

## An Analytical Study of Strong Non-Planar Shock Waves in Gas-Particle-Mixture

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**Abstract.** In present paper an analytical approach is used to derive a new exact solution of a problem of one dimensional adiabatic flow of a non planar and cylindrical strong shock wave propagating in a mixture of gas and particle when density ahead of shock front is assume to vary as power of the distance from the source of explosion. A complete investigation is made for non - planer case and for cylindrical case an expression for total energy is obtained also.

### 1. Introduction

The investigation of shock wave phenomenon in a dusty gas is important in many engineering and astrophysical problems. Assuming solid particles to behave as a pseudo - fluid Pai<sup>5</sup> has derived appropriate basic equations for gas particle mixture. Using Lie Group method Jena and Sharma<sup>1</sup> have obtained entire class of self similar solution in dusty gases. Work done by authors Vishwakarma<sup>11</sup>, Vishwakarma and Nath<sup>12</sup>, Vishwakarma and Vishwakarma<sup>13</sup>, Pandey and Shukla<sup>7</sup> and Pandey and vaish<sup>8</sup> can also be cited in this context. Such type of study has numerous applications in underground explosions also. In this context work of Mcqueen<sup>3</sup>, Nagayama<sup>4</sup> and Lamb<sup>2</sup> et-al can be cited. Sing and Husain<sup>10</sup> has investigated analytical study of strong non planer shock in magnetogasdynamics. Ray<sup>9</sup> has obtained an exact solution of a Spherical Blast wave under terrestrial conditions. Pandey and Singh<sup>6</sup> have studied non self similar solution of shock -wave in gas particle mixture. In present paper an analytical approach is used to derive a new exact solution of a problem of one dimensional adiabatic flow of a non planar and cylindrical strong shock wave propagating in a mixture of gas and particle when density ahead of shock front is assumed to vary as power of the distance from the source of explosion. A complete investigation is made for non-planar case and an expression for the total energy is obtained for spherical case also.

### 2. Basic Equations

Basic equation describing a planar, cylindrical and spherical motion for two phase flow of gas-particle mixture is given by Pai<sup>5</sup>

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \alpha \frac{\rho u}{r} = 0, \quad (2.1)$$

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$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \quad (2.2)$$

$$\frac{\partial E}{\partial t} + u \frac{\partial E}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \quad (2.3)$$

where  $\alpha = 0, 1$  and  $2$  for planer, cylindrical, spherical symmetrical cases respectively and  $u, p, \rho$  and  $E$  are resultant velocity, pressure, density and internal energy per unit mass of the mixture.

Here  $p = p_p + p_g$ ,  $p_p$  and  $p_g$  being partial pressure of solid particles and gas respectively,

$$p_g = (1-z)p \quad \text{and} \quad p_p = zp,$$

$z$  being particle volume fraction.

$$\rho = z\rho_{sp} + (1-z)\rho_g,$$

$\rho_{sp}$  and  $\rho_g$  being species density of solid particles and partial density of gas,  $r$  is the single spatial co-ordinate being either axial in flows with planer geometry, or radial in cylindrically symmetric flows.

The internal energy per unit mass is given by –

$$E = \frac{(1-z)p}{(\Gamma-1)\rho}, \quad (2.4)$$

where

$$z = \frac{K_p}{(1-K_p)G + K_p}, \quad (2.5)$$

$$G = \frac{\rho_{sp}}{\rho_g},$$

$K_p$  being mass concentration of pseudo-fluid of solid particles and is defined as

$$K_p = \frac{\bar{\rho}_p}{\rho} = \frac{z\rho_{sp}}{\rho}. \quad (2.6)$$

In equilibrium flow  $K_p$  is a constant in the whole flow field, therefore, from equation (2.6)

$$\frac{z}{\rho} = \text{Constant}. \quad (2.7)$$

Thus equation of state becomes

$$p = \frac{(1-K_p)}{(1-z)} \rho RT = \frac{\rho R_M T}{(1-z)} \quad (2.8)$$

where  $R_M = (1-K_p)R$ , and  $R_M$  may be considered as an effective gas constant of the mixture.

Here we have followed the analysis of two phase flows considered by *Pai*<sup>5</sup> in which  $p$  is total pressure,  $z$  is the particle volume fraction and  $\Gamma$  is the ratio of specific heat defined by,

$$\Gamma = \frac{c_{pM}}{c_{VM}} = \gamma \frac{\left(1 + \frac{\delta\beta}{\gamma}\right)}{1 + \delta\beta} \quad (2.9)$$

$$\gamma = \frac{c_p}{c_v}, \quad \delta = \frac{K_p}{(1 - K_p)}, \quad \beta = \frac{c_{sp}}{c_v} \quad . \quad (2.10)$$

We can write

$$\frac{\beta}{\gamma} = \frac{\frac{c_{sp}}{c_v}}{\frac{c_p}{c_v}} = \frac{c_{sp}}{c_p}.$$

Let

$$\frac{c_{sp}}{c_p} = \mu,$$

then

$$\frac{\beta}{\gamma} = \mu.$$

Thus equation (2.9) becomes

$$\Gamma = \frac{\gamma(1 + \delta\mu)}{(1 + \delta\mu\gamma)}, \quad (2.11)$$

where  $c_p$  and  $c_v$  being specific heat at constant pressure and constant volume and

$$c_{sp} = c_s + c_{vp}, \quad (2.12)$$

$c_{vp}$  being effective heat at constant volume for solid particle and  $c_s$  is the specific heat of solid particles due to internal degree of freedom.

Substituting value of  $E$  from equation (2.4) in equation (2.3) we get

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} - \frac{\Gamma p}{\rho(1-z)} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0. \quad (2.13)$$

Let,  $r = \chi(t)$  be the strong shock with the shock speed  $U = \frac{d\chi}{dt}$  propagating in to the medium characterized by

$$\rho = \rho_0(r), u = 0, p = p_0(r). \quad (2.14)$$

The conservation relations for mass, momentum and energy in present problem can be written as

$$\rho_0 = \rho_1(U - u) , \quad (2.15)$$

$$p_0 + \rho_0 U^2 = p_1 + \rho_1(U - u)^2 , \quad (2.16)$$

$$\frac{p_0}{\rho_0} + \frac{p_0(1 - z_0)}{\rho_0(\Gamma - 1)} + \frac{U^2}{2} = \frac{p_1}{\rho_1} + \frac{p_1(1 - z_1)}{\rho_1(\Gamma - 1)} + \frac{1}{2}(U - u)^2 , \quad (2.17)$$

$$\frac{z_0}{\rho_0} = \frac{z}{\rho} . \quad (2.18)$$

From equation (2.15) to (2.18), we get

$$u = \frac{2(1 - z_0)U^2}{\Gamma + 1} \left( 1 - \frac{\Lambda_0^2}{U^2} \right) , \quad (2.19)$$

$$p = \frac{2(1 - z_0)\rho_0 U^2}{\Gamma + 1} \left[ 1 - \frac{\Gamma - 1}{2\Gamma} \frac{\Lambda_0^2}{U^2} \right] , \quad (2.20)$$

$$\rho = \frac{\frac{\rho_0(\Gamma + 1)}{\Gamma - 1 + 2z_0}}{1 + \frac{2}{\Gamma - 1 + 2z_0} \left( \frac{\Lambda_0}{U} \right)^2 (1 - z_0)} . \quad (2.21)$$

where

$$z_0 = \frac{K_p}{(1 - K_p)G_0 + K_p} \quad (2.22)$$

and  $G_0$  is the ratio of density of the solid particle to the initial density of the gas,  $p_0$  and  $\rho_0$  are respectively the pressure and density of the undisturbed atmosphere and  $\Lambda_0$  is the sound velocity, given by  $\Lambda_0^2 = \frac{\Gamma p_0}{\rho_0(1 - z_0)}$ .

Since the initial energy input  $E_0$  of explosion is very large, the shock speed  $U \gg \Lambda_0$  so that  $\frac{\Lambda_0}{U} \rightarrow 0$  in case of strong shock limit.

Thus we have following strong shock conditions

$$\rho = \frac{\Gamma + 1}{\Gamma - 1 + 2z_0} \rho_0 , \quad (2.23)$$

$$u = \frac{2(1 - z_0)}{\Gamma + 1} U , \quad (2.24)$$

$$p = \frac{2(1 - z_0)}{\Gamma + 1} \rho_0 U^2 . \quad (2.25)$$

Assuming that  $\rho_0$  varies as inverse power of radial distance from source of explosion, we have

$$\rho_0 = \rho_c \chi^{-\delta} \quad (2.26)$$

where

$\delta, \rho_c$  are constants.

The total energy in this case is given by -

$$E_r = \frac{4\pi}{(1+K_p)(1-z_0)} \int_0^r \left[ \frac{1}{2} \rho u^2 + \frac{P}{\Gamma+1} \right] r^\alpha dr. \quad (2.27)$$

### 3. Analytical solution

With the help of equation (2.23) and equation (2.24) equation (2.25) becomes

$$p = \frac{1}{2} \left( \frac{\Gamma-1+2z_0}{1-z_0} \right) \rho u^2. \quad (3.1)$$

Differentiating (3.1) with respect to r and then putting value in equation (2.2) we get

$$u_{,t} + uu_{,r} + \frac{1}{\rho} \left[ \frac{\Gamma-1+2z_0}{2(1-z_0)} \right] \left[ u^2 \frac{\partial \rho}{\partial r} + 2u\rho \frac{\partial u}{\partial r} \right] = 0. \quad (3.2)$$

With help of equations (2.18) , (2.23) , (2.4) and (3.1) equation (2.3) reduces into

$$u_{,t} + uu_{,r} + \left( \frac{\Gamma-1+2z_0}{2(1-z_0)} \right) \left( u \frac{\partial u}{\partial r} + \alpha \frac{u^2}{r} \right) = 0. \quad (3.3)$$

Subtracting equation (3.2) from (3.3) and integrating it with respect to r, we get

$$\rho u r^{-\alpha} = \xi(t), \quad (3.4)$$

where  $\xi(t)$  is an arbitrary function of integration.

Differentiating equation (3.4) with respect t and r then using equation (2.1) we get

$$u_{,t} - 2\alpha \frac{u^2}{r} - \frac{u}{\xi} \frac{\partial \xi}{\partial t} = 0. \quad (3.5)$$

Subtracting equation (3.5) from equation (3.3) and after simplification , we have following linear differential equation

$$u_{,r} + \left( \frac{\Gamma+3-2z_0}{\Gamma+1} \right) \alpha \frac{u}{r} = \left( \frac{-2(1-z_0)}{\Gamma+1} \right) \frac{1}{\xi} \xi_{,t}. \quad (3.5a)$$

On solving this equation we get

$$u = -\eta \frac{r}{\xi} \frac{\partial \xi}{\partial t}, \quad (3.6)$$

where

$$\eta = \frac{2(1-z_0)}{(\Gamma+1) + (\Gamma+3) - 2z_0\alpha}.$$

Differentiating equation (3.6) with respect to  $t$  and  $r$  and putting values of  $\frac{\partial u}{\partial t}, u^2, u$  we get

$$\frac{\partial^2 \xi}{\partial t^2} - \frac{2k}{\xi} \left( \frac{\partial \xi}{\partial t} \right)^2 = 0. \quad (3.6a)$$

This is a second order linear differential equation whose solution is given by

$$\xi = \frac{\xi_0}{t^\lambda}, \quad (3.7)$$

where

$$\lambda = \frac{(\Gamma + 1) + (\Gamma + 3) - 2z_0\alpha}{(\Gamma + 1) + (\Gamma - 1)\alpha + 2z_0\alpha}.$$

Using the jump condition (2.23) to (2.25) and equations (3.6) to (3.8) and equation (2.26) we have following relations

$$r = t^{\frac{\Gamma+1}{2\omega}}, \quad (3.8)$$

$$\omega = \frac{(\Gamma + 1) + (\Gamma - 1)\alpha + 2z_0\alpha}{2}.$$

$$-\delta = \frac{(\Gamma - 3)\alpha - (\Gamma + 1) - 4\alpha z_0}{\Gamma + 1}. \quad (3.9)$$

$$u = \eta \lambda r t^{-1} = (1 - z_0) \omega^{-1} r t^{-1}$$

$$\rho = \frac{\xi_0 \omega r^{\alpha-1} t^{1-\lambda}}{1 - z_0} \quad (3.10)$$

$$p = \left( \frac{\Gamma - 1 + 2z_0}{2} \right) \xi_0 \omega^{-1} r^{\alpha+1} t^{-(1-\lambda)}$$

Using equation (2.27) and equation (3.10) the analytical expression for the total energy is given by

$$E = \frac{2\pi[2(\Gamma - 1) - z_0(\Gamma - 3)] \xi_0}{(1 + K_p)(1 - z_0)(\Gamma - 1)} \frac{r^{2\alpha-2}}{\omega (2\alpha - 2)} \quad (3.11)$$

#### 4. Result and discussion

In present article following *Singh and Husian*<sup>10</sup>, we have obtained a new exact solution of one dimensional adiabatic flow of non planer strong shock wave propagating in a mixture of gas and particle when density ahead of shock front is assumed to vary as power of the distance from the source of explosion. Variation of pressure, velocity and density is given

through figure (1,2,3) for  $z = .024, \Gamma = .97, \omega = 1.97$ . Variation of energy is given by figure (4).

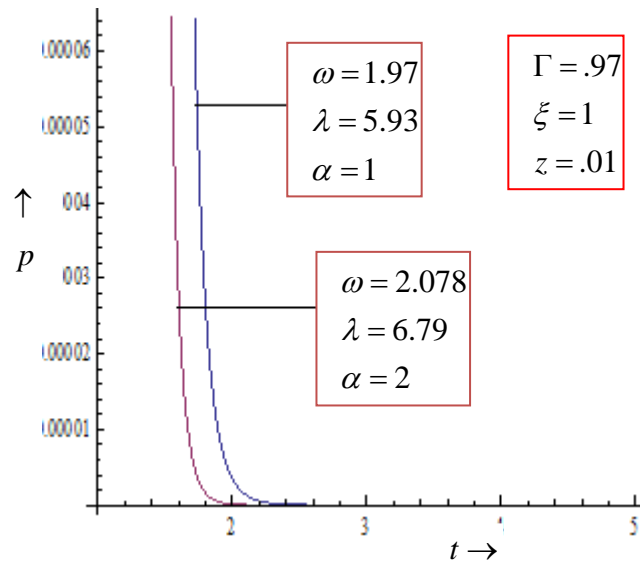


Fig1- variation of pressure for cylindrical and spherical symmetry.

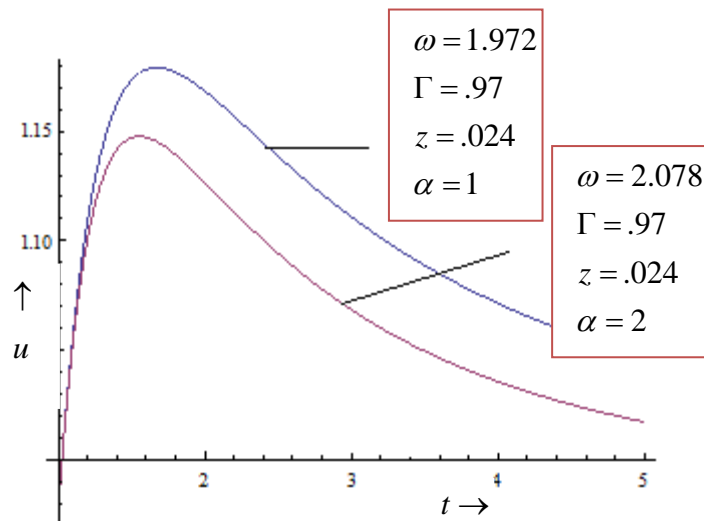


Fig2- variation of velocity for cylindrical and spherical symmetry.

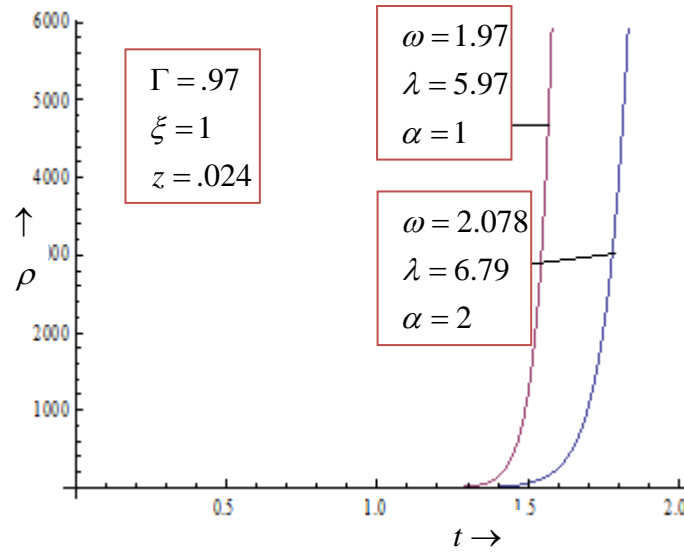


Fig3- variation of density for cylindrical and spherical symmetry.

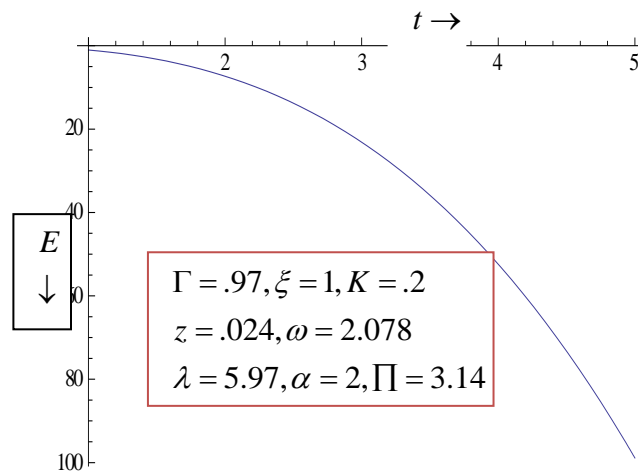


Fig4- variation of energy for spherical symmetry.

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