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On the Non-Newtonian Gadovan Sequences

Ceremnur Tetik[1], Rabia Dertli[1], Abdullah Dertli[1], Senol Eren[1]

[1]Department of Mathematics, Ondokuz Mayıs University, Samsun, Turkey

Correspondence should be addressed to Abdullah Dertli: abdullah.dertli@gmail.com

Abstract
In this study, non-Newtonian Gadovan numbers are introduced and their properties are examined within non-Newtonian calculus, a mathematical approach that has recently garnered significant attention. We also obtain the generating matrix of non-Newtonian Padovan numbers.

Keywords: Non-Newtonian calculus, non-Newtonian Gadovan numbers.

1 Introduction

The sequence of Padovan numbers was discovered by Richard Padovan. The formula for obtaining the Padovan sequences is

$$P_{n+3} = P_{n+1} + P_n$$

with $P_0 = P_1 = P_2 = 1$, [6]. Some of the terms of the Padovan sequence are

$$1, 1, 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, \ldots.$$

The sequence of Gadovan numbers was defined by Diskaya and Menken, which generalizes a new class of Padovan numbers. The formula for obtaining the Gadovan sequences is

$$GP_{n+3} = GP_{n+1} + GP_n$$

with $GP_0 = a, \quad GP_1 = b$ and $GP_2 = c$, [3]. Some of the terms of the Gadovan sequence are

$$a, b, c, a + b, b + c, a + b + c, a + 2b + c, a + 2b + 2c, \ldots.$$

The non-Newtonian calculus, introduced by Grossman and Katz in 1972, offers an alternative viewpoint to classic Newtonian and Leibnizian calculus, creating a novel branch of mathematics, [4]. This emerging discipline incorporates a wide array of innovative topics for exploration. Non-Newtonian calculus encompasses various forms of computation, geometric, bigeometric, quadratic and biquadratic approaches. Furthermore, it demonstrates considerable promise for applications across diverse domains such as technology, engineering, physics, finance, dynamic systems, and cancer therapy.

A completely ordered field is called arithmetic if its realm is a subset of $\mathbb{R}$. A generator is a one-to-one function whose domain $\mathbb{R}$ and whose range is a subset of $\mathbb{R}$. Let α be a generator with range A. The set of non-Newtonian real numbers is denoted by $\mathbb{R}_\alpha$.

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Let α be arbitrarily chosen generator which image the set $\mathbb{R}$ to A and $*$-calculus also be the ordered pairs of arithmetics. The following notations will be used

$$\dot{a} \dot{+} \dot{b} = \alpha \left\{ \alpha^{-1} (\dot{a}) + \alpha^{-1} \left(\dot{b} \right) \right\},$$

$$\dot{a} \dot{-} \dot{b} = \alpha \left\{ \alpha^{-1} (\dot{a}) - \alpha^{-1} \left(\dot{b} \right) \right\},$$

$$\dot{a} \dot{\times} \dot{b} = \alpha \left\{ \alpha^{-1} (\dot{a}) \times \alpha^{-1} \left(\dot{b} \right) \right\},$$

$$\dot{a} \dot{/} \dot{b} = \alpha \left\{ \alpha^{-1} (\dot{a}) / \alpha^{-1} \left(\dot{b} \right) \right\}, \dot{b} \neq \dot{0},$$

$$\dot{a} \dot{\leq} \dot{b} \Leftrightarrow \alpha^{-1} (\dot{a}) \leq \alpha^{-1} \left(\dot{b} \right).$$

α-zero and α-one numbers are denoted by $\dot{0} = \alpha(0)$ and $\dot{1} = \alpha(1)$, [1].

This paper expands on the concepts introduced in [2, 5] by exploring their application to Gadovan numbers. The central focus is to define and investigate the non-Newtonian forms of this well-known sequence.

2 Main Results

Definition 1. *[2], The non-Newtonian Padovan sequence is defined by the relation for $n \geqslant 0$*

$$NNP_{n+3} = NNP_{n+1} \dot{+} NNP_n,$$

with initial values $NNP_0 = NNP_1 = NNP_2 = 1$, where $NNP_n = \dot{P}_n = \alpha(P_n)$.

The non-Newtonian Padovan numbers are generated by a matrix

$$Q = \begin{bmatrix} \dot{0} & \dot{1} & \dot{0} \\ \dot{0} & \dot{0} & \dot{1} \\ \dot{1} & \dot{1} & \dot{0} \end{bmatrix}.$$

The powers of Q give

$$Q^n = \begin{bmatrix} NNP_{n-5} & NNP_{n-3} & NNP_{n-4} \\ NNP_{n-4} & NNP_{n-2} & NNP_{n-3} \\ NNP_{n-3} & NNP_{n-1} & NNP_{n-2} \end{bmatrix}.$$

The characteristic equation of the non-Newtonian Padovan sequence is

$$\dot{x}^{\dot{3}} \dot{-} \dot{x} \dot{-} \dot{1} = \dot{0},$$

so we have to solve this equation, we find three distinct roots q_1, q_2 and q_3, [2].

Theorem 2. *[2], The Binet-like formula for the n-th non-Newtonian Padovan number is*

$$NNP_n = \dot{p}_1 \dot{\times} \dot{q}_1^{\dot{n}} \dot{+} \dot{p}_2 \dot{\times} \dot{q}_2^{\dot{n}} \dot{+} \dot{p}_3 \dot{\times} \dot{q}_3^{\dot{n}}.$$

Definition 3. *The non-Newtonian Gadovan sequence is defined by the relation for $n \geq 1$*

$$NNGP_{n+3} = NNGP_{n+1} \dot{+} NNGP_n,$$

with initial values $NNGP_1 = \dot{a}$, $NNGP_2 = \dot{b}$, $NNGP_3 = \dot{c}$, where

$$NNGP_n = \alpha(GP_n).$$

The non-Newtonian Gadovan sequence, denoted by $\{NNGP_n\}$, is

$$\dot{a},\, \dot{b},\, \dot{c},\, \dot{a}\dot{+}\dot{b},\, \dot{b}\dot{+}\dot{c},\, \dot{a}\dot{+}\dot{b}\dot{+}\dot{c},\, \dot{a}\dot{+}\dot{2}\dot{\times}\dot{b}\dot{+}\dot{c},\, \dot{a}\dot{+}\dot{2}\dot{\times}\dot{b}\dot{+}\dot{2}\dot{\times}\dot{c}, \ldots \ .$$

Using the generator I, defined by $\alpha(x) = x$, we obtain Gadovan numbers concerning classical arithmetic. Also, by choosing the generator exp defined by $\alpha(x) = e^x$, we obtain Gadovan numbers with respect to geometric arithmetic, as follows:

$$e^a, e^b, e^c, e^{a+b}, e^{a+c}, e^{a+b+c}, e^{a+2b+c}, \ldots, e^{GP_n}, \ldots \ .$$

The characteristic equation of the non-Newtonian Gadovan numbers is

$$\dot{x}^{\dot{3}}\dot{-}\dot{x}\dot{-}\dot{1} = \dot{0}.$$

In the following theorem, we focus on the relation between non-Newtonian Padovan and non-Newtonian Gadovan numbers.

Theorem 4. *Let NNP_n and $NNGP_n$ be n-th non-Newtonian Padovan and n-th non-Newtonian Gadovan numbers, respectively. Then, for $n \geq 4$*

$$NNGP_n = \dot{a}\dot{\times}NNP_{n-4}\dot{+}\dot{b}\dot{\times}NNP_{n-2}\dot{+}\dot{c}\dot{\times}NNP_{n-3}\,.$$

Proof. We establish this using the principle of mathematical induction. Since

$$NNGP_4 = \dot{a}\dot{\times}NNP_0\dot{+}\dot{b}\dot{\times}NNP_2\dot{+}\dot{c}\dot{\times}NNP_1 = \dot{a}\dot{+}\dot{b}$$

and

$$NNGP_5 = \dot{a}\dot{\times}NNP_1\dot{+}\dot{b}\dot{\times}NNP_3\dot{+}\dot{c}\dot{\times}NNP_2 = \dot{b}\dot{+}\dot{c}$$

the result is true for $n = 4, 5$. Assume that the relation is true for all positive integers $n \leq k$. Then,

$$\begin{aligned}
NNGP_{k+3} &= NNGP_{k+1}\dot{+}\, NNGP_k \\
&= \dot{a}\dot{\times}NNP_{k-3}\dot{+}\dot{b}\dot{\times}NNP_{k-1}\dot{+}\dot{c}\dot{\times}NNP_{k-2} + \dot{a}\dot{\times}NNP_{k-4}\dot{+}\dot{b}\dot{\times}NNP_{k-2}\dot{+}\dot{c}\dot{\times}NNP_{k-3} \\
&= \dot{a}\dot{\times}(NNP_{k-3}\dot{+}NNP_{k-4})\dot{+}\dot{b}\dot{\times}(NNP_{k-1}\dot{+}NNP_{k-2})\dot{+}\dot{c}\dot{\times}(NNP_{k-2}\dot{+}NNP_{k-3}) \\
&= \dot{a}\dot{\times}NNP_{k-1}\dot{+}\dot{b}\dot{\times}NNP_{k+1}\dot{+}\dot{c}\dot{\times}NNP_k
\end{aligned}$$

Thus, by the strong version of the principle of mathematical induction, the formula works for all positive integers n. □

Theorem 5. *The Binet-like formula for the n-th non-Newtonian Gadovan number is*

$$NNGP_n = \dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}\dot{q}_1^{\dot{n}}\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}\dot{q}_2^{\dot{n}}\dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}\dot{q}_3^{\dot{n}},$$

where

$$\begin{aligned}
\dot{\gamma}_1 &= \dot{a}\dot{\times}\dot{q}_1^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_1^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_1^{\dot{-}\dot{3}}, \\
\dot{\gamma}_2 &= \dot{a}\dot{\times}\dot{q}_2^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_2^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_2^{\dot{-}\dot{3}}, \\
\dot{\gamma}_3 &= \dot{a}\dot{\times}\dot{q}_3^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_3^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_3^{\dot{-}\dot{3}}.
\end{aligned}$$

Proof. Using Theorem 2, we have

$$\begin{aligned} NNGP_n &= \dot{a}\dot{\times}NNP_{n-4}\dot{+}\dot{b}\dot{\times}NNP_{n-2}\dot{+}\dot{c}\dot{\times}NNP_{n-3} \\ &= \dot{a}\dot{\times}(\dot{p}_1\dot{\times}\dot{q}_1^{\dot{n}\dot{-}\dot{4}}\dot{+}\dot{p}_2\dot{\times}\dot{q}_2^{\dot{n}\dot{-}\dot{4}}\dot{+}\dot{p}_3\dot{\times}\dot{q}_3^{\dot{n}\dot{-}\dot{4}})\dot{+}\dot{b}\dot{\times}(\dot{p}_1\dot{\times}\dot{q}_1^{\dot{n}\dot{-}\dot{2}}\dot{+}\dot{p}_2\dot{\times}\dot{q}_2^{\dot{n}\dot{-}\dot{2}}\dot{+}\dot{p}_3\dot{\times}\dot{q}_3^{\dot{n}\dot{-}\dot{2}}) \\ &\quad \dot{+}\dot{c}\dot{\times}(\dot{p}_1\dot{\times}\dot{q}_1^{\dot{n}\dot{-}\dot{3}}\dot{+}\dot{p}_2\dot{\times}\dot{q}_2^{\dot{n}\dot{-}\dot{3}}\dot{+}\dot{p}_3\dot{\times}\dot{q}_3^{\dot{n}\dot{-}\dot{3}}) \\ &= \dot{p}_1\dot{\times}(\dot{a}\dot{\times}\dot{q}_1^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_1^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_1^{\dot{-}\dot{3}})\dot{\times}\dot{q}_1^{\dot{n}}\dot{+}\dot{p}_2\dot{\times}(\dot{a}\dot{\times}\dot{q}_2^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_2^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_2^{\dot{-}\dot{3}})\dot{\times}\dot{q}_2^{\dot{n}} \\ &\quad \dot{+}\dot{p}_3\dot{\times}(\dot{a}\dot{\times}\dot{q}_3^{\dot{-}\dot{4}}\dot{+}\dot{b}\dot{\times}\dot{q}_3^{\dot{-}\dot{2}}\dot{+}\dot{c}\dot{\times}\dot{q}_3^{\dot{-}\dot{3}})\dot{\times}\dot{q}_3^{\dot{n}} \\ &= \dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}\dot{q}_1^{\dot{n}}\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}\dot{q}_2^{\dot{n}}\dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}\dot{q}_3^{\dot{n}}\ . \end{aligned}$$

□

Theorem 6. *The generating function of the non-Newtonian Gadovan numbers is*

$$\mathbb{G}_{NNGP}(\dot{x}) = \frac{\dot{a}\dot{\times}\dot{x}\dot{+}\dot{b}\dot{\times}\dot{x}^{\dot{2}}\dot{+}(\dot{c}\dot{-}\dot{a})\dot{\times}\dot{x}^{\dot{3}}}{\dot{1}\dot{-}\dot{x}^{\dot{2}}\dot{-}\dot{x}^{\dot{3}}}\alpha\ .$$

Proof. Assume that the function

$$\mathbb{G}_{NNGP}(\dot{x}) = \alpha\sum_{n=1}^{\infty} NNGP_n\dot{\times}\dot{x}^{\dot{n}} = NNGP_1\dot{\times}\dot{x}\dot{+}NNGP_2\dot{\times}\dot{x}^{\dot{2}}\dot{+}\cdots\dot{+}NNGP_n\dot{\times}\dot{x}^{\dot{n}}\dot{+}\cdots$$

be the generating function of the non-Newtonian Gadovan numbers. Multiply both of side of the equality by the term $\dot{-}\dot{x}^{\dot{2}}$ such as

$$(\dot{-}\dot{x}^{\dot{2}})\dot{\times}\mathbb{G}_{NNGP}(\dot{x}) = \dot{-}NNGP_1\dot{\times}\dot{x}^{\dot{3}}\dot{-}NNGP_2\dot{\times}\dot{x}^{\dot{4}}\dot{-}\cdots\dot{-}NNGP_n\dot{\times}\dot{x}^{\dot{n}\dot{+}\dot{2}}\dot{-}\cdots$$

and that is multiplied every side with $\dot{-}\dot{x}^{\dot{3}}$ such as

$$(\dot{-}\dot{x}^{\dot{3}})\dot{\times}\mathbb{G}_{NNGP}(\dot{x}) = \dot{-}NNGP\dot{\times}\dot{x}^{\dot{4}}\dot{-}NNGP_2\dot{\times}\dot{x}^{\dot{5}}\dot{-}\cdots\dot{-}NNGP_n\dot{\times}\dot{x}^{\dot{n}\dot{+}\dot{3}}\dot{-}\cdots\ .$$

Then, we write

$$\begin{aligned} (\dot{1}\dot{-}\dot{x}^{\dot{2}}\dot{-}\dot{x}^{\dot{3}})\dot{\times}\mathbb{G}_{NNGP}(\dot{x}) &= NNGP_1\dot{\times}\dot{x}\dot{+}NNGP_2\dot{\times}\dot{x}^{\dot{2}}\dot{+}(NNGP_3\dot{-}NNGP_1)\dot{\times}\dot{x}^{\dot{3}} \\ &\quad \dot{+}(NNGP_4\dot{-}NNGP_2\dot{-}NNGP_1)\dot{\times}\dot{x}^{\dot{4}}\dot{+}\cdots\dot{+} \\ &\quad (NNGP_n\dot{-}NNGP_{n-2}\dot{-}NNGP_{n-3})\dot{\times}\dot{x}^{\dot{n}}\dot{+}\cdots\ . \end{aligned}$$

Now, by using $NNGP_1 = \dot{a}$, $NNGP_2 = \dot{b}$, $NNGP_3 = \dot{c}$, $NNGP_4 = \dot{a}\dot{+}\dot{b}$, $NNGP_5 = \dot{b}\dot{+}\dot{c}$,..., we obtain that

$$\mathbb{G}_{NNGP}(\dot{x}) = \frac{\dot{a}\dot{\times}\dot{x}\dot{+}\dot{b}\dot{\times}\dot{x}^{\dot{2}}\dot{+}(\dot{c}\dot{-}\dot{a})\dot{\times}\dot{x}^{\dot{3}}}{\dot{1}\dot{-}\dot{x}^{\dot{2}}\dot{-}\dot{x}^{\dot{3}}}\alpha\ .$$

□

If $\dot{a} = \dot{b} = \dot{c} = \dot{1}$, the generator function of non-Newtonian Padovan numbers $\mathbb{G}_{NNP}(\dot{x})$ is obtained.

Theorem 7. *Let m and n be positive integers. Then,*

$$\alpha\sum_{n=1}^{m} \dot{\binom{m}{n}}\dot{\times}NNGP_n = NNGP_{3m}\ .$$

Proof. Applying the Binet-like formula, we obtain the identity

$$\begin{aligned}
\alpha\sum_{n=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}NNGP_n &= \alpha\sum_{n=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}(\dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}\dot{q}_1^{\dot{n}}\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}\dot{q}_2^{\dot{n}}\dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}\dot{q}_3^{\dot{n}}) \\
&= \dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}\left(\alpha\sum_{n=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}\dot{q}_1^{\dot{n}}\dot{\times}\dot{1}^{(\dot{m}\dot{-}\dot{n})}\right)\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}\left(\alpha\sum_{n=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}\dot{q}_2^{\dot{n}}\dot{\times}\dot{1}^{(\dot{m}\dot{-}\dot{n})}\right) \\
&\quad \dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}\left(\alpha\sum_{n=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}\dot{q}_3^{\dot{n}}\dot{\times}\dot{1}^{(\dot{m}\dot{-}\dot{n})}\right) \\
&= \dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}(\dot{q}_1\dot{+}\dot{1})^{\dot{m}}\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}(\dot{q}_2\dot{+}\dot{1})^{\dot{m}}\dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}(\dot{q}_3\dot{+}\dot{1})^{\dot{m}} \\
&= \dot{p}_1\dot{\times}\dot{\gamma}_1\dot{\times}\dot{q}_1^{\dot{3}\dot{\times}\dot{m}}\dot{+}\dot{p}_2\dot{\times}\dot{\gamma}_2\dot{\times}\dot{q}_2^{\dot{3}\dot{\times}\dot{m}}\dot{+}\dot{p}_3\dot{\times}\dot{\gamma}_3\dot{\times}\dot{q}_3^{\dot{3}\dot{\times}\dot{m}} \\
&= NNGP_{3m}
\end{aligned}$$

□

Theorem 8. *Let m, n and k be positive integers. Then,*

$$\alpha\sum_{k=1}^{m}\dot{(}\begin{array}{c}m\\n\end{array}\dot{)}\dot{\times}NNGP_{n-k} = NNGP_{n+2m}\ .$$

Proof. The proof can be proven as in the proof 7. □

Theorem 9. *For all $n \geq 1$,*

$$\begin{bmatrix} NNGP_{n-3} & NNGP_{n-1} & NNGP_{n-2} \\ NNGP_{n-2} & NNGP_n & NNGP_{n-1} \\ NNGP_{n-1} & NNGP_{n+1} & NNGP_n \end{bmatrix} = \begin{bmatrix} NNP_{n-3} & NNP_{n-1} & NNP_{n-2} \\ NNP_{n-2} & NNP_n & NNP_{n-1} \\ NNP_{n-1} & NNP_{n+1} & NNP_n \end{bmatrix}\dot{\times}\begin{bmatrix} \dot{c}\dot{-}\dot{a} & \dot{b} & \dot{a} \\ \dot{a} & \dot{c} & \dot{b} \\ \dot{b} & \dot{a}\dot{+}\dot{b} & \dot{c} \end{bmatrix}.$$

Proof. Let

$$\begin{bmatrix} NNP_{n-3} & NNP_{n-1} & NNP_{n-2} \\ NNP_{n-2} & NNP_n & NNP_{n-1} \\ NNP_{n-1} & NNP_{n+1} & NNP_n \end{bmatrix}\dot{\times}\begin{bmatrix} \dot{c}\dot{-}\dot{a} & \dot{b} & \dot{a} \\ \dot{a} & \dot{c} & \dot{b} \\ \dot{b} & \dot{a}\dot{+}\dot{b} & \dot{c} \end{bmatrix} = \begin{bmatrix} X & Y & Z \\ K & L & M \\ A & B & C \end{bmatrix}.$$

Then, we write

$$\begin{aligned}
X &= (\dot{c}\dot{-}\dot{a})\dot{\times}NNP_{n-3}\dot{+}\dot{a}\dot{\times}NNP_{n-1}\dot{+}\dot{b}\dot{\times}NNP_{n-2} \\
&= \dot{c}\dot{\times}NNP_{n-3}\dot{-}\dot{a}\dot{\times}NNP_{n-3}\dot{+}\dot{a}\dot{\times}(NNP_{n-3}\dot{+}NNP_{n-4})\dot{+}\dot{b}\dot{\times}NNP_{n-2} \\
&= \dot{a}\dot{\times}NNP_{n-4}\dot{+}\dot{b}\dot{\times}NNP_{n-2}\dot{+}\dot{c}\dot{\times}NNP_{n-3} = NNGP_{n-3},
\end{aligned}$$

$$\begin{aligned}
Y &= \dot{b}\dot{\times}NNP_{n-3}\dot{+}\dot{c}\dot{\times}NNP_{n-1}\dot{+}(\dot{a}\dot{+}\dot{b})\dot{\times}NNP_{n-2} \\
&= \dot{b}\dot{\times}NNP_{n-3}\dot{+}\dot{c}\dot{\times}NNP_{n-1}\dot{+}\dot{a}\dot{\times}NNP_{n-2}\dot{+}\dot{b}\dot{\times}NNP_{n-2} \\
&= \dot{a}\dot{\times}NNP_{n-2}\dot{+}\dot{b}\dot{\times}NNP_n\dot{+}\dot{c}\dot{\times}NNP_{n-1} = NNGP_{n-1},
\end{aligned}$$

$$Z = \dot{a}\dot{\times}NNP_{n-3}\dot{+}\dot{b}\dot{\times}NNP_{n-1}\dot{+}\dot{c}\dot{\times}NNP_{n-2} = NNGP_{n-2},$$

$$\begin{aligned}
K &= (\dot{c}\dot{-}\dot{a})\dot{\times}NNP_{n-2}\dot{+}\dot{a}\dot{\times}NNP_n\dot{+}\dot{b}\dot{\times}NNP_{n-1} \\
&= \dot{c}\dot{\times}NNP_{n-2}\dot{-}\dot{a}\dot{\times}NNP_{n-2}\dot{+}\dot{a}\dot{\times}(NNP_{n-2}\dot{+}NNP_{n-3})\dot{+}\dot{b}\dot{\times}NNP_{n-1} \\
&= \dot{a}\dot{\times}NNP_{n-3}\dot{+}\dot{b}\dot{\times}NNP_{n-1}\dot{+}\dot{c}\dot{\times}NNP_{n-2} = NNGP_{n-2},
\end{aligned}$$

$$\begin{aligned} L &= \dot{b}\dot{\times}NNP_{n-2}\dot{+}\dot{c}\dot{\times}NNP_n\dot{+}(\dot{a}\dot{+}\dot{b})\dot{\times}NNP_{n-1} \\ &= \dot{b}\dot{\times}NNP_{n-2}\dot{+}\dot{c}\dot{\times}NNP_n\dot{+}\dot{a}\dot{\times}NNP_{n-1}\dot{+}\dot{b}\dot{\times}NNP_{n-1} \\ &= \dot{a}\dot{\times}NNP_{n-1}\dot{+}\dot{b}\dot{\times}NNP_{n+1}\dot{+}\dot{c}\dot{\times}NNP_n = NNGP_n, \end{aligned}$$

$$M = \dot{a}\dot{\times}NNP_{n-2}\dot{+}\dot{b}\dot{\times}NNP_n\dot{+}\dot{c}\dot{\times}NNP_{n-1} = NNGP_{n-1},$$

$$\begin{aligned} A &= (\dot{c}\dot{-}\dot{a})\dot{\times}NNP_{n-1}\dot{+}\dot{a}\dot{\times}NNP_{n+1}\dot{+}\dot{b}\dot{\times}NNP_n \\ &= \dot{c}\dot{\times}NNP_{n-1}\dot{-}\dot{a}\dot{\times}NNP_{n-1}\dot{+}\dot{a}\dot{\times}(NNP_{n-1}\dot{+}NNP_{n-2})\dot{+}\dot{b}\dot{\times}NNP_n \\ &= \dot{a}\dot{\times}NNP_{n-2}\dot{+}\dot{b}\dot{\times}NNP_n\dot{+}\dot{c}\dot{\times}NNP_{n-1} = NNGP_{n-1}, \end{aligned}$$

$$\begin{aligned} B &= \dot{b}\dot{\times}NNP_{n-1}\dot{+}\dot{c}\dot{\times}NNP_{n+1}\dot{+}(\dot{a}\dot{+}\dot{b})\dot{\times}NNP_n \\ &= \dot{b}\dot{\times}NNP_{n-1}\dot{+}\dot{c}\dot{\times}NNP_{n+1}\dot{+}\dot{a}\dot{\times}NNP_n\dot{+}\dot{b}\dot{\times}NNP_n \\ &= \dot{a}\dot{\times}NNP_n\dot{+}\dot{b}\dot{\times}NNP_{n+2}\dot{+}\dot{c}\dot{\times}NNP_{n+1} = NNGP_{n+1}, \end{aligned}$$

$$C = \dot{a}\dot{\times}NNP_{n-1}\dot{+}\dot{b}\dot{\times}NNP_{n+1}\dot{+}\dot{c}\dot{\times}NNP_n = NNGP_n.$$

So,

$$\begin{bmatrix} X & Y & Z \\ K & L & M \\ A & B & C \end{bmatrix} = \begin{bmatrix} NNGP_{n-3} & NNGP_{n-1} & NNGP_{n-2} \\ NNGP_{n-2} & NNGP_n & NNGP_{n-1} \\ NNGP_{n-1} & NNGP_{n+1} & NNGP_n \end{bmatrix}.$$

□

3 Conclusion

This study introduces Gadovan numbers within the context of non-Newtonian calculus and investigates their fundamental properties. Additionally, it gives the generating matrix for non-Newtonian Padovan numbers, providing a clearer understanding of their mathematical structure. Future research can further explore the use of these numbers in different disciplines, contributing to the development of non-Newtonian calculus.

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