

FIXED POINT AND COMMON FIXED POINT THEOREMS UNDER VARIOUS EXPANSIVE CONDITIONS IN PARTIAL METRIC SPACES

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Abstract: In the present paper, we prove some fixed point theorems for self-mappings satisfying various expansive type conditions in the setting of a partial metric space. The presented theorems extend, generalize and improve many existing results in the literature.

1 INTRODUCTION

Fixed point theory is one of the most popular tool in nonlinear analysis. Most of the generalizations for metric fixed point theorems usually start from Banach contraction principle [4]. It is not easy to point out all the generalizations of this principle. In 1994, Matthews [13] introduced the concept of partial metric space in which the self distance of any point of space may not be zero. In 1996, O'Neill generalized the concept of partial metric space by admitting negative distances. In 1984, Wang et.al [16] introduced the concept of expanding mappings and proved some fixed point theorems in complete metric spaces. In 1992, Daffer and Kaneko [5] defined an expanding condition for a pair of mappings and proved some common fixed point theorems for two mappings in complete metric spaces. Aage and Salunke [1] introduced several meaningful fixed point theorems for one expanding mapping. For more details on expanding mapping and related results we refer the reader to [5-8, 14, 16-18].

In this paper, we prove some fixed point theorems for surjective mappings satisfying various expansive type conditions in the setting of a partial metric space. The presented theorems extend, generalize and improve many existing results in the literature.

2 PRELIMINARIES

Throughout this paper \mathbb{R} and \mathbb{R}^+ will represents the set of real numbers and nonnegative real numbers, respectively.

The following definitions are required in the sequel.

Definition 2.1 (see [13]) Let X be a nonempty set, and $p : X \times X \rightarrow \mathbb{R}^+$ be a function. We say p is a partial metric on X if and only if for all $x, y, z \in X$ the following conditions are satisfied:

- (1) $x = y$ if and only if $p(x, x) = p(x, y) = p(y, y)$;
- (2) $p(x, x) \leq p(x, y)$;
- (3) $p(x, y) = p(y, x)$;

$$(4) p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$$

The pair (X, p) is called a partial metric space.

Remark 2.3 It is clear that the partial metric space need not be a b-metric spaces, since in a partial metric space if $p(x, y) = 0$ implies $p(x, x) = p(x, y) = p(y, y) = 0$ then $x = y$. But in a partial metric space if $x = y$ then $p(x, x) = p(x, y) = p(y, y)$ may not be equal zero. Therefore the partial metric space may not be a b-metric space.

Every partial metric p defines a metric d_p , where

$$d_p(x, y) = 2p_b(x, y) - p(x, x) - p(y, y), x, y \in X.$$

The following lemma will be useful in what follows; see [13].

Definition 2.4 A sequence $\{x_n\}_{n=1}^{\infty}$ in a partial metric space (X, p) is said to be:

1. convergent to a point $x \in X$, written as $\lim_{n \rightarrow \infty} p(x, x_n) = p(x, x)$;
2. a Cauchy sequence if $\lim_{n, m \rightarrow +\infty} p(x_n, x_m)$ exists (and is finite);

Definition 2.5 A partial metric space (X, p) is said to be complete if every Cauchy sequence in X converges to a point $x \in X$ such that

$$p(x, x) = \lim_{n \rightarrow \infty} p(x_n, x) = \lim_{n \rightarrow \infty} p(x_n, x_m)$$

Lemma 2.6 A sequence $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in a partial metric space (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p) .

Lemma 2.7 A partial metric space (X, p) is complete if and only if the metric space (X, d_p) is complete. Moreover, $\lim_{n \rightarrow \infty} d_p(x_n, x_m) = 0$ if and only if

$$p(x, x) = \lim_{n \rightarrow \infty} p(x_n, x) = \lim_{n \rightarrow \infty} p(x_n, x_m)$$

3.3 FIXED POINT THEOREMS

In this section, we prove some fixed point theorems satisfying expansive condition by considering surjective self-mappings in the context of partial metric space.

We begin with a simple but a useful Lemma.

Lemma 3.1 Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in a partial metric space (X, p) such that

$$(3.1) \quad p_b(x_n, x_{n+1}) \leq \lambda p_b(x_{n-1}, x_n)$$

where $\lambda \in [0, 1)$ and $n = 1, 2, \dots$. Then $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X .

Proof By the simple induction with the condition (3.1), we have

$$(3.2) \quad p(x_n, x_{n+1}) \leq \lambda p(x_{n-1}, x_n) \leq \lambda^2 p(x_{n-2}, x_{n-1}) \leq \dots \leq \lambda^n p(x_0, x_1)$$

On the other hand, since

$$\max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq p(x_n, x_{n+1})$$

then from (3.2), we have

$$(3.3) \quad \max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq \lambda^n p(x_0, x_1)$$

Therefore

$$(3.4) \quad \begin{aligned} d_p(x_n, x_{n+1}) &= 2p(x_n, x_{n+1}) - p(x_n, x_n) - p(x_{n+1}, x_{n+1}) \\ &\leq 2p(x_n, x_{n+1}) + p(x_n, x_n) + p(x_{n+1}, x_{n+1}) \end{aligned}$$

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$$\leq 4\lambda^n p(x_0, x_1)$$

This show that $\lim_{n \rightarrow +\infty} d_p(x_n, x_{n+1}) = 0$. Now we have

$$\begin{aligned} (3.5) \quad d_p(x_n, x_{n+m}) &\leq d_p(x_n, x_{n+1}) + d_p(x_{n+1}, x_{n+m}) \\ &\leq d_p(x_n, x_{n+1}) + d_p(x_{n+1}, x_{n+2}) + d_p(x_{n+2}, x_{n+m}) \\ &\leq d_p(x_n, x_{n+1}) + d_p(x_{n+1}, x_{n+2}) + \dots \\ &\quad + d_p(x_{n+m-2}, x_{n+m-1}) + d_p(x_{n+m-1}, x_{n+m}) \\ &\leq 4\lambda^n p(x_0, x_1) + 4\lambda^{n+1} p(x_0, x_1) + \dots \\ &\quad + 4\lambda^{n+m-2} p(x_0, x_1) + 4\lambda^{n+m-1} p(x_0, x_1) \\ &\leq 4\lambda^n \{1 + \lambda + \lambda^2 + \dots \dots \dots\} p(x_0, x_1) \\ &\leq \frac{4\lambda^n}{1-\lambda} p(x_0, x_1). \end{aligned}$$

Note that $\lambda < 1$. This show that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in metric space (X, d_p) , then from Lemma 2.6, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in partial metric space (X, p) .

Theorem 3.2 Let (X, p) be a complete partial metric space. Assume that $T: X \rightarrow X$ is surjection and satisfies

$$(3.6) \quad p(Tx, Ty) \geq \lambda p(x, y)$$

$\forall x, y \in X$, where $\lambda > 1$. Then T has a unique fixed point in X .

Proof; Let $x_0 \in X$, since T is surjection, then there exists $x_1 \in X$ such that $x_0 = Tx_1$. By continuing this process, we get

$$(3.7) \quad x_n = Tx_{n+1}, \quad \forall n \in \mathbb{N} \cup \{0\}.$$

In case $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N} \cup \{0\}$, then it is clear that x_{n_0} is a fixed point of T .

Without loss of generality, we assume that $x_n \neq x_{n-1}$ for all n . Consider,

$$(3.8) \quad p(x_{n-1}, x_n) = p(Tx_n, Tx_{n+1})$$

Now by (3.7) and definition of the sequence

$$\begin{aligned} p(x_{n-1}, x_n) &= p(Tx_n, Tx_{n+1}) \\ &\geq \lambda p(x_n, x_{n+1}) \end{aligned}$$

and so

$$(3.9) \quad p(x_n, x_{n+1}) \leq \frac{1}{\lambda} p(x_{n-1}, x_n) = hp(x_{n-1}, x_n)$$

where $h = \frac{1}{\lambda} < \frac{1}{s}$. Then by Lemma 3.1, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X . Since (X, p) is a complete, then from Lemma 2.7, (X, d_p) is complete and so the sequence $\{x_n\}_{n=1}^{\infty}$ is converges in the metric space (X, d_p) , that is there exists $x^* \in X$ such that

$$\lim_{n \rightarrow +\infty} d_p(x_n, x^*) = 0.$$

Again from Lemma 2.7, we have

$$(3.10) \quad p(x^*, x^*) = \lim_{n \rightarrow \infty} p(x_n, x^*) = \lim_{n \rightarrow \infty} p(x_n, x_m)$$

Moreover, since $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in the metric space (X, d_p) ,

$$\lim_{n \rightarrow \infty} d_p(x_n, x_m) = 0,$$

On the other hand, since

$$\max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq p(x_n, x_{n+1})$$

then by the simple induction with (3.9), we have

$$(3.11) \quad \max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq h^n p(x_0, x_1)$$

Hence, we have $\lim_{n \rightarrow \infty} p(x_n, x_n) = 0$. Thus from the definition of d_p , we have

$$\lim_{n \rightarrow \infty} p(x_n, x_m) = 0.$$

Therefore, from (3.10), we have

$$p(x^*, x^*) = \lim_{n \rightarrow \infty} p(x_n, x^*) = \lim_{n \rightarrow \infty} p(x_n, x_m) = 0.$$

Since T is surjection on X , there exists $z \in X$ such that $x^* = Tz$. From (3.6), we have

$$(3.12) \quad \begin{aligned} p(x_n, x^*) &= p(Tx_{n+1}, Tz) \\ &\geq \lambda p(x_{n+1}, z) \end{aligned}$$

Taking limit as $n \rightarrow +\infty$ in the above inequality, we get

$$0 = p(x^*, x^*) \geq \lambda p(x^*, z)$$

This implies that $p(x^*, z) = 0$. Also from (3.6), we have

$$0 = p(x^*, x^*) = p(Tz, Tz) \geq \lambda p(z, z)$$

and so $p(z, z) = 0$. Thus $p(x^*, x^*) = p(x^*, z) = p(z, z)$ implies that $x^* = z = Tz$. Hence x^* is a fixed point of T . Finally, assume $x^* \neq y^*$ is also another fixed point of T . From (3.6), we get

$$(3.13) \quad \begin{aligned} p(x^*, y^*) &= p(Tx^*, Ty^*) \\ &\geq \lambda p(x^*, y^*) \end{aligned}$$

This is true only when $p(x^*, y^*) = 0$. Also $p(x^*, x^*) = 0 = p(y^*, y^*)$. So $x^* = y^*$. Hence T has a unique fixed point in X .

Corollary 3.3 Let (X, p) be a complete partial metric space and $T: X \rightarrow X$ be a surjection. Suppose that there exist a positive integer n and a constant $\lambda > 1$ such that

$$(3.14) \quad p(T^n x, T^n y) \geq \lambda p(x, y)$$

$\forall x, y \in X$. Then T has a unique fixed point in X .

Proof From Theorem 3.2, T^n has a unique fixed point x^* . But $T^n(Tx^*) = T(T^n x^*) = Tx^*$. So Tx^* is also a fixed point of T^n . Hence $Tx^* = x^*$, x^* is a fixed point of T . Since the fixed point of T is also fixed point of T^n , the fixed point of T is unique.

3.3 COMMON FIXED POINT THEOREMS

Now, we give a common fixed point theorem of two weakly compatible mappings in partial metric spaces.

In [14] Jungck introduced the concept of commuting maps. In [15] Jungck introduced the concept of compatible mappings which generalize the concept of commuting maps. Jungck in [16] further generalized the concept of weakly compatible maps as follows.

Definition 4.1 Let S and T be two self-mappings on a nonempty set X . Then S and T are said to be weakly compatible if they commute at all of their coincidence points; that is, $Sx = Tx$ for some $x \in X$ and then $STx = TSx$.

Theorem 4.2 Let (X, p) be a complete partial metric space. Let S and T be two self-mappings of X and $T(X) \subseteq S(X)$. Suppose that $a, b, c \geq 0$ with $a + b + c > 1$ such that

$$(4.1) \quad p(Sx, Sy) \geq ap(Tx, Ty) + bp(Sx, Tx) + cp(Sy, Ty)$$

$\forall x, y \in X$. If one of the subspaces $T(X)$ or $S(X)$ is complete, then S and T have a point of coincidence in X . Moreover, if $a > 1$, then point of coincidence is unique. If S and T be weakly compatible and $a > 1$, then S and T have a unique common fixed point in X .

Proof: Let $x_0 \in X$. Since $T(X) \subseteq S(X)$, choose $x_1 \in X$ such that $y_1 = Sx_1 = Tx_0$. In general, choose $x_{n+1} \in X$ such that $y_{n+1} = Sx_{n+1} = Tx_n$. Now by (4.1), we have

$$\begin{aligned} p(y_n, y_{n+1}) &= p(Sx_n, Sx_{n+1}) \\ &\geq ap(Tx_n, Tx_{n+1}) + bp(Sx_n, Tx_n) + cp(Sx_{n+1}, Tx_{n+1}) \\ &= ap(y_{n+1}, y_{n+2}) + bp(y_n, y_{n+1}) + cp(y_{n+1}, y_{n+2}) \end{aligned}$$

and so

$$(1 - b)p(y_n, y_{n+1}) \geq (a + c)p(y_{n+1}, y_{n+2})$$

If $a + c = 0$, then $b > 1$. The above inequality implies that a negative number is greater than or equal to zero. That is impossible. So, $a + c \neq 0$ and $1 - b > 0$. Therefore,

$$(4.2) \quad p(y_{n+1}, y_{n+2}) \leq hp(y_n, y_{n+1})$$

where $h = \frac{1-b}{a+c} < \frac{1}{s}$. Then by Lemma 3.1, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence. Since $T(X) \subseteq S(X)$ and $T(X)$ or $S(X)$ is a complete subspace of X . Then from Lemma 2.7, $(S(X), d_p)$ is complete and so the sequence $\{y_n\} = \{Tx_{n-1}\} \subseteq S(X)$ is converges in the metric space $(S(X), d_p)$, that is, there exists $z^* \in X$ such that

$$\lim_{n \rightarrow +\infty} d_p(y_n, z^*) = 0.$$

Consequently, we can find $u \in X$ such that $Su = z^*$. Again from Lemma 2.7, we have

$$(4.3) \quad p(Su, z^*) = p(z^*, z^*) = \lim_{n \rightarrow \infty} p(y_n, z^*) = \lim_{n \rightarrow \infty} p(y_n, y_m)$$

Moreover, since $\{y_n\}_{n=1}^{\infty}$ is a Cauchy sequence in the metric space $(S(X), d_p)$,

$$\lim_{n \rightarrow \infty} d_p(y_n, y_m) = 0,$$

On the other hand, since

$$\max\{p(y_n, y_n), p(y_{n+1}, y_{n+1})\} \leq p(y_n, y_{n+1})$$

then by the simple induction with (4.2), we have

$$(4.4) \quad \max\{p(y_n, y_n), p(y_{n+1}, y_{n+1})\} \leq h^n p(y_0, y_1)$$

Hence, we have $\lim_{n \rightarrow \infty} p(y_n, y_n) = 0$. Thus from the definition of d_p , we have

$$\lim_{n \rightarrow \infty} p(y_n, y_m) = 0.$$

Therefore, from (4.3), we have

$$p(Su, z^*) = p(z^*, z^*) = \lim_{n \rightarrow \infty} p(y_n, z^*) = \lim_{n \rightarrow \infty} p(y_n, y_m) = 0.$$

Now to show that $Tu = z^*$. From (4.1), we have

$$(4.5) \quad p(Su, Sx_n) \geq ap(Tu, Tx_n) + bp(Su, Tu) + cp(Sx_n, Tx_n)$$

Taking limit as $n \rightarrow +\infty$ in the above inequality, we get

$$\begin{aligned} 0 = p(Su, z^*) &\geq ap(Tu, z^*) + bp(z^*, Tu) \\ &= (a + b)p(Tu, z^*) \end{aligned}$$

This implies that $p(Tu, z^*) = 0$ and so $Tu = z^*$. Therefore, $Su = Tu = z^*$. Therefore, z^* is a point of coincidence of S and T .

Now we suppose that $a > 1$. Let w^* be another point of coincidence of S and T . So $Sv = Tv = w^*$ for some $v \in X$. Then from (4.1), we have

$$\begin{aligned} p(z^*, w^*) &= p(Su, Sv) \\ &\geq ap(Tu, Tv) + bp(Su, Tu) + cp(Sv, Tv) \\ &= ap(z^*, w^*) \end{aligned}$$

This is true only when $p(z^*, w^*) = 0$. Also $p(z^*, z^*) = 0 = p(w^*, w^*)$. So $z^* = w^*$.

Since S and T be weakly compatible, $STu = TSu$, that is, $Sz^* = Tz^*$. Now we show that z^* is a common fixed point of S and T . If $a > 1$, then from condition (4.1), we have

$$p(Sz^*, Sx_n) \geq ap(Tz^*, Tx_n) + bp(Sz^*, Tz^*) + cp(Sx_n, Tx_n)$$

Proceeding to the limit as $n \rightarrow +\infty$, we have $p(Sz^*, z^*) \geq ap(Tz^*, z^*) = ap(Sz^*, z^*)$, which implies that $p(Sz^*, z^*) = 0$. Also $p(Sz^*, Sz^*) = 0 = p(z^*, z^*)$. Hence $Sz^* = z^*$ and so $Sz^* = Tz^* = z^*$. Hence S and T have a unique fixed point in X . This completes the proof.

Remark 4.3 If we take, $S = T, T = I$ in Theorem 4.2, then we get Theorem 2.1 of Huang et al. [26].

Now, we prove the following common fixed point theorem, which is generalization of Theorem 2.2 of Shatanawi et al. [14] in the setting of partial b-metric space.

Theorem 4.4 Let $T, S: X \rightarrow X$ be two surjective mappings of a complete partial metric space (X, p) . Suppose that T and S satisfying inequalities

$$(4.6) \quad p(T(Sx), Sx) + kp(T(Sx), x) \geq ap(Sx, x)$$

$$(4.7) \quad p(S(Tx), Tx) + kp(S(Tx), x) \geq bp(Tx, x)$$

for $x \in X$ and some nonnegative real numbers a, b and k with $a > 1 + 2k$ and $b > 1 + 2k$.

If T or S is continuous, then T and S have a common fixed point in X .

Proof Let x_0 be an arbitrary point in X . Since T is surjective, there exists $x_1 \in X$ such that $x_0 = Tx_1$. Also, since S is surjective, there exists $x_2 \in X$ such that $x_2 = Sx_1$. Continuing this process, we construct a sequence $\{x_n\}$ in X such that

$$(4.8) \quad x_{2n} = Tx_{2n+1} \text{ and } x_{2n+1} = Sx_{2n+2}$$

for all $n \in \mathbb{N} \cup \{0\}$, Now for $n \in \mathbb{N} \cup \{0\}$, by (4.6) we have

$$p(T(Sx_{2n+2}), Sx_{2n+2}) + kp(T(Sx_{2n+2}), x_{2n+2}) \geq ap(Sx_{2n+2}, x_{2n+2})$$

Thus, we have

$$p(x_{2n}, x_{2n+1}) + kp(x_{2n}, x_{2n+2}) \geq ap(x_{2n+1}, x_{2n+2})$$

which implies that

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$$\begin{aligned} p(x_{2n}, x_{2n+1}) + k[p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2}) - p(x_{2n+1}, x_{2n+1})] \\ \geq ap(x_{2n+1}, x_{2n+2}) \end{aligned}$$

That is,

$$p(x_{2n}, x_{2n+1}) + k[p(x_{2n}, x_{2n+1}) + p(x_{2n+1}, x_{2n+2})] \geq ap(x_{2n+1}, x_{2n+2})$$

Hence

$$(4.9) \quad p(x_{2n+1}, x_{2n+2}) \leq \frac{1+k}{a-k} p(x_{2n}, x_{2n+1})$$

On other hand, we have (from (4.7))

$$p(S(Tx_{2n+1}), Tx_{2n+1}) + kp(S(Tx_{2n+1}), x_{2n+1}) \geq bp(Tx_{2n+1}, x_{2n+1})$$

Thus we have

$$p(x_{2n-1}, x_{2n}) + kp(x_{2n-1}, x_{2n+1}) \geq bp(x_{2n}, x_{2n+1})$$

which implies that

$$\begin{aligned} p(x_{2n-1}, x_{2n}) + k[p(x_{2n-1}, x_{2n}) + p(x_{2n}, x_{2n+1}) - p(x_{2n}, x_{2n})] \\ \geq bp(x_{2n}, x_{2n+1}) \end{aligned}$$

That is,

$$p(x_{2n-1}, x_{2n}) + k[p(x_{2n-1}, x_{2n}) + p(x_{2n}, x_{2n+1})] \geq bp(x_{2n}, x_{2n+1})$$

Hence

$$(4.10) \quad p(x_{2n-1}, x_{2n}) \leq \frac{1+k}{b-k} p(x_{2n-1}, x_{2n})$$

Let $h = \max\left\{\frac{1+k}{a-k}, \frac{1+k}{b-k}\right\} < 1$.

Then by combining (4.9) and (4.10), we have

$$(4.11) \quad p(x_n, x_{n+1}) \leq h p(x_{n-1}, x_n)$$

where $h \in [0, 1)$, $\forall n \in \mathbb{N} \cup \{0\}$. Then by Lemma 3.1, $\{x_n\}_{n=1}^{\infty}$ is p -Cauchy sequence in the complete partial metric space. Then there exists $x^* \in X$ such that $x_n \rightarrow x^*$ as $n \rightarrow +\infty$. Therefore $x_{2n+1} \rightarrow x^*$ and $x_{2n+2} \rightarrow v$ as $n \rightarrow +\infty$. Without loss of generality, we may assume that T is continuous, then $Tx_{2n+1} \rightarrow Tx^*$ as $n \rightarrow +\infty$. But $Tx_{2n+1} = x_{2n} \rightarrow x^*$ as $n \rightarrow +\infty$. Thus, we have $Tx^* = x^*$. since S is surjective, there exists $z \in X$ such that $Sz = x^*$. Now

$$p(T(Sz), Sz) + kp(T(Sz), z) \geq ap(Sz, z)$$

implies that

$$kp(x^*, z) \geq ap(x^*, z)$$

Then $p(x^*, z) \leq \frac{k}{a} p(x^*, z)$. Since $a > k$, we conclude that $p(x^*, z) = 0$. so $x^* = z$. Hence $Tx^* = Sx^* = x^*$. Therefore x^* is a common fixed point of T and S .

By taking $b = a$ in theorem 4.4, we have the following result.

Corollary 4.5 Let $T, S: X \rightarrow X$ be two surjective mappings of a complete partial metric space (X, p) . Suppose that T and S satisfying inequalities

$$(4.12) \quad p(T(Sx), Sx) + kp(T(Sx), x) \geq ap(Sx, x)$$

$$(4.13) \quad p(S(Tx), Tx) + kp(S(Tx), x) \geq ap(Tx, x)$$

for $x \in X$ and some nonnegative real numbers a and k with $a > 1 + 2k$. If T or S is continuous, then T and S have a common fixed point in X .

By taking $S = T$ in Corollary 4.5, we have the following Corollary.

Corollary 4.6 Let $T: X \rightarrow X$ be a surjective mappings of a complete partial metric space (X, p) . Suppose that T satisfying inequality

$$(4.14) \quad p(T(Tx), Tx) + kp(T(Tx), x) \geq ap(Tx, x)$$

for $x \in X$ and some nonnegative real numbers a and k with $a > 1 + 2k$. If T is continuous, then T has a fixed point in X .

Now, we present an example to illustrate the usability of Corollary 4.6.

Example 4.7 Let $X = [0, \infty)$ and define $p: X \times X \rightarrow \mathbb{R}^+$ by

$$d(x, y) = p(x, y) = \max\{x, y\}, \forall x, y \in X.$$

Then (X, p) is a complete partial metric space. Define $T: X \rightarrow X$ by $T(x) = 2x$. Then T has a fixed point.

Proof Note that

$$\begin{aligned} p(T(Tx), Tx) + p(T(Tx), x) &= p(4x, 2x) + p(4x, x) \\ &= \max\{4x, 2x\} + \max\{4x, x\} \\ &= 4x + 4x \\ &= 8x \\ &> \frac{7}{2} \max\{2x, x\} \\ &= \frac{7}{2} p(Tx, x) \end{aligned}$$

for all $x \in X$. Here $k = 1$ and $a = \frac{7}{2}$. Clearly $\frac{7}{2} = a > 1 + 2k = 1 + 2 = 3$. Also T is surjection on X . Thus T satisfies all the hypotheses of Corollary 4.5 and hence T has a fixed point. Here $0 \in X$ is the fixed point of T .

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

AUTHOR'S CONTRIBUTIONS

All authors contributed equally and significantly to writing this paper. All authors read and approved the final manuscript.

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