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NATURAL CONVECTION FLOW PAST AN IMPERMEABLE VERTICAL PLATE EMBEDDED IN NANOFLUID SATURATED POROUS MEDIUM WITH TEMPERATURE DEPENDENT VISCOSITY

MAKUNGU JAMES, DR. E.W. MUREITHI AND DMITRY KUZNETSOV

ABSTRACT. The paper investigates a natural convection flow past an impermeable vertical plate embedded in nanofluid saturated porous medium. The base fluid is assumed to have variable viscosity. The self-similar boundary layer equations are solved using standard numerical techniques. The validity of the model is checked against other previous works and is found to be in good agreement. The results obtained show that the nanoparticle volume fraction and the variable viscosity parameter significantly enhance the heat transfer properties from the wall to the fluid saturated porous medium.

1. Introduction

The study of convective heat transfer in nanofluids flow through porous medium is gaining a lot of attention. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers with sizes typically of 1-100nm suspended in liquid [14]. On the other hand, nanofluids are engineered by suspending nanoparticles in traditional heat transfer fluids such as water, oil and ethylene glycol. The nanoparticles used in nanofluids are typically made of metal (Au, Ag, Cu, and Fe), metal oxides (CuO, SiO2, Al2O3, TiO2, ZnO, and Fe3O4), carbides or carbon nanotubes (diamond, graphite, single/multi wall carbon nanotubes) [20]. Nanofluids have attracted great interest recently because of reports of greatly enhanced thermal properties. For instance, when less than 1% of the solid particles (copper or carbon nanotubes) are added to the base liquids of lower thermal conductivity eg., ethylene glycol or oil, solid particles improve the heat conductivity

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of the liquid by 40% and 150% respectively. Convectional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such enhancement. [18] observed that dispersion of nanoparticles in the base fluid increase the thermal conductivity of the fluid and therefore, the presence of porous media enhances the effective thermal conductivity of the base fluid . On the other hand, the effective thermal conductivity produced by a combination of base fluid, nanoparticle and porous medium plays a major role in heat transfer enhancement. Modeling of convection of heat by a nanofluid flow through a porous media is a phenomenon of great interest from both theory and application points of view. Nanofluids flow in porous media have numerous thermal engineering applications such as electronic cooling, vehicle cooling, transformer cooling and help to improve thermal properties of oil and lubricants [2].

The theoretical and applied research in flow, heat and mass transfer in porous media has received attention during the past three decades ([19];[8];[7]; [21];[10]). Since the early work of Darcy in the nineteenth century, several and intensive investigations have been conducted on the laminar flow and heat transfer through porous medium, [12]. Darcy's experiment discovered that, in laminar flow through porous medium, the pressure drop caused by the frictional drag is proportional to the velocity at the low Reynolds number range. The Darcy's experiment leads to a law called Darcy law which states that, the velocity of flow of a liquid through a porous medium due to difference in pressure is proportional to the pressure gradient in the direction of flow. The law is a phenomenologically derived constitutive equation that describes the flow through a porous medium ([13];[3]). Nanofluids are found to exhibit higher conductive, boiling point and convective heat transfer performances compared to base fluids [17]. It has showed that an increase in the thermal conductivity of a solid-liquid depends on the shape, size and thermal properties of the nanoparticles. Nanofluids, when used as coolants can provide dramatic improvements in the thermal properties at smallest possible concentrations by uniform dispersion and stable suspension of nanoparticles in host fluids [2]. By combining nanofluid with biotechnology components, nanotechnology can have numerous potential applications across a wide range of practical applications such as agriculture, pharmaceuticals and biological sensors. The potential forms of nanomaterials available for use in biotechnological applications includes nanowires, nanofibres, nanostructures and nanomachines [5]. The commercialization of nanobiotechnological products seems to have a potential future and within next a few years many new products of this nature are likely to be used [17]. Since, nanofluid flow and heat transfer in the porous medium has many applications and still a new area with potential for engineering as mentioned above, then, the study has attracted more attention from the research communities to show mathematically what really happen in the system of flow of nanofluid.

Various evidences of nanofluid flow and heat transfer in the porous medium have been reported in the literatures by several researchers. The first work on convection flow and heat transfer of nanofluids inside a 10.66mm-diameter tube by Park and Cho [2] observed a substantial increase in the heat transfer coefficient in the tube regime. The boundary layer flows of nanofluids have been analyzed by [11] and [6] solved the non-similar problem of free convection heat transfer from a vertical plate embedded in a saturated porous medium with an arbitrary varying surface temperature or heat flux. [15]investigated the mixed convection flow of non-Newtonian fluid from vertical surface saturated in a porous medium filled with a nanofluid. [18] investigated the natural convection heat transfer of nanofluid along a vertical plate embedded in porous medium.
However the impact of variable viscosity of a natural convection flow of nanofluid along a vertical impermeable plate embedded in a porous medium has received little attention. Hence in the study presented here we investigate the effects of variable viscosity of the base fluid with nanoparticles, volume fraction of the nanoparticles and the power law wall temperature on steady state two-dimensional natural convection flow along a vertical plate embedded on the porous medium. The problem has an important application to convective flow on the heated plate, especially in the compact design devices that cannot be cooled by the use of traditional methods rather than using nanoparticle material for enhancing thermal conductivity of the base fluids. Furthermore, the paper is organised in the following manner. Section two presents the problem formulation, Section three is presented the numerical technique. Section four will discuss the results and discussion and finally conclusion is made in Section five.

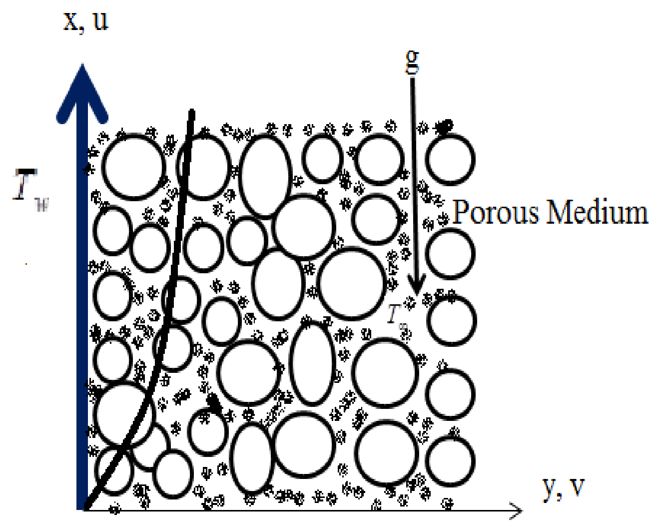

FIGURE 1. Physical model and coordinate system.

TABLE 1. Physical properties of base fluid water and copper at 20 °C (293K), (Anjali,2011)

	$\rho(kg/m^3)$	$c_p(J/kgK)$	k(w/mK)	β/K	m^2/s
water	1000.52	4181.8	0.598	$210*10^{-6}$	$0.143*10^{-6}$
copper	8954	383.1	385	$51*10^{-6}$	$1.12*10^{-4}$

2. Problem Formulation

The current problem considers the steady convection boundary layer flow of a nanofluid over an impermeable plate embedded in porous medium with temperature dependent viscosity. The impermeable plate is placed vertically in the porous medium filled with a nanofluid as shown in Fig 1. The system is governed by continuity, momentum and energy equations in two dimensional laminar flow of saturated nanofluid. The Darcy-model is used to describe the flow of nanofluid in porous medium with small porosity and low flow along a vertical plate. Let the convecting nanofluid and the porous medium be at the local thermodynamic equilibrium with no viscous dissipation effect to the system. Also, it is assumed that the physical properties of the base fluid and that of the nanoparticles are constant except the density of the fluid at the buoyancy term, so that the Boussinesq approximation holds. Furthermore, the base (pure) fluid viscosity varies inversely with temperature and the flow of nanofluid past a vertical plate is assumed to satisfy slip condition. Following the modified equations studied by [4] along with the Boussinesq technique and boundary layer approximation, then the modified model becomes;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\mu_{nf}}{K}(T)\,u=-\frac{\partial p}{\partial x}+(\rho\beta)_{nf}\,g\,(T-T_\infty) \tag{2}$$

$$0=\frac{\partial p}{\partial y} \tag{3}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{eff}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Noting that $v << u$ in the boundary layer and this shows that there are no body forces (including gravity) in the y-direction and therefore, the force balance in the y-direction is zero. Apart form that, the pressure forces within the boundary layer is established by the hydrostatic condition outside the boundary layer and therefore, the unbalance of pressure and gravitational forces within the boundary layer due to the variation of density of the fluid creates the fluid motion adjacent to the vertical plate. Here, u and v are seepage velocity components along x and y axes. Noting that, x and y are the cartesian coordinates measured along the vertical plate and normal to it, respectively. Furthermore, T is the temperature of the nanofluid within the boundary layer and $\mu_{nf}(T),(\rho c_p)_{nf},k_{eff}$ and $(\rho c_p)_{eff}$ are the dynamic viscosity of the nanofluid, heat capacitance, effective thermal conductivity and effective heat capacitance of the porous medium, while p is the pressure and K is the permeability of the porous medium. The subscript "nf" defines all the thermophysical quantities of nanofluid in porous medium and these quantities are defined as follows-([18],[9]);

$$\begin{cases} k_{nf}=\frac{k_p+2k_f-2\varphi\left(k_p-k_f\right)}{k_p+2k_f+\varphi\left(k_p-k_f\right)},(\rho\beta)_{nf}=\varphi(\rho\beta)_p+(1-\varphi)(\rho\beta)_f\,, \\ k_{eff}=\frac{k_s k_{nf}}{\epsilon k_s+(1-\epsilon)k_{nf}},(\rho c_p)_{eff}=\epsilon(\rho c_p)_{nf}+(1-\epsilon)(\rho c_p)_m\,, \\ \mu_{nf}(T)=\frac{\mu_f(T)}{(1-\varphi)^{2.5}},(\rho c_p)_{nf}=(1-\varphi)(\rho c_p)_f+\varphi(\rho c_p)_p\,. \end{cases} \tag{5}$$

In eqn.(5), $\varphi,\beta_p,\rho_p,(\rho c_p)_p$ and k_p are the nanoparticle volume fraction, volumetric thermal expansion, density, heat capacitance and thermal conductivity of the nanoparticles in the base fluid, respectively, while, $\beta_f,\rho_f,(\rho c_p)_f$ and k_f are volumetric thermal expansion, density, heat capacitance and thermal conductivity of the base fluid, respectively. In the model, it is assumed that the vertical plate satisfies the power law temperature T_w and the temperature far away from the plate is at the ambient temperature T_∞. The ambient temperature T_∞ is assumed to be less than temperature at any arbitrary reference point in the boundary layer formed along the plate. Therefore, under these situations the boundary conditions associated with this model are as follows:

$$v = 0, T = T_\infty + Cx^\lambda \text{ at } y = 0,\ x \geq 0 \tag{6}$$

$$u = 0, T = T_\infty \text{ at } y \to \infty,\ x \geq 0 \tag{7}$$

Furthermore, the temperatute of the plate is a power function of distance along the plate measured from its leading edge of the plate and is defined as;

$$T_w = T_\infty + Cx^\lambda \tag{8}$$

where, $C > 0$ for heated vertical plate and $C < 0$ for cooled vertical plate, whereas $\lambda = 0$ for isothermal/constant temperature along the vertical plate. The pressure in Eqns (2), (3) is eliminated using the cross-differentiation approach and therefore the continuity, momentum and heat equations are written as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{\partial}{\partial y}\left(\mu_{nf}\left(T\right) u\right) = \left(\rho\beta\right)_{nf} gK\frac{\partial T}{\partial y} \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{eff}}{\left(\rho c_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

For the present base fluid, following ([1]; [16]), the dynamic viscosity of the base fluid is considered in the following form:

$$\mu_f\left(T\right) = \frac{\mu_\infty}{1 + \gamma\left(T - T_\infty\right)} = \frac{1}{a\left(T - T_e\right)} \tag{12}$$

where μ_∞ is a dynamic viscosity of the ambient base fluid; γ is a constant value, it implies that when $\gamma = 0$, the dynamic viscosity of the base fluid flow remains constant throughout the system. On the other hand a and T_e are constants, given by;

$$a = \frac{\gamma}{\mu_\infty},\ T_e - T_\infty = -\frac{1}{\gamma} \tag{13}$$

We now look for similarity solutions of Equations (10) and (11) with the boundary conditions (6) and (7) of the following form;

$$\Psi = \alpha_{eff} Ra_x^{1/2} f(\eta),\ \eta = \frac{y}{x} Ra_x^{1/2},\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \theta_r = \frac{T_e - T_\infty}{T_w - T_\infty}, \tag{14}$$

provided that $Ra_x = g\beta K(T_w - T_\infty)x/\nu\alpha_{eff}$ is the local Rayleigh number of the porous medium, ν, α_{eff} and θ_r are the kinetic viscosity, effective thermal diffusivity of the porous medium with nanofluid and viscosity-variable parameter. Also, Ψ is the stream function which is expressed in terms of derivatives of the seepage velocity as $u = \partial\Psi/\partial y$ and $v = -\partial\Psi/\partial x$, apart from that, the seepage velocities usually satsify the continuity equation of the flow. Using, similarity solutions, the seepage velocities may be expressed in the following form;

$$u = \frac{g\beta K\left(T_w - T_\infty\right)}{\nu} f' \tag{15}$$

$$v = -\frac{1}{2}\left[\frac{\alpha_{eff} g\beta K(T_w - T_\infty)}{\nu x}\right]^{1/2}\left[(1+\lambda)f - (1-\lambda)\eta f'\right] \tag{16}$$

where, primes represent the derivative with respect to η.
Substituting the seepage velocities Egns (15) and (16) in Eqns (10) and (11) together with Eqns (12-14), we get the following system of non-linear ordinary differential equations:

$$f'' + \frac{f'\theta'}{\theta_r - \theta} - (1-\varphi)^{2.5}\left(\varphi\frac{\rho_p\beta_p}{\rho_f\beta_f} + (1-\varphi)\right)\theta'\left(1 - \frac{\theta}{\theta_r}\right) = 0 \tag{17}$$

$$\theta'' + \frac{(1+\lambda)}{2} f\theta' - \lambda f'\theta = 0 \tag{18}$$

Subjected to the boundary conditions:

$$\begin{gathered} y = 0,\ \eta = 0,\ f(0) = 0,\ \theta(0) = 1, \\ y \to \infty,\ \eta \to \infty,\ f'(\infty) = 0,\ \theta(\infty) = 0 \end{gathered} \tag{19}$$

It has showned that θ_r is determined by the viscosity/temperature characteristics of the base fluid and the operating temperature difference is $\Delta T = T_w - T_\infty$. It is also shown that, if θ_r is small then either the base fluid viscosity changes with temperature difference or the temperature difference is high. It may be noted that θ_r is negative for liquids and positive for gases [1].

The quantity of physical interest is local Nusselt number Nu_x, which is defined as follows:

$$
\begin{aligned}
Nu_x &= \frac{hx}{k_{eff}},\ h = \frac{q_w^{''}}{T_w - T_\infty},\ q_w^{''} = -k_{eff}\left(\frac{\partial T}{\partial y}\right)_{y=0} \\
q_w^{''} &= -k_{eff}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k_{eff}C^{3/2}\theta^{'}\left(\frac{g\beta K}{\nu\alpha_{eff}}\right)^{1/2} x^{3\lambda/2}x^{-1/2}
\end{aligned} \tag{20}
$$

Therefore, using the similarity solutions Egn (14), we get the local Nusselt number equation in dimensionless form;

$$Nu_x = -\left[\frac{g\beta K(T_w - T_\infty)x}{\nu\alpha_{eff}}\right]^{1/2}\theta^{'}(0) = Ra_x^{1/2}\left[-\theta^{'}(0)\right] \tag{21}$$

$$\frac{Nu_x}{Ra_x^{1/2}} = \left[-\theta^{'}(0)\right] \tag{22}$$

where by $q_w^{''}$ and h are the local heat flux and heat transfer coefficient from the vertical plate, respectively. Hence, the effects of the nanoparticle volume fraction (φ), viscosity-variable parameter (θ_r) and parameter (λ) on the velocity profile, temperature profile and convective heat transfer (local Nusselt number) from a vertical plate to the nanofluid are discussed in the results and discussion part.

3. Numerical technique

The dimensionless flow Equations (17) and (18) are non-linear coupled ordinary differential equations, which have no exact solution/closed-form solution. Thus, they must be computed numerically together with the boundary values provided in Egn (19). The solutions of Eqns (17) and (18) subject to the boundary condition (19) were obtained by applying the fourth-order Runge-Kutta integration with a shooting method. In general, the shooting method transforms the original boundary value problem into initial value problem. It is an iterative technique which attempts to find an appropriate initial conditions of the original boundary value problem . Furthermore, it attempts to assume some of missing initial conditions which are not provided in the original boundary value problem. Therefore, the systems of the first order differential equations formed after transforming the original boundary value problem using shooting method were solved by fourth order Runge-Kutta method. The accuracy of the assumed initial condition is then checked by comparing the approximated value of the dependent variable at the end point of the domain with its known value at that end point. If it is happen that the difference still exists, then an improvement is done by choosing another initial conditions and this process repeated until the difference become very small between the calculated and the know value at the end point of the domain. Furthermore, the different step sizes were selected to satisfy the convergence criterion of 10^{-6}. From the process of numerical computation, the velocity, temperature profiles and local Nusselt number were observed and their numerical values were presented in the graphical and tabular forms.

TABLE 2. Numerical values of $[-\theta'(0)]$ for Cu nanoparticles when $\varphi \to 0$ and $\theta_r \to -\infty$

λ	$[-\theta'(0)]$- Present results	$[-\theta'(0)]$- Nield and Bejan (2006) results
-1/3	0.0247	0
-1/4	0.1755	0.162
0	0.448	0.444
1/4	0.6286	0.630
1/3	0.6793	0.678
1/2	0.7715	0.761
3/4	0.8931	0.892
1	1.0005	1.001

4. Results and Discussion

In our present study, we have computed and analyzed a theoretical solution of the dimensionless equations subject to the boundary conditions. The computations have been done for the vertical plate embedded in saturated porous medium with nanofluid. The temperature of the vertical plate assumed to be a power function of distance along the plate from its leading edge. Also, the values of the physical properties of the base fluid water and copper were taken from ([2]), whereas the nanofluid correlation formulas were taken from [9]. The effects of nanoparticle volume fraction φ, variable-viscosity paramete θ_r and parameter λ on the dimensionless velocity, temperature and local Nusselt number were discussed and shown graphically. Furthermore, the present results have been compared with [4] and it is found that they are in good agreement as the variable viscosity parameter goes to infinity and nanoparticles in the base fluid approaches to zero, Table 2 and Fig 9.

The velocity profile distribution for different values of nanoparticle volume fraction φ, with fixed parameter $\lambda = -1/3$ and variable viscosity parameter $\theta_r = -1$ is depicted in Fig 2. It is observed that as the nanoparticle volume fraction increases, dimensionless velocity of the nanofluid along a vertical plate decreases. We suggested that the decline of the dimensionless velocity of the nanofluid is due to the increase of surface drag caused by more addition of nanoparticle in the base fluid water.

In Fig 3, the dimensionless temperature plotted against the similarity variable η for different values of nanoparticle volume fraction. From the figure, it is observed that with the increase in nanoparticle volume fraction, the dimensionless temperature profile increases in thermal boundary layer. Here, the nanoparticle volume fraction has shown a significant in enhancing heat transfer from the heated plate to the fluid in motion adjacent to the plate.

Figures 4-5 show the dimensionless velocity and temperature profiles for different parameter λ with fixed nanoparticle volume fraction and variable viscosity parameter. It is observed that both of them show a common behavior. It means that, as the parameter λ increases, velocity and temperature profiles decrease. Also, through different computations we have observed that the problem becomes realistic when the parameter λ range is $-1/3 \leq \lambda \leq 1$.

The dimensionless velocity profile plotted against the similarity variable η was shown in Fig 6, with fixed nanoparticle volume fraction φ and parameter λ. It is observed that as the variable viscosity parameter θ_r decreases, velocity profile increases. This is due to the fact that, as the difference in temperature between a vertical plate and ambient become smaller, the variable viscosity parameter becomes large, this leads to lower the velocity of the fluid. However, when the difference in temperature increases the variable viscosity parameter decrease as a result the velocity profile of the nanofluid do increase. Furthermore, the velocity profiles drop faster as you move away from the vertical plate as the variable viscosity parameters decrease.

In Fig 7, the effects of variable viscosity parameter θ_r on temperature profiles are depicted. It is observed that, as variable viscosity parameter decreases, temperature profiles decrease. The decline of the temperature profile is due to the overhauling of heat in the boundary layer caused by convection of fluid at high velocity adjacent to the heated plate.

Fig 8 displays the effects of the ratio of local Nusselt number and Rayleigh number on the surface of heated plate for various values of nanoparticle fraction volume with fixed λ and variable viscosity parameter θ_r. It is observed that, the ratio of local Nusselt number and Rayleigh number with nanoparticles are linearly relationship. This indicates that as nanoparticle increases in the base fluid water, the ratio number increases. This indicates that, more heat is removed from a vertical plate and this action proves that nanofluids are efficient coolants than the traditional fluids.

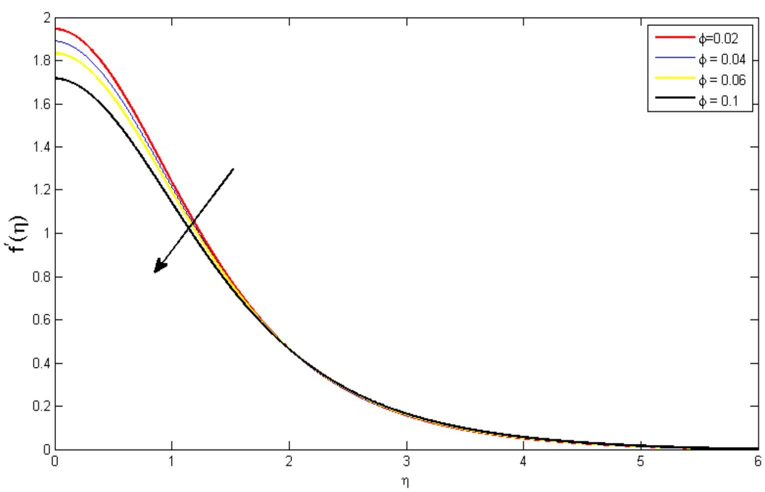

FIGURE 2. Effects of nanoparticle fraction volume on velocity profiles $f^{'}(\eta)$ when $\lambda = -1/3, \theta_r = -1$.

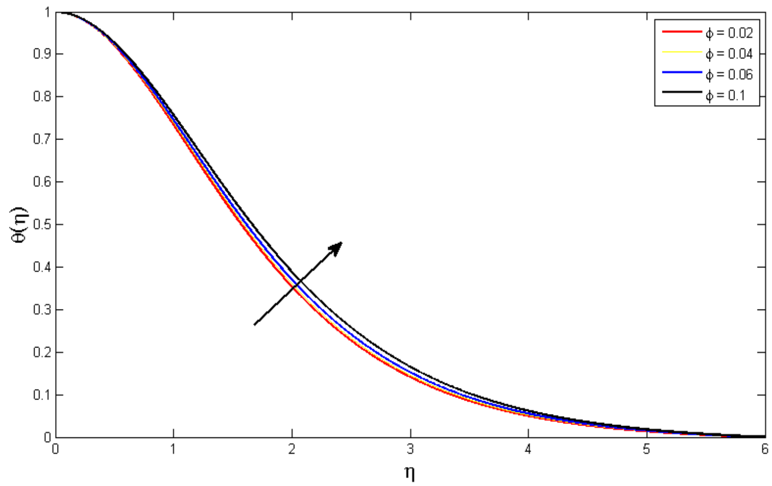

FIGURE 3. Effects of nanoparticle fraction volume on temperature profiles $\theta(\eta)$ when $\lambda = -1/3, \theta_r = -1$.

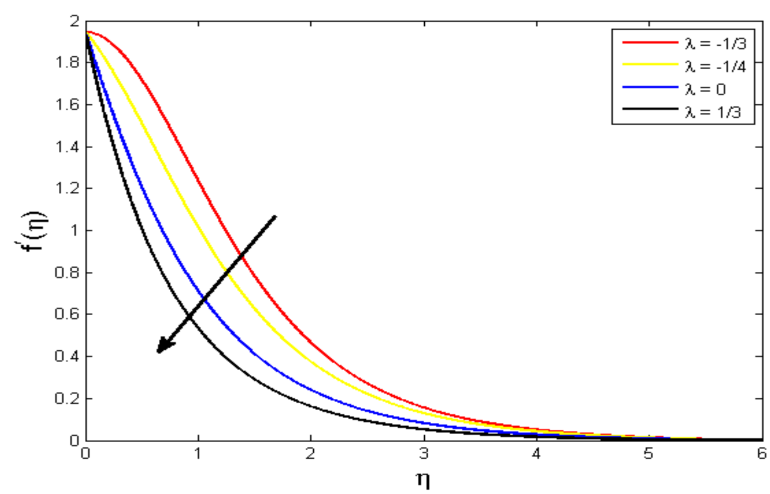

FIGURE 4. Effects of parameter λ on velocity profiles f' when $\varphi = 0.02, \theta_r = -1$.

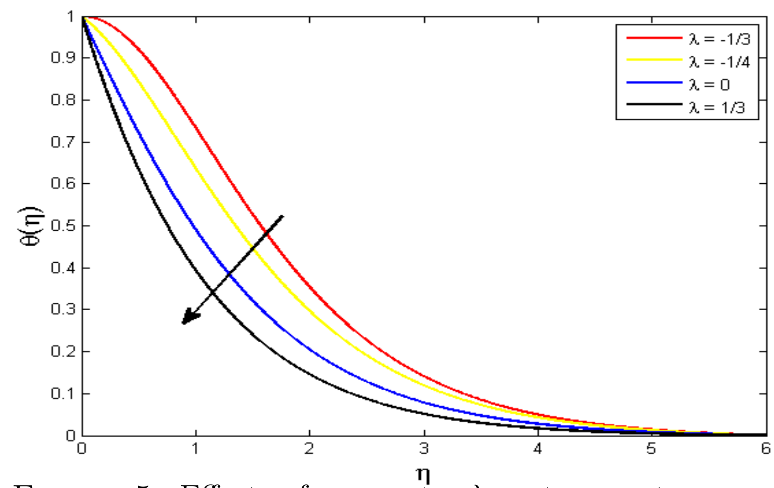

FIGURE 5. Effects of parameter λ on temperature profiles $\theta(\eta)$ when $\varphi = 0.02, \theta_r = -1$.

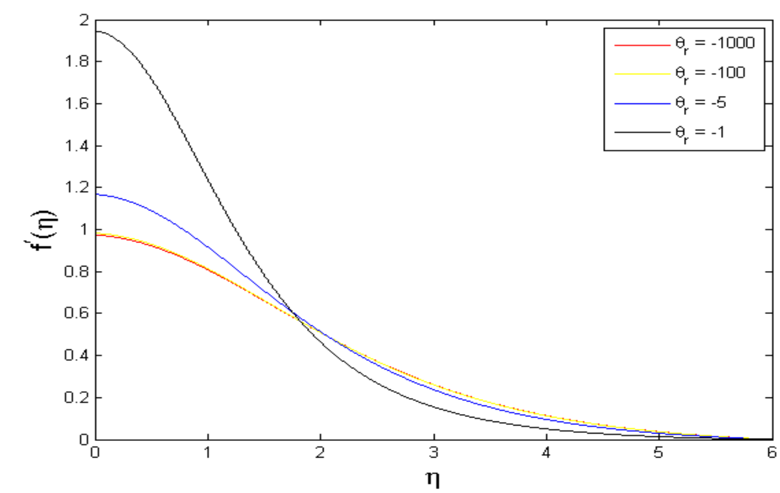

FIGURE 6. Effects of variable -viscosity parameter on velocity profiles f' when $\lambda = -1/3, \phi = 0.02$.

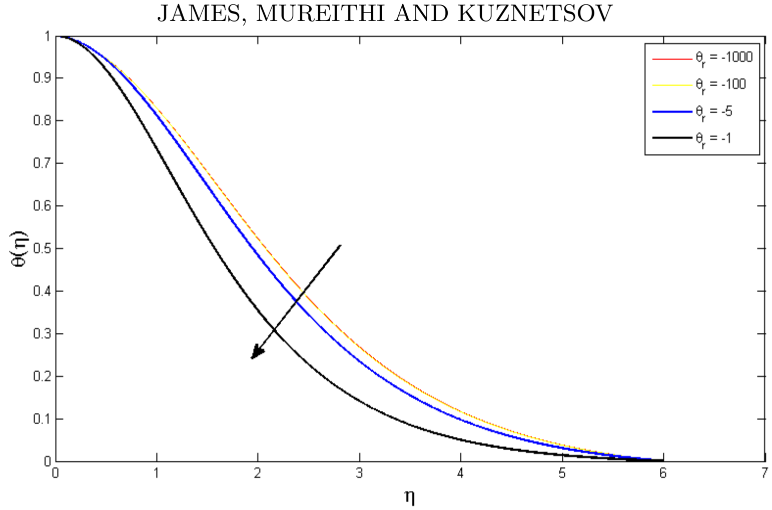

FIGURE 7. Effects of variable -viscosity parameter on temperature profiles $\theta(\eta)$ when $\lambda = -1/3, \phi = 0.02$.

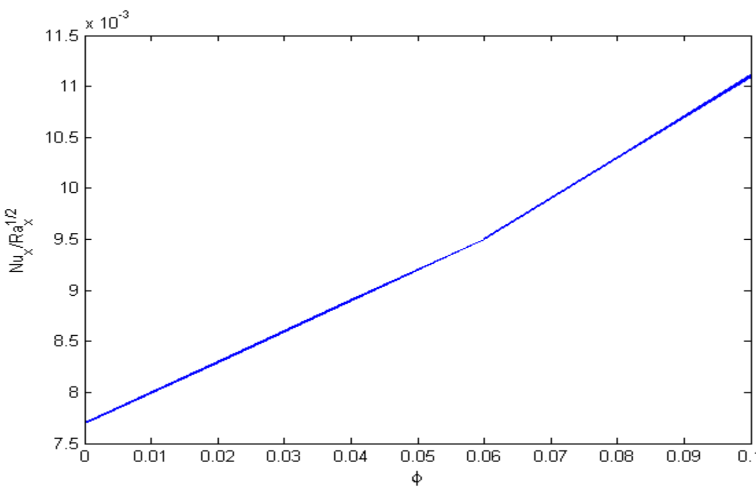

FIGURE 8. Effects of heat transfer from the heated surface for various values of nanoparticle fraction volume when $\lambda = -1/3$ and $\theta_r = -1$.

5. Conclusion

Natural convection flow past an impermeable vertical plate embedded in nanofluid saturated porous medium with temperature dependent viscosity is studied numerically using fourth order Runge-Kutta integration with shooting method. The numerical solutions of the model are compared with previous works and found to be in good agreement. The effects of the nanoparticle volume fraction, variable-viscosity parameter and the power parameter of temperature along the plate are discussed and presented in graphical forms. In general a copper-water nanofluid has shown an improvement in heat transfer from a heated vertical plate into convective fluid in porous medium.

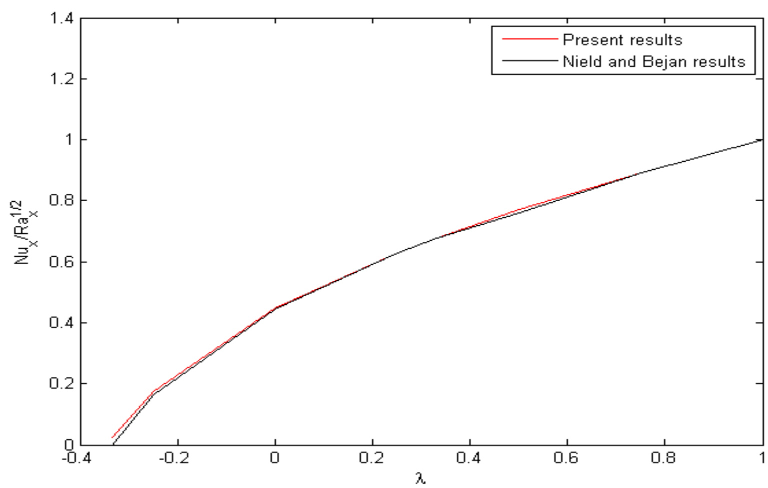

Figure 9. Comparison between the present results and Nield and Bejan results on the heat transfer from the heated plate when $\phi = 0$ and $\theta_r \to -\infty$.

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Makungu James, School of Computational and Communication Science and Engineering, The Nelson Mandela African Institution of Science and Technology (NM-AIST), P. O. Box 447, Tengeru, Arusha, Tanzania

E.W. Mureithi, School of Computational and Communication Science and Engineering, The Nelson Mandela African Institution of Science and Technology (NM-AIST), P. O. Box 447, Tengeru, Arusha, Tanzania

Dmitry Kuznetsov, NM-AIST, Tengeru, Arusha, Tanzania