators, which check and categorizes the collected solid wastes. After sorting, recoverable wastes will be sent to recycling plants $J, H$
and $G$ for remanufacturing and the products will be sold, humid material will be sent to $L$ composting plants in which organic fertilizer will be sold and dry material will be sent to $M$ incinerator plants in which energy recovery will be sold for electricity power generation, $N$ potential landfill sites, which will receive rejected wastes from $K$ separators. The solid waste flow structure is illustrated in Figure 1. In this work, we consider minimization of economic cost, minimization of final disposed quantities to the landfill and environmental impact (GHGs) as optimization objectives and find out the best solution with respect to the location of recycling plants, separating plants, composting plants, incinerating plants, landfill and waste flow allocation.

![Figure 1. Solid waste flow structure of the model](image)

### 3. Mathematical Programming Formulation

#### 3.1 Indices

$i$ — Potential sources of solid waste index, $i = 1, 2, \ldots, I$

$k$ — Potential separation plant index, $k = 1, 2, \ldots, K$

$j$ — Potential plastic recycling plants index, $j = 1, 2, \ldots, J$

$h$ — Potential metal recycling plants index, $h = 1, 2, \ldots, H$

$g$ — Potential paper recycling plants index, $g = 1, 2, \ldots, G$

$l$ — Potential composting plants index, $l = 1, 2, \ldots, L$
3.2 Decision Variables

The decision variables in the model are the amounts of solid waste transferred from one point to another. In this mathematical formulation, these are defined as follows:

- $AW_{ik}$ - Amount of daily solid waste to be transferred from source $i$ to separation plant $k$
- $AS_{kj}$ - Amount of daily solid waste to be transferred from separation plant $k$ to plastic recycling plant $j$
- $BS_{kh}$ - Amount of daily solid waste to be transferred from separation plant $k$ to metal recycling plant $h$
- $CS_{kg}$ - Amount of daily solid waste to be transferred from separation plant $k$ to paper recycling plant $g$
- $DS_{kl}$ - Amount of daily solid waste to be transferred from separation plant $k$ to composting plant $l$
- $ES_{km}$ - Amount of daily solid waste to be transferred from separation plant $k$ to incinerator plant $m$
- $LS_{kn}$ - Amount of daily solid waste to be transferred from separation plant $k$ to landfill $n$
- $R_j$ - Binary variable, it takes the value 1 if a plastic recycling plant is to be set up at candidate location $j$ ($j = 1, ..., J$) and 0 otherwise.
- $S_h$ - Binary variable, it takes 1 if a metal recycling plant is to be set up at candidate location $h$ ($h = 1, ..., H$) and 0 otherwise.
- $U_g$ - Binary variable, it takes 1 if a paper recycling plant is to be set up at candidate location $g$ ($g = 1, ..., G$) and 0 otherwise.
- $V_k$ - Binary variable, it takes 1 if a separation plant is to be set up at candidate location $k$ ($k = 1, ..., K$) and 0 otherwise.
- $W_l$ - Binary variable, it takes 1 if a composting plant is to be set up at candidate location $l$ ($l = 1, ..., L$) and 0 otherwise.
- $X_m$ - Binary variable, it takes 1 if an incinerator plant is to be set up at candidate location $m$ ($m = 1, ..., M$) and 0 otherwise.
- $Y_n$ - Binary variable, it takes 1 if a landfill is to be set up at candidate location $n$ ($n = 1, ..., N$) and 0 otherwise.
3.3 Parameters

The parameters are the known values (data) that are required by the model as inputs data to calculate the decision variables. These parameters are:

- **$k_{FS}$**: Fixed cost of the separation plant represented as per unit weight.
- **$j_{FR}$**: Fixed cost of the plastic recycling plant represented as per unit weight.
- **$h_{FM}$**: Fixed cost of the metal recycling plant represented as per unit weight.
- **$g_{FP}$**: Fixed cost of the paper recycling plant represented as per unit weight.
- **$l_{FC}$**: Fixed cost of the composting plant represented as per unit weight.
- **$m_{FE}$**: Fixed cost of the incinerator plant represented as per unit weight.
- **$n_{FL}$**: Fixed cost of the landfill represented as per unit weight.
- **$k_{CS}$**: Daily capacity of the separation plant
- **$j_{CR}$**: Daily capacity of the plastic recycling plant
- **$h_{CM}$**: Daily capacity of the metal recycling plant
- **$g_{CP}$**: Daily capacity of the paper recycling plant
- **$l_{CC}$**: Daily capacity of the composting plant
- **$m_{CE}$**: Daily capacity of the incinerator plant
- **$n_{CL}$**: Daily capacity of the landfill
- **$k_{VS}$**: Cost per unit weight processed at the separation plant $k$
- **$j_{VR}$**: Cost per unit weight processed at the plastic recycling plant $j$
- **$h_{VM}$**: Cost per unit weight processed at the metal recycling plant $h$
- **$g_{VP}$**: Cost per unit weight processed at the paper recycling plant $g$
- **$l_{VC}$**: Cost per unit weight processed at the composting plant $l$
- **$m_{VE}$**: Cost per unit weight processed at the incinerator plant $m$
- **$n_{VL}$**: Cost per unit weight processed at the landfill $n$
- **$i_{TW}$**: Transportation cost per unit weight of waste from source $i$ to separation plant $k$
- **$k_{TS}$**: Transportation cost per unit weight of waste from separator $k$ to plastic recycling plant $j$
- **$k_{TS}$**: Transportation cost per unit weight of waste from separator $k$ to metal recycling plant $h$
- **$k_{TS}$**: Transportation cost per unit weight of waste from separator $k$ to paper recycling plant $g$
TS_{ki} - Transportation cost per unit weight of waste from separator $k$ to composting plant $i$

TS_{km} - Transportation cost per unit weight of waste from separator $k$ to incinerator $m$

TS_{kn} - Transportation cost per unit weight of waste from separator $k$ to landfill $n$

Q_j - Revenue generated per unit weight of product from plastic recycling plant $j$

Q_h - Revenue generated per unit weight of product from metal recycling plant $h$

Q_g - Revenue generated per unit weight of product from paper recycling plant $g$

Q_l - Revenue generated per unit weight of product from composting plant $l$

Q_m - Revenue generated per unit weight of product from incinerator plant $m$

$\alpha_1$ - Fractional of plastic material in the waste.

$\alpha_2$ - Fractional of metal material in the waste.

$\alpha_3$ - Fractional of paper material in the waste.

$\beta$ - Fractional of compostable material in the waste

$\gamma$ - Fractional of dry material in the waste.

G_{jGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from plastic recycling plant $j$

G_{hGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from metal recycling plant $h$

G_{gGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from paper recycling plant $g$

G_{lGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from composting plant $l$

G_{mGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from incinerator plant $m$

G_{nGHE} - Emission coefficients for greenhouse effect in ton of $CO_2$ and $CH_4$ per unit weight of waste from landfill $n$

A_i - Amount of daily waste generated at source $i$

### 3.4 Objective Functions

To accommodate for the need of low greenhouse gas emission, three goals are set in this model formulation: 1) Total Cost Minimization, which contains the cost for transportation, recycling, separation, composting, incineration, and recovered from the
disposal of waste. The costs for every operating facility and capital costs are contained; 2) Minimization of total environment impact (GHG emissions), which includes carbon and methane emissions due to recycling, composting, incineration and disposal to the landfill; and 3) Minimize the final disposal to the landfill that is the total amount of waste per day disposed to all landfills from separation facilities. The multi-objective optimization model is as follows:

\[ \text{Min } Z_1 = FC + VC + TC - R \]  

(1)

where

\[ FC = \sum_{j=1}^{I} FR_j R_j + \sum_{h=1}^{H} FM_{h} S_{h} + \sum_{g=1}^{G} FP_{g} U_{g} + \sum_{k=1}^{K} FS_{k} V_{k} + \sum_{i=1}^{l} FC_i W_i \]  

+ \sum_{m=1}^{M} FE_m X_m + \sum_{n=1}^{N} FL_n Y_n \]  

(2)

\[ VC = VS_k \sum_{k=1}^{K} \sum_{i=1}^{I} AW_{ik} + VR \sum_{j=1}^{J} \sum_{k=1}^{K} AS_{kj} + VM \sum_{h=1}^{H} \sum_{k=1}^{K} BS_{kh} + VP \sum_{g=1}^{G} \sum_{k=1}^{K} CS_{kg} \]  

+ VC \sum_{l=1}^{L} \sum_{k=1}^{K} DS_{kl} + VE_m \sum_{m=1}^{M} \sum_{k=1}^{K} ES_{km} + VL_n \sum_{n=1}^{N} \sum_{k=1}^{K} LS_{kn} \]  

(3)

\[ TC = \sum_{i=1}^{I} \sum_{k=1}^{K} TW_{ik} AW_{ik} + \sum_{j=1}^{J} \sum_{k=1}^{K} TS_{kj} AS_{kj} + \sum_{h=1}^{H} \sum_{k=1}^{K} TS_{kh} BS_{kh} + \sum_{g=1}^{G} \sum_{k=1}^{K} TS_{kg} CS_{kg} \]  

+ \sum_{k=1}^{K} \sum_{i=1}^{I} TS_{ki} DS_{kl} + \sum_{k=1}^{K} \sum_{m=1}^{M} TS_{km} ES_{km} + \sum_{k=1}^{K} \sum_{n=1}^{N} TS_{kn} LS_{kn} \]  

(4)

\[ R = Q_k \sum_{k=1}^{K} AS_{kj} + Q_h \sum_{h=1}^{H} BS_{kh} + Q_g \sum_{g=1}^{G} CS_{kg} + Q_l \sum_{l=1}^{L} DS_{kl} + Q_m \sum_{m=1}^{M} ES_{km} \]  

(5)

\[ \text{Min } Z_2 = G_{j}^{GHE} \sum_{j=1}^{I} \sum_{k=1}^{K} AS_{kj} + G_{h}^{GHE} \sum_{h=1}^{H} \sum_{k=1}^{K} BS_{kh} + G_{g}^{GHE} \sum_{g=1}^{G} \sum_{k=1}^{K} CS_{kg} \]  

+ \sum_{l=1}^{L} \sum_{k=1}^{K} DS_{kl} + \sum_{m=1}^{M} \sum_{k=1}^{K} ES_{km} + \sum_{n=1}^{N} \sum_{k=1}^{K} LS_{kn} \]  

(6)

\[ \text{Min } Z_3 = \sum_{k=1}^{K} \sum_{n=1}^{N} LS_{kn} \]  

(7)

subject to the constraints:

\[ \sum_{k=1}^{K} AW_{ik} = A_i, \text{for } i = (1, \ldots, I) \]  

(8)

\[ \sum_{j=1}^{J} \sum_{k=1}^{K} AS_{kj} = \sum_{i=1}^{I} \sum_{k=1}^{K} \alpha_i AW_{ik} \]  

(9)
\[
\sum_{k=1}^{K} \sum_{h=1}^{H} BS_{bh} = \sum_{i=1}^{I} \sum_{k=1}^{K} \alpha_i AW_{ik} \tag{10}
\]
\[
\sum_{k=1}^{K} \sum_{g=1}^{G} CS_{kg} = \sum_{i=1}^{I} \sum_{k=1}^{K} \alpha_i AW_{ik} \tag{11}
\]
\[
\sum_{k=1}^{K} \sum_{l=1}^{L} DS_{kl} = \sum_{i=1}^{I} \sum_{k=1}^{K} \beta AW_{ik} \tag{12}
\]
\[
\sum_{k=1}^{K} \sum_{m=1}^{M} ES_{km} = \sum_{i=1}^{I} \sum_{k=1}^{K} \gamma AW_{ik} \tag{13}
\]
\[
\sum_{k=1}^{K} \sum_{n=1}^{N} LS_{kn} = \sum_{i=1}^{I} \sum_{k=1}^{K} (1 - \alpha_1 - \alpha_2 - \alpha_3 - \beta - \gamma) AW_{ik} \tag{14}
\]
\[
\sum_{i=1}^{I} AW_{ik} \leq CS_k V_k, \text{ for } k = (1, \ldots, K) \tag{15}
\]
\[
\sum_{k=1}^{K} AS_{kj} \leq CR_j R_j, \text{ for } j = (1, \ldots, J) \tag{16}
\]
\[
\sum_{k=1}^{K} BS_{bh} \leq CM_h S_h, \text{ for } h = (1, \ldots, H) \tag{17}
\]
\[
\sum_{k=1}^{K} CS_{kg} \leq CP_g U_g, \text{ for } g = (1, \ldots, G) \tag{18}
\]
\[
\sum_{k=1}^{K} DS_{kl} \leq CC_l W_l, \text{ for } l = (1, \ldots, L) \tag{19}
\]
\[
\sum_{k=1}^{K} ES_{kl} \leq CC_l W_l, \text{ for } l = (1, \ldots, L) \tag{20}
\]
\[
\sum_{k=1}^{K} LS_{kn} \leq CL_n Y_n, \text{ for } n = (1, \ldots, N) \tag{21}
\]
\[
AW_{ik} \geq 0, AS_{kj} \geq 0, BS_{bh} \geq 0, CS_{kg} \geq 0, DS_{kl} \geq 0, ES_{km} \geq 0, LS_{kn} \geq 0 \tag{22}
\]
for \(i = (1, \ldots, I); \ j = (1, \ldots, J); \ k = (1, \ldots, K); \ l = (1, \ldots, L); \ m = (1, \ldots, M); \ n = (1, \ldots, N)\)
\[
R_j = 0 \text{ or } 1, \text{ for } j = (1, \ldots, J) \tag{23}
\]
\[
S_h = 0 \text{ or } 1, \text{ for } h = (1, \ldots, H) \tag{24}
\]
\[
U_g = 0 \text{ or } 1, \text{ for } g = (1, \ldots, G) \tag{25}
\]
\[
V_k = 0 \text{ or } 1, \text{ for } k = (1, \ldots, K) \tag{26}
\]
\[
W_l = 0 \text{ or } 1, \text{ for } l = (1, \ldots, L) \tag{27}
\]
\[
X_m = 0 \text{ or } 1, \text{ for } m = (1, \ldots, M) \tag{28}
\]
\[
Y_n = 0 \text{ or } 1, \text{ for } n = (1, \ldots, N) \tag{29}
\]
The sum of equations (1) through (5) is the cost objective function, which is the sum of daily fixed cost of constructing technology facilities, variable cost of running the selected waste facilities, transportation cost from waste sources (wards) to the separation plants, and transportation cost from the separation plants to various waste facilities. Equation (6) is the environmental impact objective function, which considers GHG emissions from all waste facilities. Equation (7) is the final disposal objective function, which minimizes the waste to the landfill. Equations (8) – (14) are mass balance constraints, which ensure that all solid waste from each ward are sorted at separation plants and properly assigned to all waste facilities. Inequalities (15) – (21) are capacity limitation constraints, which ensure that the total waste sent to various waste facilities does not exceed their capacity. Inequality (22) is a non-negativity inequality, which ensures that the values of decision variable are either zero or positive. Equations (23) – (29) are binary variables, which determine whether facilities have been selected or not.

4. Summary of Results
The multi-objective optimization model above has been converted to lexicographic preemptive goal programming (PGP) model for the model test. Goals for the PGP model are provided in Table 1 along with their target, current and the priority of each goal.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Goal</th>
<th>Target</th>
<th>Current</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimize</td>
<td>Total Cost</td>
<td>195,000,000 Tsh</td>
<td>200,000,000 Tsh</td>
<td>1</td>
</tr>
<tr>
<td>Minimize</td>
<td>GHG emissions</td>
<td>168 $CO_2$ eq</td>
<td>376 $CO_2$ eq</td>
<td>2</td>
</tr>
<tr>
<td>Minimize</td>
<td>Final Disposal</td>
<td>280 Tons</td>
<td>1050 Tons</td>
<td>3</td>
</tr>
</tbody>
</table>

The aim of goal programming is to launch a goal level of attainment for each criterion. Goal programming (GP) method needs the decision maker to set goals for each objective that he/she desires to obtain. A favored solution is then well-defined as the one which reduces the deviations from the set goals. The purpose of GP is to minimize the deviations between the attainment of goals [16]. In this model, the primary objective of the problem is to minimize the total cost for the SWM system. The second priority is to minimize total environmental impact, that is, GHG emissions and to minimize the final disposal from various separation plants, which is the third priority goal as suggested by the decision maker.

The developed model was solved using GNU Linear Programming Kit (GLPK). The GLPK is a software package intended for solving large scale linear programming (LP), Mixed Integer Programming (MIP), and other related problems [17]. The first priority goal was
solved as single objective. Thereafter, the second priority goal was formulated and solved in which the first priority goal has been taken as a constraint. The third priority goal was formulated and solved in which the first and second priority goals have been taken as the constraints. Finally the last objective was formulated and solved, Tables 2, 3, 4, and 4 shows the solution obtained after the last formulated objective was solved which comprises of only deviation variables. The final value of the objective function and all deviation variables is zero as shown in Table 2, which indicates that both goals are perfectly satisfied. That is, all the waste in the separation plants are evacuated with minimum cost and all the waste in the recycling, composting, incineration as well as in the landfill are processed with minimum GHG emissions.

Table 2: Deviation Variable Values

<table>
<thead>
<tr>
<th>Deviation Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1^+$</td>
<td>0.008</td>
</tr>
<tr>
<td>$d_1^-$</td>
<td>0</td>
</tr>
<tr>
<td>$d_2^+$</td>
<td>0</td>
</tr>
<tr>
<td>$d_2^-$</td>
<td>0</td>
</tr>
<tr>
<td>$d_3^+$</td>
<td>0</td>
</tr>
<tr>
<td>$d_3^-$</td>
<td>0</td>
</tr>
</tbody>
</table>

Furthermore, the model proposed 10 separation plants and 2 recycling plants for plastics, metals and paper respectively. In addition to that, the model proposed 2 composting plants, 2 incineration plants as well as 2 landfills. Table 3, shows the values for the three objectives which gives significant reduction of both cost, GHG emissions and final disposal waste to the landfill as compared to the current situation as shown in Table 1 above. In this model test, we took data from some wards in Dar es Salaam with 1050 tons per day as their total solid waste generation. Currently, Dar es Salaam has no formal waste diversion rather about 40% of the generated wastes were transferred to the landfill. This shows that about 60% remaining are left on the open dump which favors for GHG emissions [18]. The formulated model reduced amount of waste to the landfill and GHG emissions by 76% and 55.2% respectively. The remaining Table 4 and 5 shows the amount of solid waste flow from separation plants to various recycling, composting, incineration plants and landfills respectively.
### Table 3: Objective Functions Value

<table>
<thead>
<tr>
<th>Priority Goal</th>
<th>Objective Function</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Z_1$</td>
<td>192,550,594.1 Tsh</td>
</tr>
<tr>
<td>2</td>
<td>$Z_2$</td>
<td>168 $CO_2$ eq</td>
</tr>
<tr>
<td>3</td>
<td>$Z_3$</td>
<td>273 Tons</td>
</tr>
<tr>
<td>-</td>
<td>$Z$</td>
<td>0.008</td>
</tr>
</tbody>
</table>

### Table 4: Waste Amount (ton) Flow from Separation to Various Recycling Plants

<table>
<thead>
<tr>
<th>Separation Plants</th>
<th>Plastic Recycling Plant</th>
<th>Metal Recycling Plant</th>
<th>Paper Recycling Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Malapa</td>
<td>Kisutu</td>
<td>Barakuda</td>
</tr>
<tr>
<td>Buguruni</td>
<td>17.6</td>
<td>-</td>
<td>4.6</td>
</tr>
<tr>
<td>Ilala</td>
<td>17.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Segerea</td>
<td>11.76</td>
<td>5.84</td>
<td>6.6</td>
</tr>
<tr>
<td>Kariakoo</td>
<td>-</td>
<td>17.6</td>
<td>-</td>
</tr>
<tr>
<td>Pugu</td>
<td>17.6</td>
<td>-</td>
<td>6.6</td>
</tr>
<tr>
<td>Kipawa</td>
<td>-</td>
<td>13.76</td>
<td>5.16</td>
</tr>
<tr>
<td>Ukonga</td>
<td>13.44</td>
<td>-</td>
<td>5.4</td>
</tr>
<tr>
<td>Gerezani</td>
<td>-</td>
<td>17.6</td>
<td>-</td>
</tr>
<tr>
<td>Jangwani</td>
<td>-</td>
<td>17.6</td>
<td>-</td>
</tr>
<tr>
<td>Mchafukogge</td>
<td>-</td>
<td>17.6</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>78</strong></td>
<td><strong>90</strong></td>
<td><strong>28</strong></td>
</tr>
</tbody>
</table>
Table 5: Waste Amount (ton) Flow from Separation to Composting, Incineration Plants and Landfill

<table>
<thead>
<tr>
<th>Separation Plants</th>
<th>Composting Plant</th>
<th>Incineration Plant</th>
<th>Landfill</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kamata</td>
<td>Mombasa</td>
<td>Amana</td>
</tr>
<tr>
<td>Buguruni</td>
<td>42.9</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td>Ilala</td>
<td>-</td>
<td>42.9</td>
<td>-</td>
</tr>
<tr>
<td>Segerega</td>
<td>-</td>
<td>42.9</td>
<td>-</td>
</tr>
<tr>
<td>Kariakoo</td>
<td>42.9</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td>Pugu</td>
<td>-</td>
<td>42.9</td>
<td>-</td>
</tr>
<tr>
<td>Kipawa</td>
<td>0.5</td>
<td>33.4</td>
<td>2.5</td>
</tr>
<tr>
<td>Ukonga</td>
<td>-</td>
<td>32.76</td>
<td>-</td>
</tr>
<tr>
<td>Gerezani</td>
<td>42.9</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td>Jangwani</td>
<td>42.9</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td>Mchafukogge</td>
<td>42.9</td>
<td>-</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>215</strong></td>
<td><strong>195</strong></td>
<td><strong>30</strong></td>
</tr>
</tbody>
</table>

5. Conclusions and Future Research Directions

In this paper, the new model proposed is formulated as a multi-objective optimization model (MOOM), which simultaneously solves the three objectives. The multi-objective programming algorithm was considered in developing the proposed model using the principle of mixed-integer programming. The model objectives functions focused on minimizing the costs, environmental impact and final disposal to the landfill. The lexicographic goal programming technique is used to solve the formulated MOOM. This approach allows the analyst to assign different priorities given by decision maker to the goals considered, first of all looking for a solution that meets the most important of these priorities. The model has been tested in a real SWM system of some wards in Dar es Salaam city. This approach provides solutions that are consistent with the decision maker’s preferences.

The model is coded in GNU Linear Programming Kit (GLPK) software for Linux and has been run to optimality. The developed model provides a reduced amount of waste to the landfill and GHG emissions by 76% and 55.2% respectively. Moreover, the model proposed 10 separation plants and 2 recycling plants for plastics, metals and paper respectively. Two composting plants, 2 incineration plants as well as 2 landfills were also proposed by the model.

Future research needs an extension of analysis of the model to cover the whole city of Dar es Salaam. This will give a better picture of possible further cost saving and environmental impact reductions. Since the model has a vast number of parameters, sensitivity analysis of these parameters is another area of further research. Due to the huge funds invested in
waste disposal in Dar es Salaam, implementations and compliance to regulations should be considered for a successful waste management relief in the future.

Acknowledgements
The authors would like to thank the Government of the Republic of Tanzania through the Commission for Science and Technology (COSTECH) for the full financial support of this study.

References


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