ISSN: 2307-7743
www.scienceasia.asia

Asian J. Math. Appl. (2025) 2025:7

Modified Laplace-Variational Iteration Method for Solving System of Fractional Partial Differential Equations

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Abstract
In this seminar, Modified Laplace-Variational Iteration Method is introduced for solving system of nonlinear fractional partial differential equations (FPDEs). A detailed procedure of the method was constructed where recurrence relation are explicitly developed. Numerical examples that were previously solved by other well-known methods were again solved by the proposed method to test for its validity, accuracy and efficiency. The approximate solutions are obtained in the form of rapidly convergent series which demonstrate applicability, reliability and efficiency of the technique. Some of the obtained results were compared with the results of the well-known existing methods in the literature. It is observed from the results that the method is accurate and efficient for solving system of nonlinear FPDEs since the approximate solutions obtained are in good agreement to a reasonable decimal with exact solutions.

Keywords: Caputo Derivatives, Lagrange Multiplier, Laplace Transforms, Fractional Partial Derivative.

1 Introduction

Fractional calculus was formulated through the Leibnitz's letter to L'Hospital about 300 years ago where Leibnitz first discussed the meaning of one-half order derivative [13]. In the 20^{th} century, fractional derivatives proved itself to be an essential instrument in problems arising in rheology and fluid flow, frequency-dependent acoustic wave propagation in porous media, modeling of behaviour of visco-elasticity materials, micro-grids and decentralized wireless networks, electrochemistry of corrosion, epidemiology etc [4]. According to [16], FPDEs have been widely employed and developed in engineering and physics domains, and are better suited for representing complex phenomena and process.

As a result of rapid advancements in science and engineering, fractional differential equations have become subject of interest and rapidly growing areas of research [1]. This is because, according to [11], it has been shown that in the last few decades, derivatives of arbitrary order are convenient for describing properties of real materials. In [2], fractional calculus has been useful in the fields of mathematical physics, interface chaos and probability. Hence, it is used to describe a wide range of complex phenomena in the fields like anomalous diffusion, systems identification, continuous-time random walk dynamic systems, fractional electrical circuits, control theory, signal processing, fluid flow, etc. Fractional order partial differential equations are increasingly used to model problems in fluid flow, finance, physical and biological processes and systems [17]. Furthermore, the nonlinear oscillation of an earthquake is modelled using fractional derivatives and a fluid dynamic traffic

Received: 2025-02-13

model using fractional derivatives can eliminate the deficiency caused by an assumption of continuous traffic flow.

According to [4], fractional differential equations allow for the study of properties of materials while the models of fractional order allows for the modelling of memory and hereditary properties. FPDEs are strong tools for modelling and analyzing a wide range of phenomena, including heat conduction, wave propagation, image processing and data analysis [11]. In [8], Mathematicians have become increasingly interested in FPDEs due to their numerous applications in engineering, applied sciences, biology, mathematical physics, neurology, geophysics physics, financial mathematics, computing, containment transport, etc.

2 Preliminaries

Definition 2.1 (The Fractional Derivative)
The fractional derivative of $f(x)$ in the Caputo sense is defined as:

$$D_*^{\alpha} f(x) = \frac{1}{\Gamma(m-x)} \int_0^x \frac{f^{(m)}(t)}{(x-t)^{m-\alpha-1}} dt; m-1 < \alpha < m, m \in N \tag{1}$$

(See [17])

Definition 2.2 (The Laplace Transform, f(t))

The Laplace transform denoted by $f(t)$ is defined as:

$$L[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t) dt; \tag{2}$$

where s is either real or complex.

Definition 2.3 (The Laplace Transform of the Caputo Derivative)
According to [5] and [9], the Laplace transform of the Caputu derivative is defined as:

$$L[D_*^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=0}^{m-1} s^{\alpha-1-k} f^{(k)}(0); m-1 < \alpha < m \tag{3}$$

Definition 2.4 (Mittag-Leffler Function)
According to [6], [10], [10] and [7], one parameter function commonly known as Mittag-Leffler function, denoted by $E_{\alpha}(z)$, and valid in the whole complex plane is defined by the series:

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)}; \alpha > 0, z \in N \tag{4}$$

Definition 2.5 (Euler Gamma Function)
According to [15], Gamma function of $f(x)$ denoted by $\Gamma(x)$ is defined as:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \tag{5}$$

3 Description of Modified Laplace-Variational Iteration Method (MLVIM)

In this work, the system of NFPDEs considered is of the form:

$$\left.\begin{aligned} D_t^{\alpha} u(x,t) + L_1(u,v) + N_1(u,v) = g_1(x,t) \\ D_t^{\alpha} v(x,t) + L_2(u,v) + N_2(u,v) = g_2(x,t) \\ D_t^{\alpha} w(x,t) + L_3(u,v) + N_3(y,z) = g_n(x,t) \end{aligned}\right\} \tag{6}$$

with initial conditions

$$u(x,0) = f_1(x); v(x,0) = f_2(x); w(x,0) = f_3(x) \tag{7}$$

Procedure for the Modified Laplace-Variational Iteration Method

Step 1

The series solutions of (6) are:

$$u(x,t) = \sum_{n=0}^{\infty} u(x,t); v(x,t) = \sum_{n=0}^{\infty} v(x,t), w(x,t) \tag{8}$$

Step 2

Taking the Laplace transform of equation (6) with the initial conditions in (7) and using differentiation property of Laplace transform gives:

$$\left.\begin{aligned} L\left[u(x,t)\right] - \textstyle\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k u(x,t)}{\partial t^k}|_{t=0} = L\left[g_1(x,t) - L_1(u,v,w) - N_1(u,v,w)\right] \\ L\left[v(x,t)\right] - \textstyle\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k v(x,t)}{\partial t^k}|_{t=0} = L\left[g_2(x,t) - L_2(u,v,w) - N_2(u,v,w)\right] \\ L\left[w(x,t)\right] - \textstyle\sum_{k=0}^{n-1} s^{\alpha-1-k} \frac{\partial^k w(x,t)}{\partial t^k}|_{t=0} = L\left[g_3(x,t) - L_3(u,v,w) - N_3(u,v,w)\right] \end{aligned}\right\} \tag{9}$$

Step 3

The iteration formula obtained in equation (9) is used to suggest the main iterative scheme that involves the Lagrange multiplier defined as, $\lambda(s) = \frac{-1}{s^{\alpha}}$. Thus, $\lambda(s)$ is introduced to equation (9) to obtain:

$$\left.\begin{aligned} L\left[u_{n+1}(x,t)\right] = \tfrac{1}{s^{\alpha}} \left\{\textstyle\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k u(x,t)}{\partial t^k}|_{t=0} - L\left[g_1(x,t)\right] + L\left[L_1(u_n,v_n,w_n)\right] + \left[N_1(u_n,v_n)\right]\right\} \\ L\left[v_{n+1}(x,t)\right] = \tfrac{1}{s^{\alpha}} \left\{\textstyle\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k v(x,t)}{\partial t^k}|_{t=0} - L\left[g_2(x,t)\right] + L\left[L_2(u_n,v_n,w_n)\right] + \left[N_2(u_n,v_n)\right]\right\} \\ L\left[w_{n+1}(x,t)\right] = \tfrac{1}{s^{\alpha}} \left\{\textstyle\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k z(x,t)}{\partial t^k}|_{t=0} - L\left[g_n(x,t)\right] + L\left[L_n(u_n,v_n,w_n)\right] + \left[N_3(u_n,v_n,w_n)\right]\right\} \end{aligned}\right\} \tag{10}$$

Step 4

Find the inverse Laplace transform of equation (10) to obtain the iterative formula

$$\begin{aligned} u_{n+1}(x,t) =& L^{-1}\left[\frac{1}{s^{\alpha}}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k u(x,t)}{\partial t^k}|_{t=0} - L\Big[g_1(x,t)\Big]\right]\right] \\ &+ L^{-1}\left[\frac{1}{s^{\alpha}}\Big[L\Big[L_1(u_n,v_n,w_n)\Big] + \Big[N_1(u_n,v_n,w_n)\Big]\Big]\right] \end{aligned} \tag{11}$$

$$\begin{aligned} v_{n+1}(x,t) =& L^{-1}\left[\frac{1}{s^{\alpha}}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k v(x,t)}{\partial t^k}|_{t=0} - L\Big[g_2(x,t)\Big]\right]\right] \\ &+ L^{-1}\left[\frac{1}{s^{\alpha}}\left[L\left[L_2(u_n,v_n,w_n)\right] + \left[N_2(u_n,v_n,w_n)\right]\right]\right] \end{aligned} \tag{12}$$

$$\begin{aligned} w_{n+1}(x,t) =& L^{-1}\left[\frac{1}{s^{\alpha}}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k} \frac{\partial^k w(x,t)}{\partial t^k}|_{t=0} - L\left[g_3(x,t)\right]\right]\right] \\ &+ L^{-1}\left[\frac{1}{s^{\alpha}}\Big[L\Big[L_3(u_n,v_n,w_n)\Big] + \Big[N_2(u_n,v_n,w_n)\Big]\Big]\right] \end{aligned} \tag{13}$$

Step 5

The zero iterations are determined by

$$\left.\begin{array}{l} u_0(x,t) = L^{-1}\left[\frac{1}{s^\alpha}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k}\frac{\partial^k u(x,t)}{\partial t^k}|_{t=0}\right]\right] \\ v_0(x,t) = L^{-1}\left[\frac{1}{s^\alpha}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k}\frac{\partial^k v(x,t)}{\partial t^k}|_{t=0}\right]\right] \\ w_0(x,t) = L^{-1}\left[\frac{1}{s^\alpha}\left[\sum_{k=0}^{m-1} s^{\alpha-1-k}\frac{\partial^k z(x,t)}{\partial t^k}|_{t=0}\right]\right] \end{array}\right\} \tag{14}$$

Hence, the system of equations obtained in equations (11), (12), (13) and (14) are solved with MAPPLE 18.

4 Applications of Modified Laplace-Variational Iteration Method

In this section, the proposed Modified Laplace-Variational Iteration Method introduced is applied to obtain the numerical solutions of some system of nonlinear fractional partial differential equations.

Example 1

Solve the system of inhomogeneous fractional order nonlinear partial differential equations in three unknowns

$$\left.\begin{array}{r} \frac{\partial^\beta u}{\partial\tau^\beta} - \frac{\partial w}{\partial\xi}\frac{\partial v}{\partial\tau} - \frac{1}{2}\frac{\partial w}{\partial\tau}\frac{\partial^2 u}{\partial\xi^2} = -4\xi\tau \\ \frac{\partial^\beta v}{\partial\tau^\beta} - \frac{\partial w}{\partial\tau}\frac{\partial^2 u}{\partial\xi^2} = 6\tau \\ \frac{\partial^\beta w}{\partial\tau^\beta} - \frac{\partial^2 u}{\partial\xi^2} - \frac{\partial v}{\partial\xi}\frac{\partial w}{\partial\tau} = 4\xi\tau - 2\tau - 2 \end{array}\right\} \tag{15}$$

with initial conditions

$$u(\xi,0) = \xi^2 + 1; v(\xi,0) = \xi^2 - 1; w(\xi,0) = \xi^2 - 1 \tag{16}$$

[3] and [9].

Solution

The zero iterations $u_0(x,t)$, $v_0(x,t)$ and $w_0(x,t)$ are determined by (14) thus:

$$L\left[u_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1-k}\frac{\partial^k u}{\partial t^k}|_{t=0}\right]; L\left[v_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1-k}\frac{\partial^k v}{\partial t^k}|_{t=0}\right];$$

$$L\left[w_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1-k}\frac{\partial^k v}{\partial t^k}|_{t=0}\right] \tag{17}$$

Hence,

$$L\left[u_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1}u(\xi,0)\right] = \frac{1}{s}(\xi^2+1) \tag{18}$$

Taking the inverse Laplace transform of equation (18) to obtain:

$$u_0(\xi,\tau) = \xi^2 + 1 \tag{19}$$

Similarly,

$$L\left[v_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1}v_0(\xi,0)\right]\frac{1}{s}(\xi^2-1) \tag{20}$$

Taking the inverse Laplace transform of equation (20) to obtain:

$$v_0(\xi,\tau) = \xi^2 - 1 \tag{21}$$

and

$$L\left[w_0(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\sum_{k=0}^{n-1} s^{\beta-1}w_(\xi,0)\right] = \frac{1}{s}(\xi^2-1) \tag{22}$$

Taking the inverse Laplace transform of equation (22) to obtain:

$$w_0(\xi,\tau) = \xi^2 - 1 \tag{23}$$

$$N_{10} = w_0 v_0 + w_0 v_0$$

$$L\left[u_1(\xi,\tau)\right] = \frac{1}{s^\beta} L\left[w_{0\xi} v_{0\tau} + \frac{1}{2} w_{0\tau} u_{0\xi\xi} - 4\xi\tau\right]$$

$$L\left[u_1(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}(\xi^2-1) + \frac{1}{2}\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}(\xi^2+1) - 4\xi\tau\right] = \frac{1}{s^\beta}(-4\xi\tau) \tag{24}$$

Taking the inverse Laplace transform of equation (24) to obtain:

$$u_1(\xi,\tau) = \frac{-4\xi\tau^\beta}{\Gamma(\beta+2)} \tag{25}$$

$$N_{20} = w_0 u_0$$

$$L\left[v_1(\xi,\tau)\right] = \frac{1}{s^\beta} L\left[w_{0\tau} u_{0\xi\xi} + 6\tau\right] = \frac{1}{s^\beta}\left[\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}(\xi^2+1) + 6\tau\right] = \frac{1}{s^\beta}(6\tau) \tag{26}$$

Taking the inverse Laplace transform of equation (26) to obtain:

$$v_1(\xi,\tau) = \frac{6\tau^{\beta+1}}{\Gamma(\beta+2)} \tag{27}$$

$$N_{30} = u_0 + v_0 w_0$$

$$L\left[w_1(\xi,\tau)\right] = \frac{1}{s^\beta} L\left[u_{0\xi\xi} + v_{0\xi} w_{0\tau} + 4\xi\tau - 2\tau - 2\right]$$

$$= \frac{1}{s^\beta}\left[\frac{\partial^2}{\partial\xi^2}(\xi^2+1) + \frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}(\xi^2-1) + 4\xi\tau - 2\tau - 2\right] = \frac{1}{s^\beta}\left[2 + 4\xi\tau - 2\tau - 2\right] \tag{28}$$

Taking the inverse Laplace transform of equation (28):

$$w_1(\xi,\tau) = \frac{4\xi\tau^\beta}{\Gamma(\beta+2)} - \frac{2\tau^{\beta+1}}{\Gamma(\beta+2)} \tag{29}$$

$$N_{11} = v_1 w_0 + v_0 w_1 + w_0 u_1 + w_1 u_0$$

$$L\left[u_2(\xi,\tau)\right] = \frac{1}{s^\beta} L\left[w_{0\xi} v_{0\tau} + w_{1\xi} v_{0\tau} + \frac{1}{2} w_{0\tau} u_{1\xi\xi} + \frac{1}{2} w_{1\xi} u_{0\xi\xi}\right]$$

$$L\left[u_2(\xi,\tau)\right] = \frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}\left(\frac{6\tau^\beta}{\Gamma(\beta+2)}\right) + \frac{\partial}{\partial\xi}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}(\xi^2-1)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{1}{2}\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}\left(-4\xi\tau^{\beta+2}\right)\right] = \frac{1}{s^\beta}\left[\frac{12\xi\tau^\beta}{\Gamma(\beta+2)} - \frac{2\tau^\beta}{\Gamma(\beta+2)}\right] \tag{30}$$

Taking the inverse Laplace transform of equation (30),

$$u_2(\xi,\tau) = \frac{12\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{2\tau^{2\beta}}{\Gamma(\beta+2)} \tag{31}$$

$$N_{21} = w_1 u_0 + w_0 u_1$$

$$L\left[v_2(\xi,\tau)\right]=\frac{1}{s^\beta}L\left[w_{0\tau}u_{1\xi\xi}+w_{1\tau}u_{0\xi\xi}\right]$$

$$v_2(\xi,\tau)=L^{-1}\left[\frac{1}{s^\beta}\left[\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}\left(\frac{-4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}\right)+\frac{\partial}{\partial\tau}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}-\frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial^2}{\partial\xi^2}(\xi^2+1)\right]\right] \tag{32}$$

$$v_2(\xi,\tau)=\frac{-4\tau^{2\beta}}{\Gamma(\beta+2)} \tag{33}$$

$$N_{31}=u_1+v_0w_1+v_1u_0$$

$$L\left[w_2(\xi,\tau)\right]=\frac{1}{s^\beta}\left[u_{0\xi\xi}+v_{0\xi}w_{1\tau}+v_{1\xi}w_1\tau\right]$$

$$w_2(\xi,\tau)=L^{-1}\left[\frac{1}{s^\beta}\left[\frac{\partial^2}{\partial\xi^2}\left(\frac{-4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}\right)+\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}-\frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\right]\right]$$

$$+Ł^{-1}\left[\frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}\left(\frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}(\xi^2-1)\right]\right]$$

$$w_2(\xi,\tau)=\frac{-4\xi\tau^{2\beta}}{\Gamma(\beta+2)} \tag{34}$$

$$N_{12}=w_0v_2+w_1v_1+w_2v_0+w_0u_2+w_1u_1+w_2u_0$$

$$L\left[u_3(\xi,\tau)\right]=\frac{1}{s^\beta}L\left[w_{0\xi}v_{2\tau}+w_{1\xi}v_{1\tau}+w_{2\xi}v_{0\tau}+\frac{1}{2}w_{0\tau}u_{2\xi\xi}+\frac{1}{2}w_{1\tau}u_{1\xi\xi}+\frac{1}{2}w_{2\tau}u_{0\xi\xi}\right]$$

$$L\left[u_3(\xi,\tau)\right]=\frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}\left(\frac{-4\tau^{\beta}}{\Gamma(\beta+2)}\right)+\frac{\partial}{\partial\xi}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}-\frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}\left(\frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}\left(\frac{-4\xi\tau^{2\beta}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}(\xi^2+1)+\frac{1}{2}\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}\left(\frac{12\xi\tau^{2\beta}}{\Gamma(\beta+2)}-\frac{2\tau^{2\beta}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{1}{2}\frac{\partial}{\partial\tau}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}-\frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial^2}{\partial\xi^2}\left(\frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}\right)+\frac{1}{2}\frac{\partial}{\partial\tau}\left(\frac{-4\xi\tau^{2\beta}}{\Gamma(\beta+2)}\right)\frac{\partial^2}{\partial\xi^2}(\xi^2-1)\right]$$

Taking the inverse Laplace transform to obtain:

$$u_3(\xi,\tau)=-\frac{8\xi\tau^{2\beta-1}}{\Gamma(\beta+2)}-\frac{8\xi\tau^{3\beta-1}}{\Gamma(\beta+2)} \tag{35}$$

$$N_{22}=w_0u_2+w_1u_1+w_2u_0$$

$$L\left[v_3(\xi,\tau)\right]=\frac{1}{s^\beta}L\left[w_{0\tau}u_{2\xi\xi}+w_{1\tau}u_{1\xi\xi}+w_{2\tau}u_{0\xi\xi}\right]$$

$$L\left[v_3(\xi,\tau)\right]=\frac{1}{s^\beta}\left[\frac{\partial}{\partial\tau}(\xi^2-1)\frac{\partial^2}{\partial\xi^2}\left(\frac{12\xi\tau^{2\beta}}{\Gamma(\beta+2)}-\frac{2\tau^{2\beta}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{\partial}{\partial\tau}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}-\frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial^2}{\partial\xi^2}\left(-\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{\partial}{\partial\tau}\left(-\frac{4\xi\tau^{2\beta}}{\Gamma(\beta+2)}\right)\frac{\partial^2}{\partial\xi^2}(\xi^2+1)\right]=\frac{1}{s^\beta}\left(\frac{-16\xi\tau^{2\beta-1}}{\Gamma(\beta+2)}\right)$$

Taking the inverse Laplace transform to obtain:

$$v_3(\xi,\tau)=\frac{-16\xi\tau^{3\beta-1}}{\Gamma(\beta+2)} \tag{36}$$

From equation (31),

$$N_{32} = u_2 + v_0 w_2 + v_1 w_1 + v_2 w_0$$

$$L\left[w_3(\xi, \tau)\right] = \frac{1}{s^\beta} L\left[u_{2\xi\xi} + v_{0\xi} w_{2\tau} + v_{1\xi} w_{1\tau} + v_{2\xi} u_{0\tau}\right]$$

$$w_3(\xi, \tau) = \frac{1}{s^\beta}\left[\frac{\partial^2}{\partial\xi^2}\left(\frac{12\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{2\tau^{2\beta}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\tau}\left(-\frac{4\xi\tau^{2\beta}}{\Gamma(\beta+2)}\right) + \frac{\partial}{\partial\xi}\left(\frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}\left(\frac{4\xi\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{2\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\right]$$

$$+\frac{1}{s^\beta}\left[\frac{\partial}{\partial\xi}\left(-\frac{4\tau^{2\beta}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}(\xi^2-1) + \frac{\partial}{\partial\xi}\left(\frac{6\tau^{\beta+1}}{\Gamma(\beta+2)}\right)\frac{\partial}{\partial\tau}(\xi^2-1)\right]$$

Taking the inverse Laplace transform to obtain:

$$w_3(\xi, \tau) = \frac{-16\xi^2\tau^{3\beta-1}}{\Gamma(\beta+2)} \tag{37}$$

Hence,

$$\sum_{i=1}^{n} u_i = u(\xi, \tau) = \xi^2 + 1 + 2\left[-\frac{2\xi\tau^{\beta+1}}{\Gamma(\beta+2)} + \frac{6\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{4\xi\tau^{2\beta-1}}{\Gamma(\beta+2)} - \frac{4\xi\tau^{3\beta-1}}{\Gamma(\beta+2)} + \cdots\right] \tag{38}$$

$$\sum_{i=1}^{n} v_i = v(\xi, \tau) = \xi^2 - 1 - 4\left[\frac{\tau^{2\beta}}{\Gamma(\beta+2)} + \frac{4\xi\tau^{3\beta-1}}{\Gamma(\beta+2)} + \cdots\right] \tag{39}$$

$$\sum_{i=1}^{n} w_i = w(\xi, \tau) = \xi^2 - 1 + 2\left[\frac{2\xi\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{\tau^{\beta+1}}{\Gamma(\beta+2)} - \frac{2\xi\tau^{2\beta}}{\Gamma(\beta+2)} - \frac{8\xi^2\tau^{3\beta-1}}{\Gamma(\beta+2)} + \cdots\right] \tag{40}$$

The approximate solution obtained is in agreement with that of [3].

Table 1: Exact Solution of $u(\xi, \tau)$ for equation (43) for $\tau = 0.0, 1.0, 2.0$;
$0.1 \le \xi \le 0.5$, $\beta = 0.4, 0.6, 0.8, 1.0$ in comparison with MLVIM used in this work and the results of [9].

		$\beta = 1.0$	$\beta = 1.0$	$\beta = 1.0$	$\beta = 1.0$	$\beta = 1.0$
τ	ξ	u_{Exact}	$MLVIM$	$\lvert Error\rvert$	Khan *et al.* 2020	$\lvert Error\rvert$
0.0	0.1	1.010000000	1.010000000	0.000000000	1.010000000	0.000000000
	0.2	1.040000000	1.040000000	0.000000000	1.040000000	0.000000000
	0.3	1.090000000	1.090000000	0.000000000	1.090000000	0.000000000
	0.4	1.160000000	1.160000000	0.000000000	1.160000000	0.000000000
	0.5	1.250000000	1.250000000	0.000000000	1.250000000	0.000000000
1.0	0.1	0.010000000	0.010000046	-4.6000×10^{-8}	0.010000057	-5.7000×10^{-8}
	0.2	0.040000000	0.040000050	-5.0000×10^{-8}	0.040000068	-6.8000×10^{-8}
	0.3	0.090000000	0.090000053	-5.3000×10^{-8}	0.090000070	-7.0000×10^{-8}
	0.4	0.160000000	0.16000600	-6.0000×10^{-7}	0.160000089	-8.9000×10^{-8}
	0.5	0.250000000	0.250000937	-8.3700×10^{-7}	0.250000572	-5.7200×10^{-7}
2.0	0.1	−2.990000000	−2.990006039	6.0390×10^{-6}	−2.990007669	7.6690×10^{-6}
	0.2	−2.960000000	−2.960008372	8.3720×10^{-6}	−2.96001943	1.9430×10^{-5}
	0.3	−2.910000000	−2.91003211	3.2110×10^{-5}	−2.910079347	7.9347×10^{-5}
	0.4	−2.840000000	−2.840043286	4.3286×10^{-5}	−2.840092312	9.2312×10^{-5}
	0.5	−2.750000000	−2.750176930	1.7693×10^{-4}	−2.750472810	4.7281×10^{-4}

Table 1 is the comparison of the exact and approximate solutions of $u(\xi, \tau)$ obtained for equation (43) and the results of [9] for values of $\tau = 0.0$, 1.0 and 2.0; $\beta = 1.0$. The errors are equally shown on the table. The results show that the approximate solutions are converging rapidly when using Modified Laplace-Variational Iteration Method.

Table 2: Approximate Solution of $v(\xi, \tau)$ for equation (43) for $\tau = 0.0, 1.0, 2.0$; $0.1 \leq \xi \leq 0.5$; $\beta = 0.4, 0.6, 0.8$, and 1.0

		$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1.0$
τ	ξ	$v(\xi, \tau)$	$v(\xi, \tau)$	$v(\xi, \tau)$	$v(\xi, \tau)$
0.0	0.1	−0.990000000	−0.990000000	−0.990000000	−0.990000000
	0.2	−0.960000000	−0.960000000	−0.960000000	−0.960000000
	0.3	−0.910000000	−0.910000000	−0.910000000	−0.910000000
	0.4	−0.840000000	−0.840000000	−0.840000000	−0.840000000
	0.5	−0.750000000	−0.750000000	−0.750000000	−0.750000000
1.0	0.1	−0.667982715	−0.7102062615	−0.7514063836	−0.790000000
	0.2	−1.926051855	−1.799381215	−1.675780849	−1.560000000
	0.3	−3.164120995	−2.868556169	−2.580155315	−2.310000000
	0.4	−4.382190135	−3.917731123	−3.464529780	−3.040000000
	0.5	−5.580259275	−4.946906077	−4.328904246	−3.750000000
2.0	0.1	−2.111604135	−2.261604135	−3.249977669	−3.790000000
	0.2	−2.579884980	−2.699884980	−3.863552366	−4.560000000
	0.3	−3.048165822	−3.118165822	−3.118165822	−5.310000000
	0.4	−3.516446664	−4.201505943	−4.201505943	−6.040000000
	0.5	−3.894727517	−4.648385906	−4.648385906	−6.750000000
3.0	0.1	12.13796211	10.19849416	6.585944425	0.810000000
	0.2	10.56337452	7.533269413	2.172819057	−6.360000000
	0.3	9.008786943	4.888044660	−2.220306317	−13.51000000
	0.4	7.474199357	2.262819913	−6.593431685	−20.64000000
	0.5	5.959611780	−0.342404841	−10.94655706	−27.75000000

Table 3: Approximate Solution of $w(\xi, \tau)$ for equation (43) for $\tau = 0.0, 1.0, 2.0, 3.0$; $0.1 \leq \xi \leq 0.5$, $\beta = 0.4, 0.6, 0.8$ and 1.0

τ	ξ	$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1.0$
0.0	0.1	−0.990000000	−0.990000000	−0.990000000	−0.990000000
	0.2	−0.960000000	−0.960000000	−0.960000000	−0.960000000
	0.3	−0.910000000	−0.910000000	−0.910000000	−0.910000000
	0.4	−0.840000000	−0.840000000	−0.840000000	−0.840000000
	0.5	−0.750000000	−0.750000000	−0.750000000	−0.750000000
1.0	0.1	−2.728893339	−2.500886188	−2.278405529	−2.278405529
	0.2	−3.085314081	−2.806638674	−2.534717868	−2.534717868
	0.3	−4.069262226	−2.917257459	−2.768937019	−2.768937019
	0.4	−4.510997049	−4.029648619	−3.559967227	−3.559967227
	0.5	−5.580259275	−4.946906077	−4.328904246	−4.328904246
2.0	0.1	−5.097859789	−5.220360928	−5.288466440	−5.288466440
	0.2	−5.222596607	−5.569561166	−5.906498600	−5.906498600
	0.3	−1.374210455	−2.047600711	−2.854096480	−2.854096480
	0.4	−6.299831985	−7.377119704	−8.593728510	−8.593728510
	0.5	−7.252330545	−8.835478007	−10.66292626	−10.66292626
3.0	0.1	−7.922601914	−8.795872401	−9.713091665	−9.713091665
	0.2	−7.650304868	−8.997406288	−10.67600285	−10.67600285
	0.3	−8.1831088610	−9.604601643	−13.88873356	−13.88873356
	0.4	−8.008463323	−10.95760890	−15.20770045	−15.20770045
	0.5	−8.638918825	−12.71627763	−18.77648686	−18.77648686

Table 2 is the approximate solutions of $v(\xi, \tau)$ for equation (43) for the values of $\tau = 0.0, 1.0, 2.0$ and 3.0; $0.1 \leq \xi \leq 0.5$ and $\beta = 0.4, 0.6, 0.8$ and 1.0. It is observed that the results are in good agreement with the results of [9]; results of the existing literature as used in this seminar for the given values of ξ, τ and β.

Table 3 is the approximate solution $w(\xi, \tau)$ of equation (43) for the values of $\tau = 0.0, 1.0, 2.0$ and 3.0; $0.1 \leq \xi \leq 0.5$ and $\eta = 0.4, 0.6, 0.8$ and 1.0. The results are also in good agreement with the existing literature as used in this work and converge to the exact solution.

Table 4: The comparison of approximate solutions of $v(\xi, \tau)$ for system in equation (43) between MLVIM and [9] for $\tau = 0.0; 0.1 \leq \xi \leq 0.5$, $\beta = 0.4$ and $\beta = 1.0$

		$\beta = 0.4$	$\beta = 0.4$	$\beta = 1.0$	$\beta = 1.0$
τ	ξ	$MLIVM$	$Khan2020$	$MLIVM$	$Khan2020$
0.0	0.1	−0.990000000	−0.990000000	−0.990000000	−0.990000000
	0.2	−0.960000000	−0.960000000	−0.960000000	−0.960000000
	0.3	−0.910000000	−0.910000000	−0.910000000	−0.910000000
	0.4	−0.840000000	−0.840000000	−0.840000000	−0.840000000
	0.5	−0.750000000	−0.750000000	−0.750000000	−0.750000000

In Table 4, the results of $v(\xi, \tau)$ for the system of equation (43) shows that the solutions of MLIVM and K [9] are in good agreement with the exact solution for values of $\tau = 0.0$, $\xi = 0.4$ and $\xi = 1.0$.

Example 2

Consider the system of nonlinear fractional partial differential equations

$$\left.\begin{array}{c} D_t^{\alpha} u + v_x w_y - v_y w_x = -u; \\ D_t^{\beta} v + u_x w_y - u_y w_x = v; \\ D_t^{\gamma} w + u_x v_y - u_y v_x = w; \end{array}\right\} \quad (41)$$

with initial condition:

$$u(x, y, 0) = e^{x+y}, v(x, y, 0) = e^{x-y}, w(x, y, 0) = e^{-x+y} \quad (42)$$

(See [7], and [18]).

Solution

The zero iterations are determined by equation (14) thus:

$$L\left[u_0(x, y, t)\right] = \frac{1}{s^{\alpha}}\left[\sum_{k=0}^{n-1} s^{\alpha-1} u(x, y, t)\right] = \frac{1}{s}\left[e^{x+y}\right] \quad (43)$$

Taking the inverse Laplace transform of equation (43),

$$u_0(x, y, t) = e^{x+y} \quad (44)$$

$$v_0(x, y, t) = L^{-1}\left[\frac{1}{s^{\beta}}\left[\sum_{k=0}^{n-1} s^{\beta-1} v(x, y, t)\right]\right] = e^{x-y} \quad (45)$$

$$w_0(x, y, t) = L^{-1}\left[\frac{1}{s^{\gamma}}\left[\sum_{k=0}^{n-1} s^{\gamma-1} w(x, y, t)\right]\right] = e^{-x+y} \quad (46)$$

$$N_{10} = v_{0x} w_{0y} + v_{0y} w_{0x} - u_0$$

$$L\left[u_1(x, y, t)\right] = \frac{1}{s^{\beta}}\left\{-\frac{\partial}{\partial x}(e^{x-y})\frac{\partial}{\partial y}(e^{-x+y}) + \frac{\partial}{\partial y}(e^{x-y})\frac{\partial}{\partial x}(e^{-x+y}) - e^{x+y}\right\}$$

$$L^{-1}\left[L\left[u_1(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^{\alpha}}\left[e^{-x+y}\right]\right]=\frac{e^{-x+y}t^{\alpha}}{\Gamma(\alpha+1)} \tag{47}$$

$$N_{20}=-u_{0x}w_{0y}-v_{0y}w_{0x}+v_0$$
$$L\left[v_1(x,y,t)\right]=\frac{1}{s^{\beta}}\left\{-\frac{\partial}{\partial x}(e^{x+y})\frac{\partial}{\partial y}(e^{-x+y})-\frac{\partial}{\partial y}(e^{x+y})\frac{\partial}{\partial x}(e^{-x+y})+e^{x-y}\right\}$$
$$v_1(x,y,t)=L^{-1}\left[\frac{1}{s^{\beta}}\left[e^{x-y}\right]\right]=\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)} \tag{48}$$

$$N_{30}=-u_{0x}v_{0y}-u_{0y}v_{0x}+w_0$$
$$L\left[w_1(x,y,t)\right]=\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial x}(e^{x+y})\frac{\partial}{\partial y}(e^{x-y})-\frac{\partial}{\partial y}(e^{x+y})\frac{\partial}{\partial x}(e^{x-y})+e^{-x+y}\right\}$$
$$L^{-1}\left[L\left[w_1(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^{\gamma}}\left[e^{-x+y}\right]\right]=\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)} \tag{49}$$

$$N_{11}=-v_{0x}w_{1y}-v_{1y}w_{0x}+v_{0y}w_{1x}+v_{1x}w_{0y}-u_1$$
$$L\left[u_2(x,y,t)\right]=\frac{1}{s^{\alpha}}\left\{-\frac{\partial}{\partial x}(e^{x-y})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})\right\}$$
$$+\frac{1}{s^{\alpha}}\left\{\frac{\partial}{\partial y}(e^{x-y})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)+\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial y}(e^{x-y})-\frac{\partial}{\partial y}\left(\frac{-e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\right\}$$
$$L^{-1}\left[L\left[u_2(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^{\alpha}}\left\{\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right\}\right]=\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)} \tag{50}$$

$$N_{21}=-u_{0x}w_{1y}-u_{1y}w_{0x}+v_0$$
$$L\left[v_2(x,y,t)\right]=\frac{1}{s^{\beta}}\left\{-\frac{\partial}{\partial x}(e^{x+y})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})\right\}$$
$$+\frac{1}{s^{\beta}}\left\{-\frac{\partial}{\partial y}(e^{-x+y})\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial y}(e^{x+y})\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)+\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\alpha+1)}\right)\right\}$$
$$L^{-1}\left[v_2(x,y,t)\right]=L^{-1}\left[\frac{1}{s^{\beta}}\left\{\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right\}\right]=\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)} \tag{51}$$

$$N_{31}=-u_{0x}v_{1y}-u_{1y}v_{0x}+w_1$$
$$L\left[w_2(x,y,t)\right]=\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial x}(e^{x+y})\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)-\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}(e^{x-y})\right\}$$
$$+\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial y}(e^{x-y})\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)-\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial y}(e^{x+y})+\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)\right\}$$
$$L^{-1}\left[L\left[w_2(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^{\gamma}}\left\{\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right\}\right]=\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)} \tag{52}$$

$$N_{12}=-v_{1x}w_{1y}-v_{1y}w_{1x}+v_{1x}w_{1y}+v_{1y}w_{1x}-v_{0x}w_{1y}-v_{1y}w_{0x}+v_{0x}w_{1y}+v_{1y}w_{0x}-v_{0x}w_{2y}-v_{2y}w_{0x}$$
$$+v_{0x}w_{2y}+v_{2y}w_{0x}-v_{2x}w_{0y}-v_{0y}w_{2x}+v_{2x}w_{0y}+v_{0y}w_{2x}$$

$$L\left[u_3(x,y,t)\right]=\frac{1}{s^{\alpha}}\left\{-\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{\gamma}}{\Gamma(\gamma+1)}\right)\right\}$$

$$+\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^\beta}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)+\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^\beta}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial x}(e^{x-y})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^\beta}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})+\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$+\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^\beta}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial x}(e^{-x+1})-\frac{\partial}{\partial x}(e^{x-1})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial y}(e^{-x+1})+\frac{\partial}{\partial y}(e^{-x+1})+\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right)\right\}$$

$$+\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})-\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial y}(e^{-x+y})-\frac{\partial}{\partial y}(e^{x-y})\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right)\right\}$$

$$+\frac{1}{s^\alpha}\left\{\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial y}(e^{-x+y})+\frac{\partial}{\partial y}(e^{x-y})\left(\frac{\partial}{\partial x}\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right)-\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right\}$$

$$L^{-1}\left[L\left[u_3(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^\beta}\left\{\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right\}\right]=\frac{-e^{x+y}t^{3\alpha}}{\Gamma(3\alpha+1)} \tag{53}$$

$$N_{22}=-u_{1x}w_{1y}-u_{1y}w_{1x}-u_{1x}w_{1y}-u_{1y}w_{0x}-u_{0x}w_{1x}-u_{1y}w_{0x}-u_{0x}w_{1y}-u_{1x}w_{1y}-u_{1y}w_{0x}-u_{0y}w_{1x}$$

$$-u_{1x}w_{0y}-u_{0x}w_{2y}-u_{2y}w_{0x}-u_{0y}w_{2x}-u_{2x}w_{0y}-u_{2x}w_{0y}-u_{0y}w_{2x}-u_{2y}w_{0x}-u_{0y}w_{2x}+v_2$$

$$L\left[v_3(x,y,t)\right]=\frac{1}{s^\beta}\left\{-\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\beta+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)-\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}(e^{-x+y})\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)-\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\frac{\partial}{\partial y}(e^{x+y})\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\frac{\partial}{\partial x}(e^{x+1})-\frac{\partial}{\partial x}(e^{x+1})\frac{\partial}{\partial y}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial y}(e^{-x+1})-\frac{\partial}{\partial y}(e^{x+1})-\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^\gamma}{\Gamma(\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^\alpha}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}(e^{x+y})-\frac{\partial}{\partial y}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})-\frac{\partial}{\partial y}(e^{x+y})\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial x}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial y}(e^{-x+y})-\frac{\partial}{\partial y}(e^{-x+y})\frac{\partial}{\partial x}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial y}(e^{x+y})\frac{\partial}{\partial x}\left(\frac{e^{-x+y}t^{2\gamma}}{(2\Gamma+1)}\right)-\frac{\partial}{\partial y}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial x}(e^{-x+y})\right\}$$

$$-\frac{1}{s^\beta}\left\{\frac{\partial}{\partial x}(e^{x+y})\frac{\partial}{\partial y}\left(\frac{e^{-x+y\,2\gamma}}{t}\Gamma(2\gamma+1)\right)+\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right\}$$

$$L^{-1}\left[L\left[v_3(x,y,t)\right]\right]=L^{-1}\left[\frac{1}{s^\beta}\left\{\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right\}\right]=\frac{e^{x-y}t^{3\beta}}{\Gamma(3\beta+1)} \tag{54}$$

$$N_{32}=-u_{1x}v_{1y}-u_{1y}v_{1x}-u_{1y}v_{1x}-u_{1x}v_{1y}-u_{1x}v_{0y}-u_{0y}v_{1x}-u_{1y}v_{0x}-u_{0x}v_{1y}-u_{0x}v_{1y}-u_{1y}v_{0x}$$

$$-u_{0y}v_{1x}-u_{1x}v_{0y}-u_{0x}v_{2y}-u_{2y}v_{0x}-u_{0y}v_{2x}-u_{2x}v_{0y}-u_{2x}v_{0y}-u_{0y}v_{2x}-u_{2y}v_{0x}-u_{0x}v_{2y}+v_2$$

$$
\begin{aligned}
L\left[w_3(x,y,t)\right] &= \frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)-\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial y}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)-\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial x}\left(-\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial y}(e^{x-y})-\frac{\partial}{\partial y}\left(e^{x+y}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial y}\left(\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}(e^{x-y})-\frac{\partial}{\partial x}(e^{x+y})\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial x}\left(e^{x+y}\right)\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)-\frac{\partial}{\partial y}\left(\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial}{\partial x}\left(e^{x-y}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial y}\left(e^{x+y}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{\beta}}{\Gamma(\beta+1)}\right)-\frac{\partial}{\partial x}\left(\frac{e^{x+y}t^{\alpha}}{\Gamma(\alpha+1)}\right)\left(e^{x-y}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial x}\left(e^{x+y}\right)\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)-\frac{\partial}{\partial y}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial x}\left(e^{x-y}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{\frac{\partial}{\partial y}\left(e^{x+y}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)-\frac{\partial}{\partial x}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial y}\left(\frac{e^{x-y}}{\Gamma(\gamma+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial x}\left(\frac{e^{x+y}t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\frac{\partial}{\partial y}\left(e^{x-y}\right)-\frac{\partial}{\partial y}\left(e^{x+y}\right)\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\right\}\\
&-\frac{1}{s^{\gamma}}\left\{-\frac{\partial}{\partial x}\left(\frac{e^{x-y}t^{2\beta}}{\Gamma(2\beta+1)}\right)\frac{\partial}{\partial x}\left(e^{x-y}\right)\right\}+\frac{1}{s^{\gamma}}\left\{\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right\}
\end{aligned}
$$

$$
w_3(x,y,t)=L^{-1}\left[\frac{1}{s^{\gamma}}\left\{\frac{e^{-x+y}t^{2\gamma}}{\Gamma(2\gamma+1)}\right\}\right]=\frac{e^{-x+y}t^{3\gamma}}{\Gamma(3\gamma+1)} \tag{55}
$$

The series of the solutions of the equations are written thus:

$$
u(x,y,t)=e^{x+y}\left(1-\frac{t^{\alpha}}{\Gamma(\alpha+1)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right)=e^{x+y}\left(1+\sum_{k=1}^{\infty}\frac{(-t^{\alpha})^k}{\Gamma(k\alpha+1)}\right)
$$

$$
u(x,y,t)=e^{x+y}E_{\alpha}(-t^{\alpha}) \tag{56}
$$

$$
v(x,y,t)=e^{x-y}\left(1+\frac{t^{\beta}}{\Gamma(\beta+1)}+\frac{t^{2\beta}}{\Gamma(2\beta+1)}+\frac{t^{3\beta}}{\Gamma(3\beta+1)}+\cdots\right)=e^{x-y}\left(1+\sum_{k=1}^{\infty}\frac{(t^{\beta})^k}{\Gamma(k\beta+1)}\right)
$$

$$
v(x,y,t)=e^{x-y}E_{\beta}(t^{\beta}) \tag{57}
$$

$$
w(x,y,t)=e^{-x+y}\left(1+\frac{t^{\gamma}}{\Gamma(\gamma+1)}+\frac{t^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{t^{3\eta}}{\Gamma(3\gamma+1)}+\cdots\right)=e^{-x+y}\left(1+\sum_{k=1}^{\infty}\frac{(t^{\gamma})^k}{\Gamma(k\gamma+1)}\right)
$$

$$
w(x,y,t)=e^{-x+y}E_{\gamma}(t^{\gamma}) \tag{58}
$$

The results obtained with the Modified Laplace-Variational Iteration Method are the in agreement with that of [7], and [18]. This reveals that the method is effective and efficient.

Table 5: Approximate Solution of $u(x, y, t)$ for equation (58) for $t = 0.0, 1.0, 2.0, 3.0; \alpha = 0.4, 0.6, 0.8, 1.0; x = y = 0.1, 0.2, 0.3, 0.4$

			$\alpha = 0.4$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1.0$
t	x	y	$u(x, y, t)$	$u(x, y, t)$	$u(x, y, t)$	$u(x, y, t)$
0.0	0.1	0.1	1.221402758	1.221402758	1.221402758	1.221402758
	0.2	0.2	1.491824698	1.491824698	1.491824698	1.491824698
	0.3	0.3	1.822118800	1.822118800	1.822118800	1.822118800
	0.4	0.4	2.225540928	2.225540928	2.225540928	2.225540928
	0.5	0.5	2.718281828	2.718281828	2.718281828	2.718281828
1.0	0.1	0.1	0.047643369	0.2344417722	0.3546689791	0.4071342527
	0.2	0.2	0.058191743	0.2863478277	0.4331936697	0.4972748993
	0.3	0.3	0.071075555	0.349746027	0.5291039430	0.6073729333
	0.4	0.4	0.086811880	0.427180761	0.6462490138	0.7418469760
	0.5	0.5	0.106032268	0.521759761	0.7893303286	0.9060939427
2.0	0.1	0.1	−0.858549278	−0.840696858	−0.634353911	−0.407134253
	0.2	0.2	−1.048634455	−1.026829463	−0.774801616	−0.497274899
	0.3	0.3	−1.280805015	−1.254172337	−0.946344830	−0.607372933
	0.4	0.4	−1.564378778	−1.531849553	−1.155868186	−0.741846975
	0.5	0.5	−1.910736554	−1.871005269	−1.411780590	−0.906093943
3.0	0.1	0.1	−1.899613695	−2.541838966	−2.703984455	−2.442805518
	0.2	0.2	−2.320193407	−3.104609124	−3.302654075	−2.983649398
	0.3	0.3	−2.833890624	−3.791978147	−4.033870795	−3.644237600
	0.4	0.4	−3.461321826	−4.631532565	−4.926980910	−4.451081856
	0.5	0.5	−4.227668027	−5.656966656	−6.017828075	−5.436563666

The approximate solution of $u(x, y, t)$ for equation (58) is shown in Table 5 for the values of $t = 0.0, 1.0, 2.0$ and 3.0; $0.1 \leq x \leq 0.5$; $0.1 \leq y \leq 0.5$ and $\alpha = 0.4, 0.6, 0.8$ and 1.0 using the MLVIM. From the results, when $t = 0.0$; $0.1 \leq x \leq 0.5$; $0.1 \leq y \leq 0.5$, and $\alpha = 0.4, 0.6, 0.8$ and 1.0, it is observed that the results are exact solution, e^{x+y-t}. Thereafter, the results continue to vary accordingly, and converging to exact solution.

Table 6: Approximate Solution of $v(x, y, t)$ for equation (58) for $t = 0.0, 1.0, 2.0, 3.0 \beta = 0.4, 0.6, 0.8, 1.0; x = y = 0.1, 0.2, 0.3, 0.4$

			$\beta = 0.4$	$\beta = 0.6$	$\beta = 0.8$	$\beta = 1.0$
t	x	y	$v(x, y, t)$	$v(x, y, t)$	$v(x, y, t)$	$v(x, y, t)$
0.0	0.1	0.1	1.000000000	1.000000000	1.000000000	1.000000000
	0.2	0.2	1.000000000	1.000000000	1.000000000	1.000000000
	0.3	0.3	1.000000000	1.000000000	1.000000000	1.000000000
	0.4	0.4	1.000000000	1.000000000	1.000000000	1.000000000
	0.5	0.5	1.000000000	1.000000000	1.000000000	1.000000000
1.0	0.1	0.1	4.108335456	3.623262679	3.108590292	2.666666667
	0.2	0.2	4.108335456	3.623262679	3.108590292	2.666666667
	0.3	0.3	4.108335456	3.623262679	3.108590292	2.666666667
	0.4	0.4	4.108335456	3.623262679	3.108590292	2.666666667
	0.5	0.5	4.108335456	3.623262679	3.108590292	2.666666667
2.0	0.1	0.1	6.441661226	6.858555808	6.760245101	6.333333333
	0.2	0.2	6.441661226	6.858555808	6.760245101	6.333333333
	0.3	0.3	6.441661226	6.858555808	6.760245101	6.333333333
	0.4	0.4	6.441661226	6.858555808	6.760245101	6.333333333
	0.5	0.5	6.441661226	6.858555808	6.760245101	6.333333333
3.0	0.1	0.1	8.726555482	10.86486167	12.32721870	13.00000000
	0.2	0.2	8.726555482	10.86486167	12.32721870	13.00000000
	0.3	0.3	8.726555482	10.86486167	12.32721870	13.00000000
	0.4	0.4	8.726555482	10.86486167	12.32721870	13.00000000
	0.5	0.5	8.726555482	10.86486167	12.32721870	13.00000000

Table 6 shows that the approximate solution of $v(x, y, t)$ using MLVIM converges to the exact solution of equation (58). The solutions converge to the exact solution for the given values of t, x, y and α.

Table 7: Approximate Solution of $w(x, y, t)$ for equation (58) for $t = 0.0, 1.0, 2.0, 3.0; \xi = 0.4, 0.6, 0.8, 1.0; x = y = 0.1, 0.2, 0.3, 0.4$

			$\xi = 0.4$	$\xi = 0.6$	$\xi = 0.8$	$\xi = 1.0$
t	x	y	$w(x, y, t)$	$w(x, y, t)$	$w(x, y, t)$	$w(x, y, t)$
0.0	0.1	0.1	1.000000000	1.000000000	1.000000000	1.000000000
	0.2	0.2	1.000000000	1.000000000	1.000000000	1.000000000
	0.3	0.3	1.000000000	1.000000000	1.000000000	1.000000000
	0.4	0.4	1.000000000	1.000000000	1.000000000	1.000000000
	0.5	0.5	1.000000000	1.000000000	1.000000000	1.000000000
1.0	0.1	0.1	4.108335456	3.623262679	3.108590292	2.666666667
	0.2	0.2	4.108335456	3.623262679	3.108590292	2.666666667
	0.3	0.3	4.108335456	3.623262679	3.108590292	2.666666667
	0.4	0.4	4.108335456	3.623262679	3.108590292	2.666666667
	0.5	0.5	4.108335456	3.623262679	3.108590292	2.666666667
2.0	0.1	0.1	6.441661226	6.858555808	6.760245101	6.333333333
	0.2	0.2	6.441661226	6.858555808	6.760245101	6.333333333
	0.3	0.3	6.441661226	6.858555808	6.760245101	6.333333333
	0.4	0.4	6.441661226	6.858555808	6.760245101	6.333333333
	0.5	0.5	6.441661226	6.858555808	6.760245101	6.333333333
3.0	0.1	0.1	8.726555482	10.86486167	12.32721870	13.00000000
	0.2	0.2	8.726555482	10.86486167	12.32721870	13.00000000
	0.3	0.3	8.726555482	10.86486167	12.32721870	13.00000000
	0.4	0.4	8.726555482	10.86486167	12.32721870	13.00000000
	0.5	0.5	8.726555482	10.86486167	12.32721870	13.00000000

Tables 6 and 7 show that the approximate solutions for $v(x, y, t)$ and $w(x, y, t)$ in equation (58) coincides for the given values of t, x, y; and at β and ξ respectively. It is observed that the results are in perfect agreement with the exact solutions at the given values of t, x, y; and β.

Table 8: Comparison of MLVIM, [7] for $v(x, y, t)$ and $w(x, y, t)$ for system of equation (58) for values $t = 0.0, x = y = 0.1, 0.2, 0.3, 0.4, 0.5$ and $\beta = 0.4, 1.0$

			$\xi = 0.4$	$\xi = 0.4$	$\xi = 1.0$	$\xi = 1.0$
t	x	y	$MLIVM$	$Jafari et al. 2013$	$MLVIM$	$Jafari et al. 2013$
0.0	0.1	0.1	1.000000000	1.000000000	1.000000000	1.000000000
	0.2	0.2	1.000000000	1.000000000	1.000000000	1.000000000
	0.3	0.3	1.000000000	1.000000000	1.000000000	1.000000000
	0.4	0.4	1.000000000	1.000000000	1.000000000	1.000000000
	0.5	0.5	1.000000000	1.000000000	1.000000000	1.000000000

The result of Table 8 shows that the values of $v(\xi, \tau)$ and $w(\xi, \tau)$ are the same and in perfect agreement with the exact solution. Equally, for the given values of $t = 0.0, x = y$ for $0.1 \leq x \leq 0.5$, the results of MLVIM and that of [7] are in perfect agreement.

Conclusion

This work introduces Modified Laplace-Variation Iteration Method as a powerful straight forward and efficient method for solving NFPDEs. The method gives room to compute approximate solutions with elegant computational terms using Caputo sense. The approximate solutions clearly illustrate the simplicity and accuracy of the method. It is obvious that the method gives rapidly convergent approximations through the use of the Lagrange multiplier. It also has more general applications. The numerical results were computed using MAPPLE 18. The obtained results are interesting, and of good agreement towards the exact solutions hence, the method is reliable and efficient.

References

[1] M.A. Abdou, An analytical method for space-time fractional nonlinear differential equations arising in plasma physics, J. Ocean Eng. Sci. 4 (2017), 288–292.

[2] A. Bekir, O. Guner, E. Aksoy, Y. Pandir, Functional variable method for the nonlinear fractional differential equations, AIP Conf. Proc. 16 (2015), 1–5.

[3] J. Biazar, M. Elslami, A new homotopy perturbation method for solving systems of partial differential equations, Comput. Math. Appl. 62 (2011), 225–234.

[4] C. Bota, B. Căruntu, D. Ţucu, M. Lăpădat, M.S. Paşca, A least squares differential quadrature method for a class of nonlinear partial differential equations of fractional order, Mathematics 8 (2020), 1336.

[5] H.K. Dass, R. Verma, Mathematical physics, S. Chand & Co. Pvt. Ltd., New Delhi (2012).

[6] D.B. Dhaigude, S.N. Kanade, C.D. Dhaigude, Solution for system of fractional partial differential equations, Int. J. Appl. Math. 13 (2018), 448–479.

[7] H. Jafari, H. Nazari, D. Baleanu, C.M. Khalique, A new approach for solving a system of fractional partial differential equations, Comput. Math. Appl. 66 (2013), 833–843.

[8] Y. Jawarneh, H. Yasmin, M.M. Al-Sawalha, R. Sha, A. Khan, Fractional comparative analysis of Camassa-Holm and Degasperis-Procesi equations, AIMS Math. 8 (2023), 25845–25862.

[9] H. Khan, R. Shah, P. Kumam, D. Baleanu, M. Arif, Laplace decomposition for solving nonlinear system of fractional order partial differential equations, Adv. Differ. Equ. 2020 (2020), 375.

[10] S. Maitama, I. Abdullahi, A new analytical method for solving linear and nonlinear fractional partial differential equations, Progr. Fract. Differ. Appl. 2 (2016), 247–256.

[11] S. Noor, A.S. Alshehry, A. Khan, I. Khan, Innovative approach for developing solitary wave solutions for the fractional modified partial differential equations, AIMS Math. 8 (2023), 27775–27819.

[12] I. Podlubny, Fractional differential equations, Academy Press, San Diego, (1999).

[13] M.H. Rahman, M.I. Bhatti, N. Dimakis, A method to solve one-dimensional nonlinear fractional differential equations using B-polynomials, Acta Sci. Appl. Phys. 2 (2022), 22–35.

[14] H. Shathar, Variable coefficient fractional partial differential equations by base-Chebyshev method, Int. J. Nonlinear Appl. 12 (2021), 1399–1411.

[15] K.A. Stroud, D.J. Booth, Engineering mathematics, 6th ed., Palgrave Macmillan (2007).

[16] M. Sultana, M. Waqar, A.H. Ali, A.A. Lupas, F. Ghanim, Z.A. Abduljabbar, Numerical investigation of systems of fractional partial differential equations by new transform iterative technique, AIMS Math. 9 (2024), 26649–26670.

[17] V. Turut, N. Guzel, On solving partial differential equations of the fractional order by using the variational iteration method and multivariate Padé approximations, Eur. J. Pure Appl. Math. 6 (2024), 147–171.

[18] A.M. Wazwaz, The variational iteration method for solving linear and nonlinear systems of partial differential equations, Comput. Math. Appl. 54 (2007), 895–902.