SUBSTITUTING GOMORY CUTTING PLANE METHOD TOWARDS BALAS ALGORITHM FOR SOLVING BINARY LINEAR PROGRAMMING

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Abstract. Application of binary linear programming can be found in many fields such as scheduling, production planning, networking and etc. Unlike linear programming problems, binary integer linear programming problem is more difficult to solve since its variables must be a binary number zero or one. Thus, solving this problem efficiently becomes an interesting issue. This paper proposes a combination of Balas additive method and Gomory cutting plane method. Balas additive method is a well-known method in binary linear programming. It systematically enumerates subset of the possible binary solution to ensure all possible solutions are examined. In this paper, a drawback of this method which is requiring long iterations is fixed. It uses the Gomory cutting plane method to eliminate non-integer partial solution. The Gomory cutting plane method generates an additional constraint to eliminate a non-integer solution. In general, this additional constraint increases the problem complexity. Therefore, combining Balas algorithm and Gomory cutting plane method can solve binary linear problem faster without facing more complex mathematical model. Substituting Gomory cutting plane to the Balas algorithm is conducted by performing the cut based on condition in Gomory algorithm towards the enumeration process conducted by Balas algorithm. The computational result using a test function proves that the proposed substituting strategy can solve binary linear programming efficiently which is shown by the number of iterations required by this method.

1. Introduction

Linear programming (LP) has been applied in many different fields. One of application of linear programming can be founded in a problem requiring a "yes-no" decision. Herein, a binary zero-one LP can be applied to solve this kind of problem. Due to its importance, many algorithms have been proposed to solve a zero-one LP. One of them is Balas algorithm proposed by Balas [1]. This method has been proven can solve zero-one LP well, yet it requires a long iterations[2]. On the other hand, A Gomory cutting plane method was proposed for solving an integer LP[3]. This method does not require too long iterations as

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proceed by Balas algorithm. The approach taken in the Cutting Plane technique is to create an additional constraint that cuts feasible space of the linear programming relaxation to eliminate a solution that is not integer. But this method has the disadvantage such a solution obtained was shaped shards or fractional integer which does not require complex simplex iteration[4]. Unfortunately, this method does not obtain an absolute zero-one solution.

This paper aims to overcome the drawback of Balas algorithm by embedding it with Gomory cutting plane method. The main concept of the proposed method is eliminating the backtracking procedure in Balas algorithm and replacing it with Gomory cutting plane.

The rest of this paper is organized as follows. Section 2 briefly reviews the literature study of Balas algorithm and Gomory cutting plane method. In Section 3, the proposed combination of Balas and Gomory algorithm is discussed. Furthermore, two numerical examples are given in Section 4. Finally, a concluding remark is given in Section 5.

2. Literature Review

This section discusses some basic theories of linear programming, Balas algorithm and Gomory cutting plane which are applied in this paper.

2.1 Binary linear programming

Linear programming is a tool for solving optimization problem. It aims to minimize or maximize the objective function with respect to some constraints. In the real-world problem, optimization problems usually involves a "yes-no" decision. These problems are modeled as binary linear programming or zero-one linear programming. The general form of binary linear programming model is as follows.

Objective function:

Maximize or Minimize

$$f(x) = \mathbf{C}^T \mathbf{X} \qquad\qquad (1)$$

Subject to:

$$\mathbf{A}\mathbf{X} + \mathbf{Y} = \mathbf{B} \qquad\qquad (2)$$

$$x_i \in \{0,1\} \qquad\qquad (3)$$

$$\mathbf{Y} \geq 0 \qquad\qquad (4)$$

where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \geq 0$, $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$, and $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

2.2   Balas Algorithm

Balas algorithm was proposed by Egon Balas [1]. This method works by successively assigning the value 1 to a certain variables. Notations used in Balas algorithm are defined as follows

J_k = Set of decision variables at iteration k. It is a   partial solution at iteration k.

R = Set of decision variables in objective function

$\bar{J}_k$ = Set of decision variables which are not included in J_k. All decision variables are assumed the be zero unless it has been defined as one in advance. $\bar{J}_k = R - J_k$.

P_k = Set of decision variables which can repair the final solution.

I_s^k = Number of violated constraints when variable $x_s = 1$ at iteration k.

f_k = Objective value at iteration k.

f^* = Final objective value

y_j^k = Slack variable at iteration k.

The Balas algorithm is as follows.

STEP 1. Initialization.

Set $k = 0$, $J_0 = \emptyset$, $Y^{(k)} = B$ with at less one component from $B < 0$, $f^* = \infty$, $f_0 = 0$, $R = \{x_1, x_2, \dots, x_n\}$ and $\bar{J}_0 = R$.

STEP 2. Determine variable in $\bar{J}_k$ which can be moved to J_k.

Choose one set of decision variables in P_k which has objective value better than J_k. Eliminate variables which have negative value.

$$y_j^{k+1} = y_j^k - a_{js}, \; y_j^k < 0 \qquad\qquad (5)$$

Variable y_j^k is feasible if $a_{js} < 0$. The $x_s \in \bar{J}_k$ with $a_{js} \geq 0$ can be eliminated and inserted to the set N_k. Eliminate all variables causing the current objective value f_k higher than f^*. Let the objective function:

$$f_k = \sum_{j \in J_k} c_j x_j \qquad\qquad (6)$$

If $x_s \in \bar{J}_k$, is set as 1, the objective function becomes $(f_k + c_s)$. Thus x_s which makes the objective value greater than f^* cannot be included in solution. Let $M_k = \{x_s | x_s \in L, \ L = f_k + c_s \geq f^*\}$, then:

$$P_k = J_k - (N_k \cup M_k) = R - J_k - (N_k \cup M_k) \qquad (7)$$

If $P_k = \emptyset$, then go to STEP 4. Otherwise, go to the next step.

Given constraints:

$$a_{j1}x_1 + a_{j2}x_2 + \ldots + a_{jn}x_n + y_1 = b_j, y_j < 0 \qquad (8)$$

which can be written as:

$$y_j^{k+1} = y_j^k - \sum_{x_s \in J_k} a_{js}x_s, y_j^k < 0 \qquad (9)$$

By considering only variables in P_k, we get:

$$\sum_{x_s \in P_k} a_{js} \leq y_j^k, \forall y_j^k < 0 \qquad (10)$$

Change variable, $x_t \in P_k$, to 1. The variable x_t is chosen based on Eqs. (11).

$$x_t = \underset{x_t \in P_k}{\arg\min} \left\{ I_t^k = \max(I_s^k) \right\} \qquad (11)$$

where,

$$I_s^k = \sum_{j=1}^{m} \min\left(0, y_j^k - a_{js}\right), x \in P_k \qquad (12)$$

Defines new solution J_{k+1}.

STEP 3. Update y_j^{k+1}:

$$y_j^{k+1} = y_j^k - a_{js} \qquad (13)$$

Update objective function:

$$f_{k+1} = f_k + c_t \qquad (14)$$

If all $y_j^{k+1} \geq 0$, then $f^* = f_{k+1}$, and go to STEP 4. Otherwise, back to STEP 2.

STEP 4. Backtracking and determining optimal solution.

The backtracking process is conducted towards partial solution J_k. It is started by assuming all variables in J_k are one. Then, successively assigning zero to certain variables. Herein, 2^n

possibilities are evaluated where n is number of variables in J_k. Then, a combination which gives the smallest value is the final objective value.

2.3 Gomory Cutting Plane

The Gomory cutting plane is an algorithm for obtaining integer solutions of the linear programming [3]. This algorithm works by examine a solution of the linear programming obtained by simplex method. If the solution is not in integers, a new constraint which can cut the search space so that non integer solution can be eliminated. The procedure of generating new constraints is as the following.

STEP 1. Given an optimal simplex tableau.

If $x_i < 0$, then

$$\sum_{j=1}^{n} a_i^j w_j = \beta_i, \qquad (15)$$

when $\lfloor a_i^j \rfloor \le a_i^j$ and $w_j \ge 0$, equation (15) can be transformed to:

$$\sum_{j=1}^{n} a_i^j w_j \le \beta_i \qquad (16)$$

and

$$\sum_{j=1}^{n} a_i^j w_j + S_i = \beta_i \qquad (17)$$

where S_i is a slack variable. Assumed β_i is non-integer, then $\lfloor a_i^j \rfloor + f_{ij} = a_i^j$ and $\lfloor \beta_i \rfloor + f_i = \beta_i$ where $0 \le f_{ij} < 1$ and $0 < f_i < 1$. The additional constraint is the differences between Eqs (15) and (17) which can be formulized as follows.

$$\sum_{j=1}^{n} \left(-f_{ij} \right) w_j + S_i = -f_i \qquad (18)$$

3. Methodology

The idea of substituting Gomory cutting plane to the Balas algorithm is proposed to accelerate the procedure of Balas algorithm. As discussed in Section 2, Balas algorithm involves a backtracking procedure which takes a long iteration. Thus, we perform Gomory cutting plane method to replace the backtracking procedure. Herein, Gomory cutting plane condition is applied to determine which variable should be set as 1. Detail steps of the proposed Balas and Gomory combination is given as follows.

STEP 1. Initialization

Set $k = 0$. Transform the LP to a standard form.

STEP 2. Solve the standard form LP using dual simplex method to obtain the optimal dual simplex tableau.

STEP 3. If all decision variables in the optimal solution are integer, then stop. Otherwise, generate a new constraint based on Gomory procedure. The new constraint is derived from the basis variable which has the biggest non-integer solution. Solve the new LP with dual simplex method and obtain new optimal tableau. If the new solution is integer, then stop and go to STEP 4. Otherwise, back to STEP 3.

STEP 4. Substitute solution the original LP form to get the final optimal solution.

4. Numerical Analysis

In order to analyze the performance of the proposed method, a numerical analysis involving a binary LPs is conducted. Herein, a binary LP is solved using Balas algorithm, Gomory cutting plane method, and the proposed method.

Problem

Objective function:

$$\text{MIN } f = -5x_1' + 7x_2' + 10x_3' - 3x_4' + x_5' \tag{19}$$

Subject to:

$$x_1' + 3x_2' - 5x_3' + x_4' + 4x_5' \le 0 \tag{20}$$

$$2x_1' + 6x_2' - 3x_3' + 2x_4' + 2x_5' \ge 4 \tag{21}$$

$$x_2' - 2x_3' - x_4' + 2x_5' \le -2 \tag{22}$$

$$x_i' \in \{0,1\} \; \forall i = 1,\dots,5 \tag{23}$$

4.1 Procedure the proposed substituting Balas and Gomory

Solve the LP using the proposed Balas and Gomory algorithm as follows:

Iteration 1.

STEP 1. Transform LP to standard form and obtained new form as follows:

$$\text{MIN } f = -5x_1 + 7x_2 + 10x_3 - 3x_4 + x_5 \tag{24}$$

Subject to:

$$-x_1 + 3x_2 - 5x_3 - x_4 + 4x_5 + y_1 = -2 \qquad (25)$$

$$2x_1 - 6x_2 + 3x_3 + 2x_4 - 2x_5 + y_2 = 0 \qquad (26)$$

$$x_2 - 2x_3 + x_4 + x_5 + y_3 = -1 \qquad (27)$$

$$x_i \in \{0,1\} \; \forall \, i = 1,\ldots,5 \qquad (28)$$

$$y_1, y_2, y_3 \geq 0 \qquad (29)$$

STEP 2. Solve the standard form using dual simplex method and obtain the optimal tableau as follows.

Basic	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	Solution
f	$-10\frac{1}{3}$	0	0	$-17\frac{1}{3}$	$-4\frac{2}{3}$	0	$-2\frac{2}{3}$	-9	9
x_3	$-\frac{2}{9}$	0	1	$-\frac{8}{9}$	$-\frac{4}{9}$	0	$-\frac{1}{9}$	$-\frac{2}{3}$	$\frac{2}{3}$
x_2	$-\frac{4}{9}$	1	0	$-\frac{7}{9}$	$\frac{1}{9}$	0	$-\frac{2}{9}$	$-\frac{1}{3}$	$\frac{1}{3}$
y_1	$-\frac{7}{9}$	0	0	$-3\frac{1}{9}$	$1\frac{4}{9}$	1	$\frac{1}{9}$	$-2\frac{1}{3}$	$\frac{1}{3}$

Figure 1. Optimal tableau 1

Since the current solution contains non-integer variables, a new constraint must be generated. According to the Gomory cutting plane rule, the x_3 is used for the new constraint. The new constraint generating procedure is as follows:

Change negative coefficient of all variables in row x_3 into positive value.

$$
\begin{aligned}
x_1 &\to -2/9 = 7/9 - 1 \\
x_4 &\to -8/9 = 1/9 - 1 \\
x_5 &\to -4/9 = 5/9 - 1 \\
y_2 &\to -1/9 = 8/9 - 1 \\
y_3 &\to -2/3 = 1/3 - 1
\end{aligned} \qquad (30)
$$

Define new coefficient for all variables: $x_1 = 7/9 \, ; x_4 = 1/9 \, ; x_5 = 5/9 \, ; y_2 = 8/9 \, ; y_3 = 1/3$

Substitute to equation x_3.

$$x_3 + \frac{7}{9}x_1 + \frac{1}{9}x_4 + \frac{5}{9}x_5 + \frac{8}{9}y_2 + \frac{1}{3}y_3 = \frac{2}{3} \qquad (31)$$

Define the constraint as follows:

$$s_1 - \frac{7}{9}x_1 - \frac{1}{9}x_4 - \frac{5}{9}x_5 - \frac{8}{9}y_2 - \frac{1}{3}y_3 = -\frac{2}{3} \qquad (32)$$

Add the new constraint to the optimal tableau and solve using simplex method. The result is shown as follows.

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Basic	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	S_1	Solution
f	-8	0	0	-17	-3	0	0	-8	-3	11
x_3	$-\frac{1}{8}$	0	1	$-\frac{7}{8}$	$-\frac{3}{8}$	0	0	$\frac{5}{8}$	$\frac{1}{8}$	$\frac{3}{4}$
x_2	$-\frac{1}{4}$	1	0	$-\frac{3}{4}$	$\frac{1}{4}$	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
y_1	$-\frac{7}{8}$	0	0	$-3\frac{1}{8}$	$1\frac{3}{8}$	1	0	$-2\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
y_2	$\frac{7}{8}$	0	1	$-\frac{1}{8}$	$\frac{5}{8}$	0	1	$-\frac{3}{8}$	$-1\frac{1}{8}$	$\frac{3}{4}$

Figure 2. Optimal tableau 2

From the current optimal tableau, the optimal solution still contains non-integer variable. Thus, return to STEP 3.

Iteration 2.

STEP 3. Generate a new constraint.

From current optimal tableau, x_3 and y_2 are variables which have the biggest integer solution. However, since coefficient s_1 in row y_2 is less than -1, the new constraint is generated from x_3. By following the same procedure in iteration 1, we get the new constraint as follow.

$$x_3 + \left(-1 + \frac{7}{8}\right)x_1 + \left(-1 + \frac{1}{8}\right)x_4 +$$

$$\left(-1 + \frac{5}{8}\right)x_5 + \left(-1 + \frac{3}{8}\right)y_3$$

$$+ \left(-1 + \frac{7}{8}\right)S_1 \qquad\qquad (33)$$

$$= \left(0 + \frac{3}{4}\right)$$

$$S_2 - \frac{7}{8}x_1 - \frac{1}{8}x_4 - \frac{5}{8}x_5 - \frac{3}{8}y_3 - \frac{7}{8}S_1 = -\frac{3}{4} \qquad (34)$$

Add the new constraint to the current LP and solve it using Simplex method. The new optimal tableau is as follows.

Basic	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	S_1	S_2	Solution
f	-5	0	0	$-16\frac{4}{7}$	$-\frac{6}{7}$	0	0	$-6\frac{5}{7}$	0	$-3\frac{3}{7}$	$13\frac{4}{7}$
x_3	0	0	1	$-\frac{6}{7}$	$-\frac{2}{7}$	0	0	$-\frac{4}{7}$	0	$-\frac{1}{7}$	$\frac{6}{7}$
x_2	0	1	0	$-\frac{5}{7}$	$\frac{3}{7}$	0	0	$-\frac{1}{7}$	0	$-\frac{2}{7}$	$\frac{5}{7}$
y_1	-1	0	0	$-3\frac{1}{7}$	$1\frac{2}{7}$	1	0	$-2\frac{3}{7}$	0	$\frac{1}{7}$	$\frac{1}{7}$
y_2	2	0	0	$\frac{2}{7}$	$1\frac{3}{7}$	0	1	$\frac{6}{7}$	0	$-1\frac{2}{7}$	$1\frac{5}{7}$
S_1	1	0	0	$\frac{1}{7}$	$\frac{5}{7}$	0	0	$\frac{3}{7}$	1	$-1\frac{1}{7}$	$\frac{6}{7}$

Figure 3. Optimal tableau 3

Since the optimal solution still involves non-integer solution, go back to STEP 3.

Iteration 3.

STEP 4. Generate new constraint.

The optimal tableau shows the highest integer values are given by x_3 and s_1. However, s_2 in row s_1 has value less than -1. Thus, x_3 is chosen for the new constraint as follows.

$$x_3 + \left(-1 + \frac{1}{7}\right)x_4 + \left(-1 + \frac{5}{7}\right)x_5$$

$$+ \left(-1 + \frac{3}{7}\right)y_3 + \left(-1 + \frac{6}{7}\right)S_2 \qquad (35)$$

$$= \left(0 + \frac{6}{7}\right)$$

$$S_3 - \frac{1}{7}x_4 - \frac{5}{7}x_5 - \frac{3}{7}y_3 - \frac{6}{7}S_2 = -\frac{6}{7} \qquad (36)$$

Add the new constraint to the LP and solve it using Simplex method. The optimal tableau is as follows

Basic	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	S_1	S_2	S_3	Solution
f	-5	0	0	$-16\frac{2}{5}$	0	0	0	$-6\frac{1}{5}$	0	$-2\frac{2}{5}$	$-1\frac{1}{5}$	$14\frac{3}{5}$
x_3	0	0	1	$-\frac{4}{5}$	0	0	0	$-\frac{2}{5}$	0	$\frac{1}{5}$	$-\frac{2}{5}$	$1\frac{1}{5}$
x_2	0	1	0	$-\frac{4}{5}$	0	0	0	$-\frac{2}{5}$	0	$-\frac{4}{5}$	$\frac{3}{5}$	$\frac{1}{5}$
y_1	-1	0	0	$-3\frac{2}{5}$	0	1	0	$-3\frac{1}{5}$	0	$-1\frac{2}{5}$	$1\frac{4}{5}$	$-1\frac{2}{5}$
y_2	2	0	0	0	0	0	1	0	0	-3	2	0
S_1	1	0	0	0	0	0	0	0	1	-2	1	0
x_5	0	0	0	$\frac{1}{5}$	1	0	0	$\frac{3}{5}$	0	$1\frac{1}{5}$	$-1\frac{2}{5}$	$1\frac{1}{5}$

Figure 4. Optimal tableau 4

The optimal tableau shows that the solution is non-integer. However, there is one variable has negative value. Thus, recalculating coefficient f and y_1 is performed instead of generating a new constraint. The ratio between coefficient f and y_1 is given as follows.

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	S_1	S_2	S_3
f	-5	0	0	$-16\frac{2}{5}$	0	0	0	$-6\frac{1}{5}$	0	$-2\frac{2}{5}$	$-1\frac{1}{5}$
y_1	-1	0	0	$-3\frac{2}{5}$	0	1	0	$-3\frac{1}{5}$	0	$-1\frac{2}{5}$	$1\frac{4}{5}$
Ratio	5	-	-	$4\frac{5}{6}$	-	-	-	2	-	$1\frac{5}{7}$	-

Figure 5. Ratio between coefficients f and y_1

Choose variable with the smallest ratio to replace y_1 is basis row. Recalculate the optimal tableau using Simplex method. The result is as follows.

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Basis	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	S_1	S_2	S_3	Solution
f	$-3\frac{2}{7}$	0	0	$-10\frac{4}{7}$	0	$-1\frac{5}{7}$	0	$-\frac{5}{7}$	0	0	$-4\frac{2}{7}$	17
x_3	$-\frac{1}{7}$	0	1	$-1\frac{2}{7}$	0	$\frac{1}{7}$	0	$-\frac{6}{7}$	0	0	$-\frac{1}{7}$	1
x_2	$\frac{4}{7}$	1	0	$1\frac{1}{7}$	0	$-\frac{4}{7}$	0	$1\frac{3}{7}$	0	0	$-\frac{3}{7}$	1
S_2	$\frac{5}{7}$	0	0	$2\frac{3}{7}$	0	$-\frac{5}{7}$	0	$2\frac{2}{7}$	0	1	$-1\frac{2}{7}$	1
y_2	$4\frac{1}{7}$	0	0	$7\frac{2}{7}$	0	$-2\frac{1}{7}$	1	$6\frac{6}{7}$	0	0	$-1\frac{6}{7}$	3
S_1	$2\frac{3}{7}$	0	0	$4\frac{6}{7}$	0	$-1\frac{3}{7}$	0	$4\frac{4}{7}$	1	0	$-1\frac{4}{7}$	2
x_5	$-\frac{6}{7}$	0	0	$-2\frac{5}{7}$	1	$\frac{6}{7}$	0	$-2\frac{1}{7}$	0	0	$\frac{1}{7}$	0

Figure 6. Final optimal tableau

This optimal tableau shows that the current solution is integer solution. Thus stop the iteration and go to the next step.

STEP 4. Substitute the optimal solution to the original LP form to get the final optimal solution.

$$x_3 = x_2 = S_2 = 1, y_2 = 3, S_1 = 2, x_5 = 0 \tag{37}$$

The final optimal solution:

$$f_{\min} = 9$$
$$x_1 = x_2 = x_3 = x_4 = 1 \tag{38}$$
$$x_5 = 0$$

4.2 Discussion

The same problem is also solved using Balas and Gomory algorithm. Since the aims of combining Balas and Gomory algorithm is to reduce the computational time in terms number of iterations, the evaluation is conducted by comparing number of iterations required by Balas, Gomory and the proposed method. The result is mentioned in Table 1. This result proves that the proposed Balas and Gomory algorithm can solve the binary LP better than Balas and Gomory algorithm. It is not only can obtain better result, but also only requires less iteration.

Table 1. Comparison between Balas, Gomory and the proposed method

Method	Number of iterations	Objective value
Balas Algorithm	5 iterations: 32 steps	9
Gomory	1 iteration: 2 steps	15
The proposed method	1 iteration: 4 steps	9

5. Conclusion

Balas algorithm is a method for solving binary LP. Although it has been proven can solve the binary LP well, this method has a drawback which is requiring a long iterations. On the other hand, Gomory method is a method for solving integer LP. Unlike Balas algorithm, this method only takes a few iterations to find an integer solution.

This paper aims to propose a method for solving binary LP with less iteration. The proposed method combines Balas algorithm with Gomory cutting plane method. In this method, the backtracking procedure in Balas algorithm is replaced with Gomory cutting plane method. Backtracking procedure is a part of Balas algorithm which requires 2^n evaluation where n is number of variables. The proposed method avoids this procedure since it requires high computational. Herein, Gomory condition is applied to generate an additional constraint which reduces the search space so that it can find a binary integer solution faster.

Furthermore, the proposed method is validated using a binary LP. Numerical computational result proves that the proposed substitution Balas and Gomory algorithm can solve binary LP better and faster than Balas and Gomory algorithm.

Acknowledgment

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