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CHARACTERIZATIONS OF SOME ALTERNATING AND SYMMETRIC GROUPS

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ABSTRACT. We suppose that $p = 2^{\alpha}5^{\beta}+1$, where $\alpha \geq 1, \beta \geq 0$, and $p \geq 11$ is a prime number. Then we prove that the simple group A_n, where $n = p, p+1$, or $p+2$ and the finite group S_n, where $n = p, p+1$, are also uniquely determined by their order components. As corollaries, the validity of a conjecture of J. G. Thompson, a conjecture of AAM (ref. [1]) and a conjecture of Shi(ref. [36]) both on A_n, where $n = p, p+1$, or $p+2$ are obtained. Also we generalized these conjectures for the groups S_n, where $n = p, p+1$.

1. Introduction

Let G be a finite group. We denote by $\pi(G)$ the set of all prime divisors of $|G|$. We construct the prime graph of G as follows. The prime graph $\Gamma(G)$ of a group G is the graph whose vertex set is $\pi(G)$, and two distinct primes p and q, considered as vertices of $\Gamma(G)$, are adjacent by an edge(we write $p \sim q$) if and only if G contains an element of order pq. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and let $\pi_1(G), \pi_2(G), \ldots, \pi_t(G)$ be the connected components of $\Gamma(G)$. Sometimes we use the notation π_i instead of $\pi_i(G)$. If $2 \in \pi(G)$ then we always suppose that $2 \in \pi_1$(ref. [38]).

Now $|G|$ can be expressed as a product of coprime positive integers $m_i, i = 1, 2, \ldots, t(G)$, where $\pi(m_i) = \pi_i$. These integers are called order components of G. The set of order components of G will be denoted by $OC(G)$. Also we call $m_2, \ldots, m_{t(G)}$ the odd-order components of G.

A finite group G is called a 2-Frobenius group if G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$, where K and G/H are Frobenius groups with kernels H and K/H, respectively. A group G is called a C_{pp} group if the centralizers of its elements of order p in G are p-groups. The non-commuting graph $\nabla(G)$ associated with a finite group G defined by Paul Erdös in 1975 is as follows: the vertex set of $\nabla(G)$ is $G \backslash Z(G)$ with two vertices x and y joined by an edge whenever the commutator of x and y is not the identity(ref. [29]).

Professor Chen first introduced the definition of order components in studying Thompson's Conjecture. It is an interesting topic to characterize finite (simple) groups by their order components. Up to now, there are many results about it. We summarize them in the following theorem([3]-[5], [7]-[12], [14]-[26], [30]).

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Theorem 1.1. *Let G be a finite group, p and r are primes, $q = p^m$, and M be one of the following groups:*

(1) *finite simple groups with $t(G) > 2$;*

(2) *Sporadic simple groups, Suzuki-Ree groups, ${}^3D_4(q)$, ${}^2D_4(q), E_6(q), {}^2E_6(q), E_8(q)$;*

(3) *$G_2(q)$, $q \equiv 0(3)$;*

(4) *$PSL_2(q), PSL_3(q), PSL_5(q)$, where $q > 5$;*

(5) *$F_4(q)$, where q is even;*

(6) *$C_2(q)$, $q > 5$;*

(7) *$PSU_r(q)$, $r \geq 3$, $q > 5$;*

(8) *${}^2D_{r+1}(2)$, where $5 \leq r \neq 2^m - 1$; $C_r(2)$; ${}^2D_n(3)$ where $9 \leq n = 2^m + 1 \neq r$); $D_{r+1}(3)$; $D_r(3)$, $D_r(5)$, $r > 5$;*

(9) *Alternating groups A_r, where r and $r - 2$ are primes; A_n where $n = r, r+1, r+2$ and $r = 2^a 3^b + 1$.*

Then $G \cong M$ if and only if $OC(G) = OC(M)$.

In this short paper, we prove the following theorems.

Theorem A Let $p = 2^\alpha 5^\beta + 1$, where $\alpha \geq 1, \beta \geq 0$, and $p \geq 11$ is a prime number. Let M be A_n, where $n = p, p+1$, or $p+2$. Then $OC(G) = OC(M)$ if and only if $G \cong M$.

Theorem B Let $p = 2^\alpha 5^\beta + 1$, where $\alpha \geq 1, \beta \geq 0$, and $p \geq 11$ is a prime number. Let M be S_n, where $n = p, p+1$. Then $OC(G) = OC(M)$ if and only if $G \cong M$.

In this paper, all groups are finite and by simple groups we mean non-abelian simple groups. All further unexplained notations are standard and we refer the reader to [38].

2. Preliminaries

Remark 2.1. Let N be a normal subgroup of G and $p \sim q$ in $\Gamma(G/N)$. Then $p \sim q$ in $\Gamma(G)$. In fact if $xN \in G/N$ has order pq then there is a power of x of order pq.

Lemma 2.1. ([38]) *If G is a finite group with prime graph having at least two components then G is one of the following groups:*

(a) a Frobenius or 2-Frobenius group;

(b) a simple group;

(c) an extension of a π_1-group by a simple group;

(d) an extension of a simple group by a π_1-group;

(e) an extension of a π_1-group by a simple group by a π_1-group.

Lemma 2.2. ([38]) *If G is a finite group with more than one prime graph component and has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$, such that H and G/K are π_1-groups and K/H is a simple group, then H is a nilpotent group.*

The next lemma follows from [6].

Lemma 2.3. *Let G be a Frobenius group of even order and let H, K be Frobenius complement and Frobenius kernel of G, respectively. Then $t(\Gamma(G)) = 2$, and the prime graph components of G are $\pi(H)$, $\pi(K)$ and G has one of the following structures:*

(a) $2 \in \pi(K)$ and all Sylow subgroups of H are cyclic;

(b) $2 \in \pi(H)$, *K is an abelian group, H is a solvable group, the Sylow subgroups of odd order of H are cyclic groups, and the 2-Sylow subgroups of H are cyclic or generalized quaternion groups;*

(c) $2 \in \pi(H)$, *K is an abelian group, and there exists $H_0 \leq H$ such that $|H : H_0| \leq 2$, $H_0 = Z \times SL(2,5), (|Z|, 2.3.5) = 1$, and the Sylow subgroups of Z are cyclic.*

The next lemma follows from [6] and Lemma 2.3.

Lemma 2.4. *Let G be a 2-Frobenius group of even order. Then $t(\Gamma(G)) = 2$ and G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that*

(a) $\pi_1 = \pi(G/K) \cup \pi(H)$ *and* $\pi(K/H) = \pi_2$;

(b) G/K and K/H are cyclic, $|G/K|$ divides $|\mathrm{Aut}\,(K/H)|$, $(|G/K|, |K/H|) = 1$, and $|G/K| < |K/H|$;

(c) H is nilpotent and G is a solvable group.

The next two lemmas follow from [7].

Lemma 2.5. *Suppose that G and M are two finite groups satisfying $t(\Gamma(G)) \geq 2$, $N(G) = N(M)$, where $N(G) = \{n | G$ has a conjugacy class of size $n\}$, and $Z(G) = 1$. Then $|G| = |M|$.*

Lemma 2.6. *Let G_1 and G_2 be finite groups satisfying $|G_1| = |G_2|$ and $N(G_1) = N(G_2)$. Then $t(G_1) = t(G_2)$ and $OC(G_1) = OC(G_2)$.*

Lemma 2.7. *([23]) Let G be a finite group and let M be a non-abelian finite group with $t(M) = 2$ satisfying $OC(G) = OC(M)$. Let $|M| = m_1m_2$, $OC(M) = \{m_1, m_2\}$, and $\pi(m_i) = \pi_i$ for $i = 1, 2$. Then $|G| = m_1m_2$ and one of the following holds:*

(a) G is a Frobenius or 2-Frobenius group;

(b) G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/K is a π_1-group, H is a nilpotent π_1-group, and K/H is a non-abelian simple group. Moreover, $OC(K/H) = \{m'_1, m'_2, \cdots, m'_s, m_2\}$, $|K/H| = m'_1m'_2\cdots m'_sm_2$, and $m'_1, m'_2, \cdots, m'_s, m_2 \big| m_1$, where

$\pi(m'_j) = \pi'_j$, $1 \leq j \leq s$ *and* $|G/K| \big| |Out(K/H)|$.

Lemma 2.8. *([28]) Let $S = A_n$ $(n \geq 4)$ or S_n $(n \geq 3)$, if H is a group such that $\nabla(H) \cong \nabla(S)$ then $\mid H \mid = \mid S \mid$.*

Lemma 2.9. *([37]) Let G and H be finite groups. If $\nabla(H) \cong \nabla(G)$, then*

$$\mid C_H(x) \backslash Z(H) \mid = \mid C_G(\phi(x)) \backslash Z(G) \mid$$

for all $x \in H \setminus Z(H)$, where ϕ is an isomorphism from $\nabla(H)$ to $\nabla(G)$.

Lemma 2.10. *([27]) Let p be a prime and $p = 2^\alpha 5^\beta + 1$, where $\alpha \geq 1, \beta \geq 0$. Then any finite simple C_{pp} group is given by Table 1.*

Table 1 finite simple C_{pp} group

p	finite simple C_{pp} group
2	A_5, A_6, $A_1(q)$,where q is a Fermat prime, a Mersenne prime, or $q=2^m$, $m\geq 3$, $A_2(2^2)$, ${}^2B_2(2^{2m+1})$ where $m\geq 1$
3	A_5, A_6; $A_1(q)$, where $q=2^3$, 3^{m+1}, or $2.3^m\pm 1$ which is a prime and $m\geq 1$, $A_2(2^2)$
5	A_5, A_6, A_7; M_{11}, M_{22}; $A_1(q)$, where $q=7^2$, 5^m, or $2.5^m\pm 1$ which is a prime, $m\geq 1$, $A_2(2^2)$, $C_2(q)$, $q=3,7$, ${}^2A_3(3)$, ${}^2B_2(q)$, $q=2^3,2^5$
11	A_{11}, A_{12}, A_{13}; M_{11}, M_{12}, M_{22}, M_{23}, M_{24} J_1, HS, Sz, On, McL, Co_2; $A_1(q)$, where $q=3^5$, 11^m, or $2.11^m\pm 1$ which is a prime, $m\geq 1$, $A_4(3)$, $A_5(3)$, $B_5(3)$, $C_5(3)$, $D_5(3)$, $D_6(3)$, ${}^2A_4(2^2)$, ${}^2A_5(2^2)$
17	A_{17}, A_{18}, A_{19}; J_3, He, F_{23}, F'_{24}; $A_1(q)$, where $q=16$, 17^m, or $2.17^m\pm 1$ which is a prime, $m\geq 1$, $C_2(4)$, $C_4(2)$, $F_4(2)$, ${}^2D_4(2)$, ${}^2D_5(2)$, ${}^2E_6(2^2)$
41	A_{41}, A_{42}, A_{43}; F_1, $A_1(q)$, where $q=3^4$, 41^m, or $2.41^m\pm 1$ which is a prime, $m\geq 1$, $B_4(3)$, $C_4(3)$, ${}^2B_2(2^5)$, ${}^2D_4(3^2)$, ${}^2D_5(3^2)$
$p=2^{2^n}+1$	A_p, A_{p+1}, A_{p+2}; $A_1(q)$, where $q=2^{2^n}$, p^m, or $2.p^m\pm 1$ which is a prime, $m\geq 1$, $C_{2^a}(2^{2^b})$, $a\geq 1, a+b=n$, ${}^2D_{2^a}(2^{2^b})$, $a\geq 2, a+b=n$, ${}^2D_{2^n+1}(2)$, $n\geq 2$, $F_4(2^m)$, $m\geq 1, 4m=2^n$
other	A_p, A_{p+1}, A_{p+2}; $A_1(q)$, where $q=p^m$, or $2p^m\pm 1$ which is a prime, $m\geq 1$

3. Characterization of some alternating and symmetric groups

In the sequel, we suppose that $p=2^\alpha 5^\beta+1$, where $\alpha\geq 1,\beta\geq 0$, and $p\geq 11$ is a prime number.

Lemma 3.1. *Let G be a finite group and let M be A_n, where $n=p,p+1$, or $p+2$, or S_n, where $n=p,p+1$. If $OC(G)=OC(M)$, then G is neither a Frobenius group nor a 2-Frobenius group.*

Proof If G is a Frobenius group, then by Lemma 2.3, $OC(G)=\{|H|,|K|\}$, where K and H are Frobenius kernel and Frobenius complement of G, respectively. Since $|H|\big|(|K|-1)$, we have $|H|<|K|$. Therefore $2\nmid |H|$, and hence $2\big||K|$. So, $|H|=p$, $|K|=|G|/p$. We claim that there exists a prime p' such that $3n/4<p'<p$. Note that $p\leq n$, and hence $p^2\nmid |A_n|$. Let $\beta(n)$ be the number of prime numbers less than or equal to n. In fact, by [22, Theorem 2] we have

$$\frac{n}{logn-\frac{1}{2}}<\beta(n)<\frac{n}{logn-\frac{3}{2}} \qquad (3.1)$$

where $n\geq 67$. Thus

$$\beta(n)-\beta(\frac{3n}{4})>\frac{n}{logn-\frac{1}{2}}-\frac{\frac{3n}{4}}{log(\frac{3n}{4})-\frac{3}{2}} \qquad (3.2)$$

When $n\geq 405$, we get $\beta(n)-\beta(3n/4)>1$, and for $n<405$, we can immediately obtain the result by checking the table of prime numbers. Now let P' be the p'- Sylow subgroup of K. Since K is nilpotent, $P'\trianglelefteq G$. Thus $P'H$ is a Frobenius group, and $p\big|p'-1$. Which is a contradiction since $p'<p$. Therefore, G is not a Frobenius group.

Now let G be a 2-Frobenius group. By Lemma 2.4, there is a normal series $1\trianglelefteq H\trianglelefteq K\trianglelefteq G$ such that $|K/H|=p$ and $|G/K|<p$. Let p' be as above, then $p'\in\pi(H)$ since $|G|=|G/K||K/H||H|$. Let P' be the p'- Sylow subgroup of H and P the p- Sylow subgroup of G. Then $[P']P$ is a Frobenius subgroups of order $p'p$. Thus $p|p'-1$, which is impossible. Hence, G is not a 2-Frobenius group. □

Theorem A Let $p=2^\alpha 5^\beta+1$, where $\alpha\geq 1,\beta\geq 0$, and $p\geq 11$ is a prime number. let M be A_n, where $n=p,p+1$, or $p+2$. Then $OC(G)=OC(M)$ if and only if $G\cong M$.

Proof By Lemma 2.7 and Lemma 3.1, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups and K/H is a simple group. Moreover, the odd-order component of M is equal to an odd-order component of K/H. Therefore, K/H is a finite simple C_{pp} group. Now using Table 1, we consider each possibility of K/H separately.

Case I $p = 11$.

(1.1) Let $K/H \cong M_{11}, M_{12}, M_{22}, HS, Mcl, M_{23}, M_{24}, J_1, Suz, O'N$, or CO_2.

If $K/H \cong M_{11}$ or M_{12}, then $|G|/|K/H| = |H||G/K| \neq 1$. Since $|G/K|\big||Out(K/H)| \leq 2$ by Lemma 2.7, $7 \in \pi(H)$. Let P be the 7-Sylow subgroup of H. Since H is nilpotent, $P \trianglelefteq G$. Hence, $11\big|(|P| - 1)$, which is a contradiction.

If $K/H \cong M_{22}$, then $5 \in \pi(H)$ since $5 \parallel |M_{22}|$, $5^2 \parallel |G|$ and $|Out(M_{22})| = 2$. Let P be the 5-Sylow subgroup of H. But since H is nilpotent, $P \trianglelefteq G$. Hence, $11\big|(|P| - 1)$, which is a contradiction.

If $K/H \cong HS$ or Mcl. It is a contradiction since $5^3\big||K/H|$, but $5^2 \parallel |G|$.

If $K/H \cong M_{23}, M_{24}, J_1, O'N$, or CO_2. It is a contradiction since there exists a prime in $\pi(K/H)$ which is not in $\pi(G)$.

If $K/H \cong Suz$. It is a contradiction since $2^{13}\big||Suz|$, but $|G|_2 \leq 2^9$.

(1.2) If $K/H \cong L_2(11^m)$ or $L_2(2 \cdot 11^m \pm 1)$, where $2 \cdot 11^m \pm 1$ is a prime and $m \geq 1$. Note $11^2 \nmid |G|$, hence $m = 1$. So $K/H \cong L_2(11)$ or $L_2(23)$. If $K/H \cong L_2(11)$, we can proceed similar to (1.1). If $K/H \cong L_2(23)$, It is obvious contradictory since $23 \in \pi(K/H)$ but $23 \notin \pi(G)$.

(1.3) If $K/H \cong L_2(3^5), L_5(3), L_6(3), B_5(3), C_5(3), D_5(3), D_6(3), {}^2A_4(2^2)$, or ${}^2A_5(2^2)$. It is a contradiction since $|K/H| \nmid |G|$ by [2].

(1.4) If K/H ia an alternating group, namely A_{11}, A_{12}, or A_{13}. First suppose that $n = 11$. Since $|K/H| \leq |A_{11}|$, $K/H \cong A_{11}$. But $|G| = |A_{11}|$, and hence $H = 1$ and $K = G \cong A_{11}$. If $n = 12$, then $K/H \cong A_{11}$ or A_{12}. But if $r \neq 6$, then $\mathrm{Aut}\,(A_r) = S_r$, and hence $|Out(A_r)| = 2$. If $K/H \cong A_{11}$, then $|G/K|\big|2$, and hence $3 \in \pi(H)$. Let P be the 3- Sylow subgroup of H. Then $|P| = 3$. Since H is nilpotent, $P \trianglelefteq G$. Hence, $11\big|(|P| - 1)$, which is a contradiction. Therefore, $K/H \cong A_{12}$, and hence $G \cong A_{12}$. If $n = 13$, we do similarly.

Case II $p = 41$.

(2.1) If $K/H \cong F_1, {}^2D_4(3^2)$, or ${}^2D_5(3^2)$. It is a contradiction since $|K/H| \nmid |G|$ by calculating their orders.

(2.2) If $K/H \cong B_4(3)$ ${}^2B_2(2^5)$ or $A_1(3^4)$. Then $37 \in \pi(H)$ since $37 \notin \pi(H/K)$ and $|Out(K/H)| \leq 10$. Let P be the 37- Sylow subgroup of H. Since H is nilpotent, $P \trianglelefteq G$. Hence, $41\big|(|P| - 1)$, which is a contradiction.

(2.3) If $K/H \cong L_2(41^m)$ or $L_2(2 \cdot 41^m \pm 1)$, where $2 \cdot 41^m \pm 1$ is a prime and $m \geq 1$. Note $41^2 \nmid |G|$, hence $m = 1$. So $K/H \cong L_2(41)$ or $L_2(83)$. If $K/H \cong L_2(41)$, we can proceed similar to (2.2). If $K/H \cong L_2(83)$, It is obvious contradictory since $83 \in \pi(K/H)$ but $83 \notin \pi(G)$.

(2.4) If K/H ia an alternating group, namely A_{41}, A_{42}, or A_{43}. Then we have $G \cong A_n$ by the same method to (1.4).

Case III $p = 17$ or other cases.

In fact the proof of this step is the same to that in [23] and we omit it. □

Theorem B Let $p = 2^\alpha 5^\beta + 1$, where $\alpha \geq 1, \beta \geq 0$, and $p \geq 11$ is a prime number. let M be S_n, where $n = p, p + 1$. Then $OC(G) = OC(M)$ if and only if $G \cong M$.

Proof Using Lemma 2.7 and Lemma 3.1, we have G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/K is a π_1-group, H is a nilpotent π_1-group, and K/H is a C_{pp} simple group. Similar to the proof of Theorem A, we have $K/H \cong A_n$. Thus $A_n \leq G/H \leq \mathrm{Aut}\,(A_n) = S_n$. Therefore, $G/H \cong A_n$ or $\mathrm{Aut}\,(A_n) = S_n$. If $G/H \cong A_n$, then $|H| = 2$ and $H \trianglelefteq G$. Hence, $H \subseteq Z(G) = 1$, which is a contradiction. Therefore, $G/H \cong S_n$, and since $|G| = |S_n|$, we have $G \cong S_n$. □

4. SOME RELATED RESULTS

In 1987, Professor J. G. Thompson gave the following conjecture.

Thompson's Conjecture If G is a finite group with $Z(G) = 1$ and M is a non-abelian simple group satisfying $N(G) = N(M)$, then $G \cong M$, where $N(G) := \{n \in \mathbb{N} \mid G$ has a conjugacy class of size $n\}$.

We can generalize this conjecture for the groups under discussion by our characterization of these groups.

Corollary 4.1. *Let G be a finite group with $Z(G) = 1$ and M be A_p, A_{p+1}, A_{p+2}, S_p or S_{p+1}. If $N(G) = N(M)$, then $G \cong M$.*

Proof By Lemma 2.5 and 2.6, if G and M are two finite groups satisfying the condition of Corollary 4.1, then $OC(G) = OC(M)$. So Theorem A and B imply this Corollary.

In 2006, A. Abdollahi, S. Akbari, and H.R. Maimani put forward a conjecture in [1] as follows.

AAM's Conjecture Let M be a finite non-abelian simple group and G a group such that $\nabla(G) \cong \nabla(M)$. Then $G \cong M$.

Corollary 4.2. *Let G be a finite group and M be A_p, A_{p+1}, A_{p+2}, S_p or S_{p+1}. Then $\nabla(G) = \nabla(M)$ if and only if $G \cong M$.*

Proof By lemma 2.8 and lemma 2.9, we have $|G| = |M|$ and $N(G) = N(M)$. Hence $OC(G) = OC(M)$ by Lemma 2.6. Thus $G \cong M$ by Theorem A and B. □

In 1987, Professor Shi put forward the following conjecture(ref.[36]).

Shi's Conjecture Let G be a finite group and M a finite nonabelian simple group. Then $G \cong M$ if and only if (1) $\pi_e(G) = \pi_e(M)$, (2) $|G| = |M|$.

This conjecture is valid for all finite simple groups except symplectic group and orthogonal group of odd dimension with order larger than 10^8(ref. [13], [31]-[36]). As a consequence of Theorems A and B, we prove a generalization of this conjecture for the groups under discussion.

Corollary 4.3. *Let G be a finite group and let M be A_p, A_{p+1}, A_{p+2}, S_p or S_{p+1}. If $|G| = |M|$ and $\pi_e(G) = \pi_e(M)$, then $G \cong M$.*

Proof By assumption, we must have $OC(G) = OC(M)$. Thus the corollary follows by Theorems A and B. □

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