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# MODELING INTERACTIONS OF NILE PERCH, NILE TILAPIA AND SMALL PELAGIC SILVER FISH WITH CONSTANT HARVESTING EFFORTS IN LAKE VICTORIA

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ABSTRACT. In this paper, we examined the effects of interactions in a predator-prey manner and harvesting at constant fishing efforts of Nile perch (*Lates niloticus*), Nile tilapia (*Oreochromis niloticus*) and Small pelagic silver (*Rastrineobola argentea*) fish species of the lake Victoria fishery, through mathematical modeling using Lotka-Volterra equation, whereby all three fish species are subjected to harvesting. The model is characterized by the system of first order non-linear ordinary differential equations.
All eight equilibrium points of the model were identified, the local stability of the co-existence equilibrium point was discussed using Routh-Hurwitz criteria and its global stability was analyzed using suitable Lyapunov function. Further, analytic solutions of the model coincided with the computed numerical solutions using fourth order Runge-Kutta method. The study revealed that as the model parameters became small the equilibrium stock level biomasses of fish species increased.

## 1. INTRODUCTION

Lake Victoria is the second largest fresh-water lake in the world and the largest in Africa. The lake is shared by Kenya (6%), Uganda (43%) and Tanzania (51%). According to [7], lake Victoria fishery is currently dominated by three fish species which are Nile perch, Nile tilapia and small pelagic silver fish. The lake is an important source of food, employment and earnings for the riparian states through fishery for millions of people.

The study aims at investigating the effects of harvesting at constant fishing efforts and interactions in a predator-prey manner to the equilibrium stock level biomasses of Nile perch, Nile tilapia and small pelagic silver fish species in lake Victoria. The Nile perch are predator to Nile tilapia and vice versa (depending on their sizes, that is, adult Nile perch eats the young Nile tilapia and adult Nile tilapia eats the young Nile perch) while small pelagic silver fishes are prey to both Nile perch and Nile tilapia.
Quantitative and qualitative understanding of the interactions of these fish species is crucial for the management of fisheries in lake Victoria. Harvesting has generally a strong impact on the population dynamics of harvested species. The severity of this impact depends on the nature of the applied harvesting strategy which, in turn,

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may range from the rapid depletion to the complete preservation of a population[10].

The problem of interspecies interactions was considered by [4] for two species obeying the law of logistic growth. [2] considered harvesting of a single species in an ecologically competing two fish population model.[1] and [11] studied the dynamics of two-species fishery by combining harvesting. [3] studied constant rate of harvesting in a predator-prey system to allow simultaneous harvesting of both species. They showed how to approximate the region of asymptotic stability in biological terms, in the initial states which lead to coexistence of the two species and their global dynamics by efficient computer simulation. [9] studied the effect of Nile perch predation to Nile tilapia and harvesting on fisheries dynamics in Lake Victoria, the study ignored the third species, the small pelagic silver fish, a prey to the first two species which has a significant contribution to the dynamics of species.To the authors' knowledge,the nature of this study has not yet been done.

## 2. MATHEMATICAL MODEL

2.1. **Assumptions of the model.** The model rely on the following assumptions:

- The fish species can grow independently in a lake and their population sizes are bounded
- Adult Nile perch are predator to both young Nile tilapia and Small pelagic silver fish
- Adult Nile tilapia are predator to both young Nile perch and Small pelagic silver fish
- Small pelagic silver fishes are prey to both Nile tilapia and Nile perch
- All three fish species have ecological interactions in a lake
- All three fish species are harvested at a rate proportional to the size of their population
- Fishing effort for all fish species is kept constant

2.2. **Definitions of variables and parameters of the model.** The following are the definitions of variables and parameters used in developing the model:
$r_1$: intrinsic growth rate of Nile perch
$r_2$: intrinsic growth rate of Nile tilapia
$r_3$: intrinsic growth rate of small pelagic silver fish
$x$: stock biomass of Nile perch
$y$: stock biomass of Nile tilapia
$z$: stock biomass of small pelagic silver fish
$\alpha$: predation rate of Nile tilapia to Nile perch
$\beta$: predation rate of Nile perch to Nile tilapia
$\gamma$: predation rate of Nile perch to small pelagic silver fish
$\psi$: predation rate of Nile tilapia to small pelagic silver fish
$E_1$: fishing effort for Nile perch
$E_2$: fishing effort for Nile tilapia
$E_3$: fishing effort for small pelagic silver fish
$q_1$: catchability coefficient of the Nile perch
$q_2$: catchability coefficient of Nile tilapia

$q_3$: catchability coefficient of small pelagic silver fish
$K_1$: carrying capacity of Nile perch
$K_2$: carrying capacity of Nile tilapia
$K_3$: carrying capacity of small pelagic silver fish

2.3. **Model equations.** The model equations for the three fish species is the set of non-linear ordinary differential equations:

$$\frac{dx}{dt} = x\left(c_1 - a_1x - \rho y + \gamma z\right) \tag{2. 1}$$

$$\frac{dy}{dt} = y\left(c_2 + \rho x - a_2 y + \psi z\right) \tag{2. 2}$$

$$\frac{dz}{dt} = z\left(c_3 - \gamma x - \psi y - a_3 z\right) \tag{2. 3}$$

where $a_i = \dfrac{r_i}{K_i} > 0$ for $i = 1, 2, 3$ , $c_j = r_j - q_j E_j > 0$ for $j = 1, 2, 3$ and $(\alpha - \beta) = \rho > 0$

## 3. EXISTENCE OF EQUILIBRIUM POINTS

The system under investigation has eight possible equilibrium points obtained by setting $\dfrac{dx}{dt} = \dfrac{dy}{dt} = \dfrac{dz}{dt} = 0$.These includes:

(i) $E_1\left(x^*, y^*, z^*\right) = (0, 0, 0)$ (absence of all fish species)

(ii) $E_2\left(x^*, y^*, z^*\right) = \left(0, 0, \dfrac{c_3}{a_3}\right)$(absence of Nile perch and Nile tilapia)

(iii) $E_3\left(x^*, y^*, z^*\right) = \left(\dfrac{c_1}{a_1}, 0, 0\right)$(absence of Nile tilapia and small pelagic silver fish)

(iv) $E_4\left(x^*, y^*, z^*\right) = \left(0, \dfrac{c_2}{a_2}, 0\right)$(absence of Nile perch and small pelagic silver fish)

(v) $E_5\left(x^*, y^*, z^*\right) = \left(\dfrac{-c_2\rho + a_2c_1}{\rho^2 + a_1a_2}, \dfrac{a_1c_2 + c_1\rho}{\rho^2 + a_1a_2}, 0\right)$(absence of small pelagic silver fish). The equilibrium point $E_5$ is positive if $a_2c_1 > c_2\rho$ hold.

(vi) $E_6\left(x^*, y^*, z^*\right) = \left(\dfrac{a_3c_1 + c_3\gamma}{\gamma^2 + a_1a_3}, 0, -\dfrac{-a_1c_3 + c_1\gamma}{\gamma^2 + a_1a_3}\right)$(absence of Niletilapia).The equilibrium point $E_6$ is positive if $a_1c_3 > c_1\gamma$ hold.

(vii) $E_7\left(x^*, y^*, z^*\right) = \left(0, \dfrac{\psi c_3 + a_3c_2}{\psi^2 + a_2a_3}, \dfrac{a_2c_3 - c_2\psi}{\psi^2 + a_2a_3}\right)$(absence of Nileperch). The equilibrium point $E_7$ is positive if $a_2c_3 > c_2\psi$ hold.

(viii) $$E_8\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = \begin{pmatrix} \frac{-\rho\psi c_3 - a_3\rho c_2 - \gamma\psi c_2 + \gamma c_3 a_2 + c_1\psi^2 + a_2a_3c_1}{a_3\rho^2 + a_1\psi^2 + a_1a_2a_3 + \gamma^2 a_2} \\ \frac{a_1a_3c_2 + a_3c_1\rho + \rho\gamma c_3 + c_2\gamma^2 - \gamma\psi c_1 + \psi c_3 a_1}{a_3\rho^2 + a_1\psi^2 + a_1a_2a_3 + \gamma^2 a_2} \\ \frac{-(-\gamma\rho c_2 + \gamma c_1 a_2 + \psi c_2 a_1 + \psi\rho c_1 - c_3\rho^2 - c_3a_1a_2)}{a_3\rho^2 + a_1\psi^2 + a_1a_2a_3 + \gamma^2 a_2} \end{pmatrix}$$

(the co-existence of all three fish species).The equiliribrium point $E_8$ is positive provided the following inequalities holds:

$\gamma c_3 a_2 + c_1\psi^2 + a_2 a_3 c_1 > \rho\psi c_3 + a_3\rho c_2 + \gamma\psi c_2$,
$a_1 a_3 c_2 + a_3 c_1\rho + \rho\gamma c_3 + c_2\gamma^2 + \psi c_3 a_1 > \gamma\psi c_1$ and
$\gamma\rho c_2 + c_3\rho^2 + c_3 a_1 a_2 > \gamma c_1 a_2 + \psi c_2 a_1 + \psi\rho c_1$

3.1. **Local stability analysis.** The local stability of the interior equilibrium point (co-existence point $E_8$) was investigated using the Routh-Hurwitz's criteria. That is, an equilibrium point is locally asymptotically stable if the characteristic equation of the Jacobian matrix evaluated at that point has all the coefficients being positive and that all of its roots have negative real parts[8].

$$J(E_8) = \begin{pmatrix} c_1 - 2a_1x^* - \rho y^* + \gamma z^* & -\rho x^* & \gamma x^* \\ \rho y^* & c_2 + \rho x^* - 2a_2 y^* + \psi z^* & \psi y^* \\ -\gamma z^* & -\psi z^* & c_3 - \gamma x^* - \psi y^* - 2a_3 z^* \end{pmatrix}$$

The characteristic equation of $J(E_8)$ is

$$\lambda^3 - (b_1 + b_2 + b_3)\lambda^2 + (b_1b_2 + b_2b_3 + b_1b_3)\lambda - (b_1b_2b_3 + b_4) = 0 \tag{3. 4}$$

where
$b_1 = c_1 - 2a_1x^* - \rho y^* + \gamma z^* = -a_1x^* < 0$ since, $x^* > 0$ and
$c_1 - a_1x^* - \rho y^* + \gamma z^* = 0$
$b_2 = c_2 + \rho x^* - 2a_2y^* + \psi z^* = -a_2y^* < 0$ since, $y^* > 0$ and
$c_2 + \rho x^* - a_2y^* + \psi z^* = 0$
$b_3 = c_3 - \gamma x^* - \psi y^* - 2a_3z^* = -a_3z^* < 0$ since, $z^* > 0$ and
$c_3 - \gamma x^* - \psi y^* - a_3z^* = 0$
$b_4 = \psi^2 y^* z^* > 0$
Hence
$(b_1 + b_2 + b_3) < 0 \Rightarrow -(b_1 + b_2 + b_3) > 0$
Similary, $(b_1b_2 + b_2b_3 + b_1b_3) > 0$
And

$$\begin{aligned} b_1b_2b_3 + b_4 &= -a_1a_2a_3x^*y^*z^* + \psi^2 y^* z^* \\ &= y^*z^*(\psi^2 - a_1a_2a_3x^*) \end{aligned}$$

Certainly, $a_1a_2a_3x^* > \psi^2$
Hence, $b_1b_2b_3 + b_4 = y^*z^*\left(\psi^2 - a_1a_2a_3x^*\right) < 0 \Rightarrow -(b_1b_2b_3 + b_4) > 0$
The conditions for Routh-Hurwitz's criteria are satisfied, that is,
$-(b_1 + b_2 + b_3)(b_1b_2 + b_2b_3 + b_1b_3) > -(b_1b_2b_3 + b_4)$

Therefore, the co-existence equilibrium point $E_8$ is locally asymptotically stable.

3.2. **Global stability analysis.** Global stability of the system was analysed by considering suitable Lyapunov function [5].

Consider the following Lyapunov function candidate,

$$V(x, y, z) = l_1[x - x^* - x^* \ln(\frac{x}{x^*})] + l_2[y - y^* - y^* \ln(\frac{y}{y^*})] + l_3[z - z^* - z^* \ln(\frac{z}{z^*})] \tag{3. 5}$$

where $l_1, l_2, l_3 > 0$.

As per, [6], it is evident that the choosen Lyapunov function candidate $V(x, y, z)$ of ( 3. 5 ) satisfy the conditions that $V(x^*, y^*, z^*) = 0$ and $V(x, y, z) > 0$ for all $(x, y, z) \neq (x^*, y^*, z^*)$. Moreover, $V(x, y, z)$ is radially unbounded.

We are required to verify that $\frac{dV}{dt} \leq 0$ for the suitable choice of $l_1 > 0$,$l_2 > 0$ and $l_3 > 0$.

$$\begin{aligned}\frac{dV}{dt} &= l_1(1-\frac{x^*}{x})\frac{dx}{dt} + l_2(1-\frac{y^*}{y})\frac{dy}{dt} + l_3(1-\frac{z^*}{z})\frac{dz}{dt} \\ &= l_1(\frac{x-x^*}{x})\frac{dx}{dt} + l_2(\frac{y-y^*}{y})\frac{dy}{dt} + l_3(\frac{z-z^*}{z})\frac{dz}{dt}\end{aligned}$$

Let

$$\frac{dV}{dt} = F + G + H \tag{3. 6}$$

where,

$$\begin{aligned}F &= l_1(x-x^*)(c_1 - a_1x - \rho y + \gamma z) \\ G &= l_2(y-y^*)(c_2 + \rho x - a_2y + \psi z) \\ H &= l_3(z-z^*)(c_3 - \gamma x - \psi y - a_3z)\end{aligned}$$

Considering,

$$\begin{aligned}F &= l_1(x-x^*)(c_1 - a_1x - \rho y + \gamma z) \\ &= l_1[x(c_1 - a_1x - \rho y + \gamma z) - x^*(c_1 - a_1x - \rho y + \gamma z)] \\ &= l_1[c_1x - a_1x^2 - \rho xy + \gamma xz - c_1x^* + a_1xx^* + \rho yx^* - \gamma zx^*]\end{aligned}$$

Since, $c_1 - a_1x^* - \rho y^* + \gamma z^* = 0$, $\Rightarrow c_1 = a_1x^* + \rho y^* - \gamma z^*$

Then

$$\begin{aligned}F &= l_1[x(a_1x^* + \rho y^* - \gamma z^*) - a_1x^2 - \rho xy + \gamma xz \\ &- x^*(a_1x^* + \rho y^* - \gamma z^*) + a_1xx^* + \rho yx^* - \gamma zx^*] \\ &= l_1[-a_1(x^2 - 2xx^* + {x^*}^2) - \rho[y(x-x^*) - y^*(x-x^*)] \\ &+ \gamma[-z^*(x-x^*) + z(x-x^*)]]\end{aligned}$$

Further algebraic simplification gives,

$$F = l_1(x-x^*)\left[-a_1(x-x^*) - \rho(y-y^*) + \gamma(z-z^*)\right] \tag{3. 7}$$

Considering,

$$\begin{aligned}G &= l_2(y-y^*)(c_2 + \rho x - a_2y + \psi z) \\ &= l_2\left[y(c_2 + \rho x - a_2y + \psi z) - y^*(c_2 + \rho x - a_2y + \psi z)\right] \\ &= l_2\left(c_2y + \rho xy - a_2y^2 + \psi yz - c_2y^* - \rho xy^* + a_2yy^* - \psi y^*z\right)\end{aligned}$$

Since, $c_2 + \rho x^* - a_2y^* + \psi z^* = 0$, $\Rightarrow c_2 = -\rho x^* + a_2y^* - \psi z^*$

Then

$$\begin{aligned}G &= l_2[y(-\rho x^* + a_2y^* - \psi z^*) + \rho xy - a_2y^2 + \psi yz \\ &- y^*(-\rho x^* + a_2y^* - \psi z^*) - \rho xy^* + a_2yy^* - \psi y^*z] \\ &= l_2[-a_2(y-y^*)^2 + \rho[-x^*(y-y^*) + x(y-y^*)] \\ &+ \psi[-z^*(y-y^*) + z(y-y^*)]]\end{aligned}$$

Further algebraic simplification gives,

$$G = l_2(y - y^*)\left[\rho(x - x^*) - a_2(y - y^*) + \psi(z - z^*)\right] \tag{3. 8}$$

Considering,

$$\begin{aligned} H &= l_3(z - z^*)(c_3 - \gamma x - \psi y - a_3 z) \\ &= l_3\left[z(c_3 - \gamma x - \psi y - a_3 z) - z^*(c_3 - \gamma x - \psi y - a_3 z)\right] \\ &= l_3\left[zc_3 - \gamma xz - \psi yz - a_3 z^2 - c_3 z^* + \gamma xz^* + \psi yz^* + a_3 zz^*\right] \end{aligned}$$

Since, $c_3 - \gamma x^* - \psi y^* - a_3 z^* = 0$, $\Rightarrow c_3 = \gamma x^* + \psi y^* + a_3 z^*$
Then

$$\begin{aligned} H &= l_3[z(\gamma x^* + \psi y^* + a_3 z^*) - \gamma xz - \psi yz - a_3 z^2 \\ &- z^*(\gamma x^* + \psi y^* + a_3 z^*) + \gamma xz^* + \psi yz^* + a_3 zz^*] \\ &= l_3[-a_3(z - z^*)^2 - \gamma(x - x^*)(z - z^*) \\ &- \psi(y - y^*)(z - z^*)] \end{aligned}$$

Further algebraic simplification gives

$$H = l_3(z - z^*)\left[-\gamma(x - x^*) - \psi(y - y^*) - a_3(z - z^*)\right] \tag{3. 9}$$

Substituting ( 3. 7 ),( 3. 8 ) and ( 3. 9 ) into ( 3. 6 ) gives,

$$\frac{dV}{dt} = l_1 X(-a_1 X - \rho Y + \gamma Z) + l_2 Y(\rho X - a_2 Y + \psi Z) + l_3 Z(-\gamma X - \psi Y - a_3 Z) \tag{3. 10}$$

where, $X = (x - x^*)$, $Y = (y - y^*)$ and $Z = (z - z^*)$

Further simplification of ( 3. 10 ) gives the following:

$$\frac{dV}{dt} = -\left[l_1 a_1 X^2 + l_2 a_2 Y^2 + l_3 a_3 Z^2\right] + \left[\rho(l_2 - l_1)XY + \gamma(l_1 - l_3)XZ + \psi(l_2 - l_3)YZ\right] \tag{3. 11}$$

If $(X, Y, Z) = (0, 0, 0)$, that is, when $[x = x^*,\ y = y^*$ and $z = z^*]$ then $\frac{dV}{dt} = 0$.
And, if $l_1 = l_2 = l_3$, then $\frac{dV}{dt} < 0$, for all $(x^*, y^*, z^*) \neq (x, y, z)$ in $\mathbb{R}^3$.
Therefore, the co-existence equilibrium point $E_8$ is globally asymptotically stable.

## 4. NUMERICAL EXAMPLES

Numerical examples and their graphical illustrations are summarized below in eight different cases. Fourth order Runge-Kutta integration algorithm was used for some cases to validate the qualitative analysis results.

TABLE 1. Parameters for figure $1a$ and figure $1b$ of co-existence equilibrium point $E_8$

| Parameters | Figure 1a | Figure 1b |
|---|---|---|
| $a_1$ | 0.0125 | 0.003 |
| $a_2$ | 0.07 | 0.055 |
| $a_3$ | 0.01 | 0.001 |
| $\rho$ | 0.04 | 0.02 |
| $\gamma$ | 0.005 | 0.001 |
| $\psi$ | 0.005 | 0.001 |
| $c_1$ | 0.95 | 0.91 |
| $c_2$ | 0.50 | 0.70 |
| $c_3$ | 0.40 | 0.21 |
| $(x^*, y^*, z^*)$ | (20,20,20) | (70,40,100) |

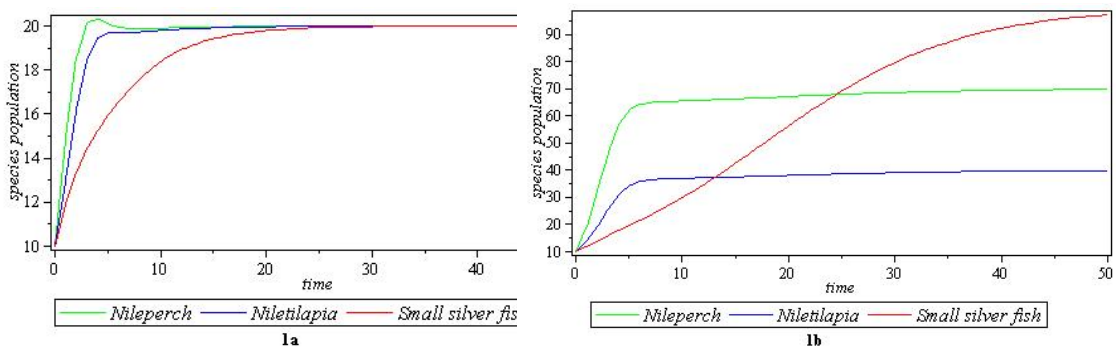


FIGURE 1. Graphical representations of the parameters in Table 1

4.1. **Examples 1&2 with $x(0) = 10$, $y(0) = 10$ and $z(0) = 10$ for both cases.**

TABLE 2. Parameters for figure $2a$ and figure $2b$ of co-existence equilibrium point $E_8$

| Parameters | Figure 2a | Figure 2b |
|---|---|---|
| $a_1$ | 0.0015 | 0.002 |
| $a_2$ | 0.005 | 0.0035 |
| $a_3$ | 0.001 | 0.0005 |
| $\rho$ | 0.002 | 0.0002 |
| $\gamma$ | 0.001 | 0.0005 |
| $\psi$ | 0.001 | 0.0005 |
| $c_1$ | 0.75 | 0.22 |
| $c_2$ | 0.60 | 0.22 |
| $c_3$ | 0.90 | 0.225 |
| $(x^*, y^*, z^*)$ | (300,300,300) | (150,100,200) |

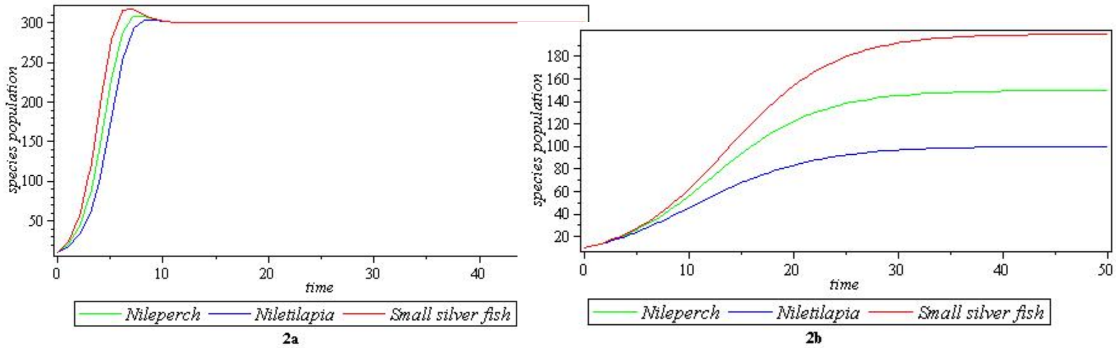


FIGURE 2. Graphical representations of the parameters in Table 2

4.2. **Examples** 3&4 **with** $x(0) = 10$**,** $y(0) = 10$ **and** $z(0) = 10$ **for both cases.**

TABLE 3. Parameters for figure 3$a$ and figure 3$b$ of co-existence equilibrium point $E_8$

| Parameters | Figure 3a | Figure 3b |
|---|---|---|
| $a_1$ | 0.0015 | 0.0008 |
| $a_2$ | 0.003 | 0.001 |
| $a_3$ | 0.004 | 0.0005 |
| $\rho$ | 0.002 | 0.0003 |
| $\gamma$ | 0.0035 | 0.0004 |
| $\psi$ | 0.001 | 0.0004 |
| $c_1$ | 0 | 0.42 |
| $c_2$ | 0 | 0.18 |
| $c_3$ | 0.85 | 0.78 |
| $(x^*, y^*, z^*)$ | (100,100,100) | (600,600,600) |

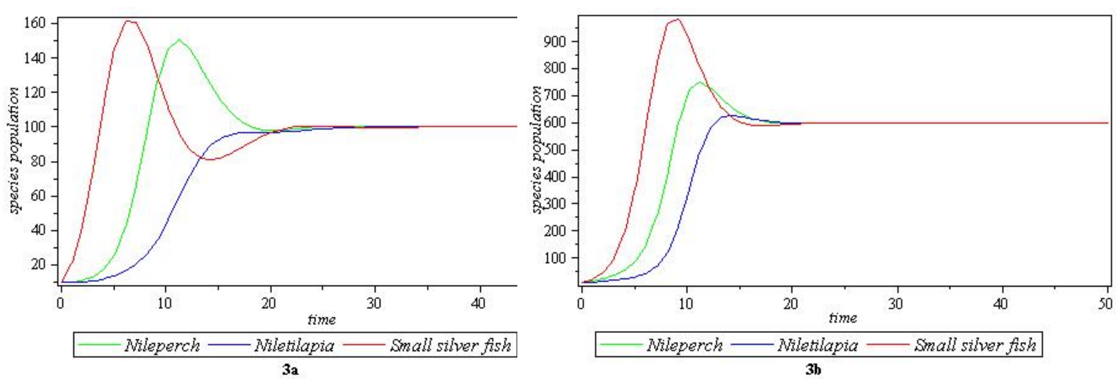


FIGURE 3. Graphical representations of the parameters in Table 3

4.3. **Examples 5&6 with $x(0) = 10$, $y(0) = 10$ and $z(0) = 10$ for both cases.**

Table 4. Parameters for figure 4$a$ and figure 4$b$ of co-existence equilibrium point $E_8$

| Parameters | Figure 4a | Figure 4b |
|---|---|---|
| $a_1$ | 0.000013 | 0.00000024 |
| $a_2$ | 0.000015 | 0.00000022 |
| $a_3$ | 0.000004 | 0.00000008 |
| $\rho$ | 0.000001 | 0.00000004 |
| $\gamma$ | 0.000004 | 0.00000008 |
| $\psi$ | 0.000004 | 0.00000008 |
| $c_1$ | 0.10 | 0.020 |
| $c_2$ | 0.10 | 0.010 |
| $c_3$ | 0.12 | 0.024 |
| $(x^*, y^*, z^*)$ | (10000,10000,10000) | (100000,100000,100000) |

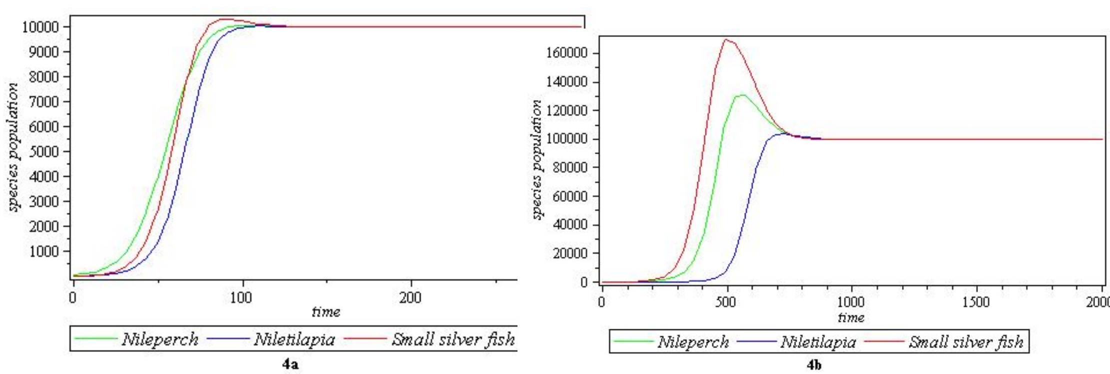


Figure 4. Graphical representations of the parameters in Table 4

4.4. **Examples 7&8 with $x(0) = 50$, $y(0) = 10$, $z(0) = 10$ and $x(0) = 10$, $y(0) = 10$, $z(0) = 10$ respectively.**

Table 5. Runge-Kutta numerical results for the parameters of figure 1*b*

| t | x(t) | y(t) | z(t) |
|---|---|---|---|
| 0 | 10 | 10 | 10 |
| 1 | 18.9543128777437318 | 14.0106457936006752 | 11.8888854688601402 |
| 2 | 31.8253617861179486 | 19.1105969078411135 | 13.8895900677599720 |
| 3 | 45.8794471255417450 | 25.2224489680994033 | 15.8820586780104948 |
| 4 | 56.5536805700466730 | 30.8599451390882572 | 17.7891887400829454 |
| 5 | 62.0946907412422107 | 34.4053377263977680 | 19.6360900260104644 |
| 10 | 65.5434280496591840 | 37.0591533853929107 | 29.9107276797517088 |
| 15 | 66.3036335404902531 | 37.5597601427863666 | 42.5936351248714118 |
| 20 | 67.1529935901520077 | 38.1175831389633544 | 56.3495160940938647 |
| 25 | 67.9670887224138056 | 38.6538868920041594 | 69.2085621375481992 |
| 30 | 68.6434584860992204 | 39.1006676479843307 | 79.6580758797434784 |
| 35 | 69.1416085841688926 | 39.4304184865951300 | 87.2220350799413211 |
| 40 | 69.4767925880774584 | 39.6526229458336062 | 92.2496849929260350 |
| 45 | 69.6888002622855112 | 39.7933057889316332 | 95.4044112768102935 |
| 50 | 69.8176894863002106 | 39.8788847025348758 | 97.3128096721919320 |
| 55 | 69.8941666570641332 | 39.9296817602021932 | 98.4417897148602919 |
| 60 | 69.9388918847710954 | 39.9593951068871366 | 99.1008750069828039 |
| 65 | 69.9648264987910836 | 39.9766269819894476 | 99.4826635148563128 |
| 70 | 69.9797909566821374 | 39.9865706090160842 | 99.7028273461452984 |
| 80 | 69.9933466703957378 | 39.9955785990514202 | 99.9021824887235254 |
| 90 | 69.9978128089895222 | 39.9985465136219318 | 99.9678459183538876 |
| 100 | 69.9992813426285636 | 39.9995224187655580 | 99.9894351820121728 |
| 150 | 69.9999972506609538 | 39.9999981729340988 | 99.999959583014500 |
| 200 | 69.9999999894831860 | 39.9999999930110804 | 99.999999845396316 |
| 250 | 69.9999999999597690 | 39.9999999999732622 | 99.999999999408586 |
| 300 | 69.9999999999998438 | 39.9999999999999006 | 99.999999999997726 |
| 400 | 70 | 39.9999999999999930 | 99.999999999999914 |
| 450 | 70 | 39.9999999999999930 | 99.999999999999914 |
| 600 | 70 | 39.9999999999999930 | 99.999999999999914 |
| 1000 | 70 | 39.9999999999999930 | 99.999999999999914 |

Table 6. Runge-Kutta numerical results for the parameters of figure $3a$

| t | x(t) | y(t) | z(t) |
|---|---|---|---|
| 0 | 10 | 10 | 10 |
| 1 | 10.1714712848859499 | 10.0495808974886156 | 21.0475841767081917 |
| 2 | 10.9124848761110318 | 10.2618637589493673 | 41.5883314435775162 |
| 3 | 12.8068331298185180 | 10.7743506720148492 | 73.6303030796512418 |
| 4 | 16.9436588893086012 | 11.7682289368287966 | 111.718258774337031 |
| 5 | 25.1301967290895085 | 13.4320617894971832 | 143.545537317488026 |
| 6 | 39.7909103204179218 | 15.9707016143782462 | 160.325630375715463 |
| 7 | 62.7767725920558064 | 19.7043297675483480 | 161.422834286784877 |
| 8 | 92.5591664654720746 | 25.1648483051793406 | 150.505268930363457 |
| 9 | 122.198951348776703 | 32.9975177648245364 | 132.918965238387841 |
| 10 | 142.847672798411224 | 43.5068883702286514 | 114.442807944181226 |
| 11 | 150.446114008380448 | 56.0273245279359600 | 99.2378765786435224 |
| 12 | 147.244649778317012 | 68.7981196794774804 | 88.8557971126201808 |
| 13 | 138.139423792913192 | 79.7804871866895270 | 83.0599633098661769 |
| 14 | 127.376912148014682 | 87.7486513562281090 | 80.9498704441656970 |
| 15 | 117.510847795193698 | 92.6528941295738804 | 81.5358933618375518 |
| 20 | 97.9345442737386236 | 96.8955070791911482 | 96.6979266202355916 |
| 25 | 100.225511128516487 | 98.8900132755399568 | 100.442281878854089 |
| 30 | 100.324803966953454 | 99.9766952411014672 | 99.7642768126507634 |
| 35 | 99.9627565982999756 | 99.9830093531780762 | 99.9406334539491184 |
| 40 | 99.9951714825362216 | 99.9847286077376226 | 100.012627291336770 |
| 45 | 100.005620192871206 | 100.000188625448374 | 99.9980692966264968 |
| 50 | 99.9995595698490406 | 100.000146478656504 | 99.9988396803452418 |
| 60 | 100.000092728494863 | 100.000000220496672 | 100.000000729379607 |
| 70 | 99.999994637777632 | 99.999996752820820 | 100.000003484676597 |
| 100 | 99.999999997910676 | 99.999999999550694 | 100.000000000103512 |
| 200 | 99.999999999999970 | 100 | 99.999999999999986 |
| 300 | 99.999999999999970 | 100 | 99.999999999999986 |
| 400 | 99.999999999999970 | 100 | 99.999999999999986 |
| 500 | 99.999999999999970 | 100 | 99.999999999999986 |

### 4.5. **The results of fourth order Runge-Kutta method.**

## 5. CONCLUSION

With reference to Tables 1, 2, 3 and 4, and Figures 1, 2, 3 and 4, as the growth rate of fish species approaches to their harvesting rate and as all other parameters involved in a model became relatively small,the population peaks (maximum population before attaining the steady state) and the equilibrium population level for each fish species increases.The fourth order Runge-Kutta numerical solutions matched with the analytical solutions of the model.

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