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Fixed Point Theorem for $(\phi, MF)-$Contraction on C^*-Algebra Valued Metric Spaces

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Abstract
This present article extends the new notion of mapping called $(\phi, MF)-$contraction in the frame work of C^*-algebra valued metric spaces and establishing the existence and uniqueness of fixed point for them. Non-trivial examples are further provided to support the hypotheses of our results.

Keywords: Fixed point; C^*-algebra valued metric spaces; $(\phi, F)-$contraction; $(\phi, MF)-C^*$ valued contraction.

Introduction

Many generalizations of the concept of metric spaces are defined and some fixed point theorems were proved in these spaces. In particular, C^*-algebra valued metric spaces were introduced by Ma et al. [5] as a generalization of metric spaces they proved certain fixed point theorems, by giving the definition of C^*-algebra valued contractive mapping analogous to Banach contraction principle. Various fixed point results were established on such spaces, see [3, 4, 8] and references therein.

In this paper, inspired by the work done in [6, 10], we introduce the notion of $(\phi, MF)-$contraction and establish some new fixed point theorems for mappings in the setting of complete C^*-algebra valued metric spaces. Moreover, some illustrative examples are presented to support the obtained results.

preliminaries

Throughout this paper, we denote $\mathbb{A}$ by an unital C^*-algebra with linear involution $*$, such that for all $x, y \in \mathbb{A}$,

$$(xy)^* = y^*x^* \text{ and } x^{**} = x.$$

We call an element $x \in \mathbb{A}$ a positive element, denote it by $x \succeq \theta$ if $x \in \mathbb{A}_h = \{x \in \mathbb{A} : x = x^*\}$ and $\sigma(x) \subset \mathbb{R}_+$, where $\sigma(x)$ is the spectrum of x.

Using positive element, we can define a partial ordering $\preceq$ on $\mathbb{A}_h$ as follows:

$$x \preceq y \text{ if and only if } y - x \succeq \theta$$

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where θ means the zero element in $\mathbb{A}$.

We denote the set $x \in \mathbb{A} : x \geq \theta$ by $\mathbb{A}_+$ and $|x| = (x^*x)^{\frac{1}{2}}$.

Remark 1.1. When $\mathbb{A}$ is an unital C^*-algebra, then for any $x \in \mathbb{A}_+$ we have

$$x \leq I \Longleftrightarrow \|x\| \leq 1$$

The following definition was given by D. Wardowski in [9].

Definition 1.2. [11] Let $\mathcal{F}$ be the family of all functions $F\colon \mathbb{R}_+ \to \mathbb{R}$ and Φ be the family of all functions $\phi :]0, +\infty[\to]0, +\infty[$ satisfying:

(i) F is strictly increasing.

(ii) For each sequence $\{x_n\}_{n\in\mathbb{N}}$ of positive numbers

$$\lim_{n\to 0} x_n = 0 \quad \text{if and only if} \quad \lim_{n\to\infty} F(x_n) = -\infty.$$

(iii) $lim\ inf_{s\to\alpha^+}\phi(s) > 0$ for all $s > 0$.

(iv) There exists $k \in]0, 1[$ such that $\lim_{x\to 0} x^k F(x) = 0$.

Definition 1.3. [11] Let (X, d) be a complete metric space. A mapping $T : X \to X$ is called a $(\phi, F)-$ contraction on (X, d) if there exists $F \in \mathcal{F}$ and $\phi \in \Phi$ such that

$$(d(Tx, Ty) > 0 \Rightarrow F(d(Tx, Ty)) + \phi(d(x, y)) \leqslant F(d(x, y))$$

for all $x, y \in X$ for which $Tx \neq Ty$

Definition 1.4. Let (X, d) be a metric space. A self-map $T : X \to X$ is said to be a $MF-$ contraction if there exists $\tau > 0$ such for $x, y \in X$

$$M(Tx, Ty) > 0 \Rightarrow \tau + F(M(Tx, Ty)) \leq F(M(x, y))$$

where

$$M(x, y) = max\{\, d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\}$$

and $F : (0, +\infty) \to (-\infty, +\infty)$ satisfies (i) and (ii) of definition 2.2

Definition 1.5. [12] Let the function $\phi : A^+ \to A^+$ be positive if having the following constraints :

(i) ϕ is continous and nondecrasing

(ii) $\phi(a) = \theta$ if and only if $a = \theta$

(iii) $lim_{n\longrightarrow\infty}\phi^n(a) = \theta$

Definition 1.6. [12] Suppose that A and B are C^*-algebras.

A mapping $\phi : A \to B$ is said to be C^*- homomorphism if:

(i) $\phi(ax + by) = a\phi(x) + b\phi(y)$ for all $a, b \in \mathbb{C}$ and $x, y \in A$

(ii) $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in A$

(iii) $\phi(x^*) = \phi(x)^*$ for all $x \in A$

(iv) ϕ maps the unit in A to the unit in B.

Lemma 1.7. *[7] Let A and B be C^*-algebras and $\phi : A \to B$ be a C^*- homomorphism, for all $x \in A$ we have*

$$\sigma(\phi(x)) \subset \sigma(x) \text{ and } \|\phi(x)\| \leq \|\phi\|.$$

Corollary 1.8. *[12] Every C^*- homomorphism is bounded.*

Corollary 1.9. *[12] Suppose that ϕ is C^*-isomorphism from A to B, then $\sigma(\phi(x)) = \sigma(x)$ and $\|\phi(x)\| = \|\phi\|$ for all $x \in A$.*

Lemma 1.10. *[12] Every $*-$ homomorphism is positive.*

Main result

Aspired by Wardowski in [2], we introduce the notion of (ϕ, MF)-contraction on a C^*-algebra valued metric space.

Definition 1.11. Let

$$F: \mathbb{A}_+ \longrightarrow \mathbb{A}$$

a function satisfying:

(i) F is continuous and nondecreasing .

(ii) $F(t) = \theta$ if and only if $t = \theta$.

A mapping $T : X \longrightarrow X$ is said to be a (ϕ, MF) C^* valued contraction if there exists $\phi : \mathbb{A}_+ \longrightarrow \mathbb{A}$ an $*-$ homomorphism such that

$$\forall x, y \in X; M(Tx, Ty) \geq \theta \Rightarrow F(M(Tx, Ty)) + \phi(M(x, y)) \leq F(M(x, y)) \ (1)$$

Where $M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty), \dfrac{d(x, Ty) + d(y, Tx)}{2}\}$.

Example 1.12. Let $X = [0, 1]$ and $\mathbb{A} = \mathbb{R}^2$ Then $\mathbb{A}$ is a C^*- algebra with norm $\|.\| : \mathbb{A} \longrightarrow \mathbb{R}$ defined by

$$\|(x, y)\| = (x^2 + y^2)^{\frac{1}{2}}.$$

Define a C^*- algebra valued metric $d : X \times X \longrightarrow \mathbb{A}$ on X by

$$d(x, y) = (|x - y|, 0)$$

With ordering on $\mathbb{A}$ by

$$(a, b) \leq (c, d) \ \Leftrightarrow a \leq c \text{ and } b \leq d$$

A mapping $T : X \longrightarrow X$ given by $Tx = \dfrac{x}{3}$ is continuous with respect to $\mathbb{A}$.
Let $F : \mathbb{A} \longrightarrow \mathbb{A}$. Defined by

$$F(x, y) = ((x - y)^2, 0)$$

It is clear that F satisfies (i) and (ii).
Now $M(x, y) = d(x, y)$ and
$M(Tx, Ty) = d(Tx, Ty) = (|Tx - Ty|, 0) = (|\dfrac{x}{3} - \dfrac{y}{3}|, 0)$
We have $F(M(Tx, Ty)) = F(d(Tx, Ty)) = F(d(\dfrac{x}{3}, \dfrac{y}{3})) = F((\dfrac{x}{3} - \dfrac{y}{3}))^2, 0)$.
And
$(\dfrac{x}{3} - \dfrac{y}{3}))^2 - (x - y)^2 \leq -\dfrac{1}{3}(x - y)^2$.
Therefore T is a valued (ϕ, MF) C^* -valued contraction with

$$\phi(M(x,y)) = (\frac{1}{3}(x-y)^2, 0).$$

Theorem 1.13. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)-C^* valued contraction mapping.*

Then T has a unique fixed point.

Proof. : Let $x_0 \in X$ be arbitrary and fixed we define a sequence $\{x_n\}_{n\in\mathbb{N}}$, $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$. Clearly, if $x_{n+1} = x_n$, then x_0 is a fixed point of T and is unique.

We have

$M(x_n, x_{n+1})$

$$= max\{d(x_n, x_{n+1}), d(x_n, Tx_n), d(x_{n+1}, Tx_{n+1}), \frac{d(x_n, Tx_{n+1}) + d(x_{n+1}, Tx_n)}{2}\}$$

$$= max\{d(x_n, x_{n+1}), d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})}{2}\}$$

$= max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\}$.

And

$$M(Tx_n, Tx_{n+1}) = max\{d(x_{n+1}, x_{n+2}), d(x_{n+2}, x_{n+3})\}.$$

If $d(x_n, x_{n+1}) \preceq d(x_{n+1}, x_{n+2})$ for all $n \in \mathbb{N}$, then

$$M(x_n, x_{n+1}) = d(x_{n+1}, x_{n+2}) \text{ and } M(Tx_n, Tx_{n+1}) = d(x_{n+2}, x_{n+3}) .$$

Then

$$F(M(Tx_n, Tx_{n+1})) + \phi(M(x_n, x_{n+1})) \preceq F(M(x_n, x_{n+1}))$$

implies

$F(d(x_{n+2}, x_{n+3}) \preceq F(d(x_{n+1}, x_{n+2}) - \phi(d(x_{n+1}, x_{n+2})) \preceq F(d(x_{n+1}, x_{n+2})$ wich a contradiction.

denote

$$d_n = d(x_{n+1}; x_n); n = 0; 1; 2; \ldots$$

Suppose that $x_{n+1} \neq x_n$ for every $n \in X$ then $d_n \succ \theta$ for all $n \in \mathbb{N}$ and using (1) the following holds for every $n \in \mathbb{N}$

$$F(d_n) \preceq F(d_{n-1}) - \phi(d_{n-1}) \prec F(d_{n-1}) \ (2)$$

Hence F is non decreasing and so the sequence (d_n) is monotonically decreasing in $\mathbb{A}$. So there exists $\theta \preceq t \in \mathbb{A}$ such that

$$d(x_n, x_{n+1}) \to t \text{ as } n \to \infty$$

From (2) we obtain $lim_{n\to\infty} F(d_{n)=\theta}$ that together with (ii) gives

$$lim_{n\to\infty} d_n = \theta \ (3)$$

Now we shall show that $\{x_n\}$ is a Cauchy sequence in $(X, \mathbb{A}, d)$.

Let, $n, p \in \mathbb{N}$.Then

$$d(x_n, x_{n+p}) \preceq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+p}) \preceq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + d(x_{n+2}, x_p)$$

$$\vdots$$

$$\preceq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \ldots + d(x_{n+p-1}, x_{n+p}).$$

Taking the limit as $n \to \infty$ we get $lim_{n\to\infty} d(x_n, x_{n+p}) = \theta$.

Thus $\{x_n\}$ is a Cauchy sequence. Since the space is complete,

there exists $u \in X$ such that $lim_{n\to\infty} x_n = u$.

Again T is continous.Therefore $lim_{n\to\infty} Tx_n = Tu$ ie $lim_{n\to\infty} x_{n+1} = u = Tu$.

Thus u is a fixed point of T. Again suppose $Tu \neq u$. So $d(Tu, u) > \theta$.

Since F is strictly increasing, we take the limit as $n \to +\infty$

$$F(lim_{n\to+\infty} M(Tx_n, Tu)) \preceq F(lim_{n\to+\infty} M(x_n, u)).$$

Now

$lim_{n\to+\infty} M(x_n, u)$

$$= lim_{n\to+\infty} max\{d(x_n, u), d(x_n, Tx_n), d(u, Tu), \frac{d(x_n, Tu) + d(u, Tx_n)}{2}\}$$
$$= max\{\theta, \theta, \theta, \theta\} = \theta$$

and

$lim_{n\to+\infty} M(Tx_n, Tu)$

$$= lim_{n\to+\infty} max\{d(Tx_n, Tu), d(Tx_n, T^2x_n), d(Tu, T^2u), \frac{d(Tx_n, T^2u) + d(Tu, T^2x_n)}{2}\}$$
$$= max\{\theta, \theta, \theta, \theta\} = \theta$$

We get $F(\theta) \prec F(\theta)$ which a contradiction.

Thus $Tu = u$.

To show the uniqueness, let v be another fixed point of T.

Then by given condition

$$\phi(u, v) + F(M(Tu, Tv)) \preceq F(M(u, v))$$
$$\Rightarrow \phi(u, v) + F(M(u, v)) \preceq F(M(u, v)$$
$$\Rightarrow \phi(u, v) \preceq \theta.$$

Which is a contradiction.

Therefore T has a unique fixed point in X. □

Example 1.14. Considering the Example 3.2 , we conclude that inequality (1) remains valid for F and T constructed as above and consequently by an application of Theorem 3.3, T has a unique fixed point.
it is seen that 0 is the unique fixed point of T.

Corollary 1.15. *$(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)-C^* valued contraction mapping.*

Where

$$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty)\}.$$

Then T has a unique fixed point.

Inspired by $MF-$ contraction of Hardy Rogers type in a complete metric space [1] we give

Theorem 1.16. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF) C^* -valued contraction of Hardy Rogers type where*

$M(x, y) = \alpha_1 d(x, y) + \alpha_2 d(x, Tx) + \alpha_3 d(y, Ty) + \alpha_4 d(x, Ty) + \alpha_5 d(y, Tx)$ *and* $\alpha_i \geq 0, i \in \{1, 2, 3, 4, 5\}$ *and* $\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + 2\alpha_5 < 1$.

Then T has a unique fixed point in X.

Proof. Let x_n be a sequence in X with an initial approximation $x_0 \in X$ such that $x_{n+1} = Tx_n$ for all $n \in \mathbb{N} \cup \{0\}$. Clearly, if $x_{n+1} = x_n$, then x_0 is a fixed point of T and is unique. Now we show that $lim_{n\to\infty} d(x_n, x_{n+1}) = \theta$. We have

$$\begin{aligned} M(x_n, x_{n+1}) &= \alpha_1 d(x_n, x_{n+1}) + \alpha_2 d(x_n, Tx_n) + \alpha_3 d(x_{n+1}, Tx_{n+1}) + \alpha_4 d(x_n, Tx_{n+1}) + \alpha_5 d(x_{n+1}, Tx_n) \\ &= \alpha_1 d(x_n, x_{n+1}) + \alpha_2 d(x_n, x_{n+1}) + \alpha_3 d(x_{n+1}, x_{n+2}) + \\ &\quad \alpha_4 d(x_n, x_{n+2}) + \alpha_5 d(x_{n+1}, x_{n+1}) \\ &\preceq (\alpha_1 + \alpha_2 + \alpha_4) d(x_n, x_{n+1}) + (\alpha_3 + \alpha_4) d(x_{n+1}, x_{n+2}), \end{aligned}$$

and

$$\begin{aligned} M(Tx_n, Tx_{n+1}) &= \alpha_1 d(Tx_n, Tx_{n+1}) + \alpha_2 d(Tx_n, T^2 x_n) + \alpha_3 d(Tx_{n+1}, T^2 x_{n+1}) + \\ &\quad \alpha_4 d(Tx_n, T^2 x_{n+1}) + \alpha_5 d(Tx_{n+1}, T^2 x_n) \\ &= \alpha_1 d(x_{n+1}, x_{n+2}) + \alpha_2 d(x_{n+1}, x_{n+2}) + \alpha_3 d(x_{n+2}, x_{n+3}) + \\ &\quad \alpha_4 d(x_{n+1}, x_{n+3}) + \alpha_5 d(x_{n+2}, x_{n+2}) \\ &\preceq (\alpha_1 + \alpha_2 + \alpha_4) d(x_{n+1}, x_{n+2}) + (\alpha_3 + \alpha_4) d(x_{n+2}, x_{n+3}), \end{aligned}$$

If $d(x_n, x_{n+1}) \preceq d(x_{n+1}, x_{n+2})$,

then

$$M(x_n, x_{n+1}) \preceq (\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4) d(x_{n+1}, x_{n+2})$$

and

$$M(Tx_n, Tx_{n+1}) \preceq (\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4) d(x_{n+2}, x_{n+3})$$

Then

$F((\alpha_1+\alpha_2+\alpha_3+2\alpha_4) d(x_{n+2}, x_{n+3})) \preceq F(\alpha_1+\alpha_2+\alpha_3+2\alpha_4) d(x_{n+1}, x_{n+2}) - \phi((\alpha_1+\alpha_2+\alpha_3+2\alpha_4) d(x_{n+1}, x_{n+2}))$

Using the propreties of F and ϕ we have

$$F(d(x_{n+1}, x_{n+2})) \preceq F(d(x_n, x_{n+1})) - \phi(d(x_n, x_{n+1})).$$

Since $(\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4) < 1$.

There exists $u \in \mathbb{A}$ such that $lim_{n\to+\infty} d(x_n, x_{n+1}) = u$.

Taking $n \to +\infty$ in $F(d(x_{n+1}, x_{n+2})) \preceq F(d(x_n, x_{n+1})) - \phi(d(x_n, x_{n+1}))$ we have

$F(u) \preceq F(u) - \phi(u)$ which is a contradiction unless $u = \theta$.

Hence

$$lim_{n\to+\infty} d(x_n, x_{n+1}) = \theta.$$

We can easily show as above in theorem 3.3 that $\{x_n\}$ is a Cauchy sequence.

Since the space X is complete, there exists an $a \in X$ such that $lim_{n\to+\infty} x_n = a$.

Also T is continuous. So we have

$$lim_{n\to+\infty} Tx_n = Ta \text{ i.e, } lim_{n\to+\infty} x_n = a = Ta.$$

Thus a is a fixed point of T.

Uniqueness. Now, suppose that $z, u \in X$ are two fixed points of T such that $u \neq z$.

Therefore, we have

$$d(u, z) = d(Tu, Tz) \succ \theta$$

We have

$$F(M(z, u)) = F(M(Tz, Tu)) \preceq F(M(z, u)) - \phi(M(z, u)) \prec F(M(z, u)).$$

It is a contradiction. Therefore $u = z$. □

Corollary 1.17. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)-C^*-algebra valued of Banach -type, where*

$$M(x, y) = \alpha d(x, y) \text{ and } 0 < \alpha < 1.$$

Then T has a unique fixed point in X.

Corollary 1.18. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)-Kannan -type C^*-algebra valued contraction, where*

$$M(x, y) = \alpha d(x, Tx) + \beta d(y, Ty) \text{ and } 0 \leq \alpha + \beta < 1.$$

Then T has a unique fixed point in X.

Corollary 1.19. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)-Chaterjea type C^*-algebra valued contraction, where*

$$M(x, y) = \alpha d(x, Ty) + \beta d(y, Tx) \text{ and } \forall \alpha, \beta \geq , \alpha + \beta < 1.$$

Then T has a unique fixed point in X.

Corollary 1.20. *Let $(X, \mathbb{A}, d)$ be a complete C^*-algebra valued metric space and let $T : X \to X$ be a (ϕ, MF)- Reich type C^*-algebra valued contraction , where*

$$M(x, y) = \alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) \text{ and } \forall \alpha, \beta, \gamma \geq 0 , \alpha + \beta + \gamma < 1.$$

Then T has a unique fixed point in X.

Example 1.21. Let $X = [0.2]$ and $d : X \times X \to \mathbb{R}^2$.

Suppose that $d(x, y) = (\mid x - y \mid, \mid x - y \mid)$ for $x, y \in X$.

Then, $(X, \mathbb{R}^2, d)$ is a C^*- algebra valued metric space.

$T : X \to X$ be given by $Tx = \frac{1}{3}x$ and F is given by $F(x, y) = (x^2, 0)$.

Suppose $y < x$.

Then

$$M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty)\} = max\{(|x - y|, |x - y|), (\frac{2}{3}|x|, \frac{2}{3}|x|), (\frac{2}{3}|y|, \frac{2}{3}|y|)\}.$$

If $|x - y| \leq \frac{2}{3}|x|$,thus $M(x, y) = (\frac{2}{3}|x|, \frac{2}{3}|x|)$ and $M(Tx, Ty) = (\frac{2}{9}|x|, \frac{2}{9}|x|)$.

Therefore $F(M(x, y)) = (\frac{x^2}{9}, 0)$ and $F(M(Tx, Ty)) = (\frac{4x^2}{81}, 0)$.

Taking

$$\phi(M(x, y)) = (\frac{5x^2}{81}, 0).$$

Therefore by Corollary 3.5 T has a unique fixed point 0.

If $\frac{2}{3}|x| \leq |x - y|$, then $M(x, y) = (|x - y|, |x - y|)$ and $M(Tx, Ty) = (\frac{1}{3}|x - y|, \frac{1}{3}|x - y|)$.

Therefore,

$$F(M(x, y)) = F(|x - y|, |x - y|) = (|x - y|^2, 0)$$

and

$$F(M(Tx, Ty)) = F((\frac{1}{3}|x - y|, \frac{1}{3}|x - y|)) = ((\frac{1}{9}|x - y|^2, 0))$$

Taking,

$$\phi(M(x, y)) = (\frac{4}{9}|x - y|^2, 0)$$

Therefore by Corollary 3.5 T has a unique fixed point 0.

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