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THE HALF LOGISTIC-POISSON DISTRIBUTION

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Abstract. In this article, a new two-parameter distribution called the half Logistic Poisson distribution is proposed. Numerous properties of the new distribution are discussed, maximum likelihood estimation procedure is considered for parameter estimation; we also assessed the maximum likelihood estimators by a simulation study. Application of the new distribution to a real data set is provided for illustration purposes.

1. Introduction

Distributions with support on the positive real line are widely used in modeling of lifetime data in practical applications. For example, exponential and Weibull distributions have been used in fitting data set in various fields of science and applied sciences such as in the fields of biomedical studies, reliability, actuarial sciences, demography, engineering, public health, etc. Moreover, half-logistic distribution (HL) is another lifetime model that plays a vital role in fitting data with decreasing failure rate in practice.

In lifetime study we required more distributions with flexibility that can accommodate different kind of data set in practice. In consideration of these kinds of problems, several authors give an important attention to half logistic distribution in recent years and proposed various extensions and new forms of the half-logistic such as the generalized half logistic (GHL), power half logistic (PwHL) by [23], Olapade generalized half logistic (OGHL) by [39], exponentiated half logistic family of distributions (EHL-G) by [11], type I half-logistic family (TIHL-G) by [10] among others.

In this work, the proposed distribution is obtained via compounding the half Logistic and Poisson distributions. The new distribution serves as the complementary distribution to the Poisson-half logistic distribution (PHL) proposed by [1].

There are so many parametric models that have been successfully proposed and investigated by combining discrete and continuous distributions, for instance, [2, 24, 46] proposed the exponential geometric (EG), exponential Poisson (EP), BurrXII Poisson (BXIIP) distributions respectively, where their complementary distributions are the complementary exponential geometric (CEG), Poisson-exponential (PE) and BurrXII-zero-truncated Poisson (BXIIZTP) distributions introduced and studied by [26], [8] and [36] respectively.

Key words and phrases. Half logistic distribution; Moments; Maximum likelihood estimates.

Some others include the complementary exponentiated exponential geometric (CEEG), complementary Weibull geometric (CWG), complementary Poisson-Lindley (CPL), complementary exponentiated Inverted Weibull Power Series (CEIWPS), Complementary Burr III Poisson (CBIIIP) and complementary extended Weibull power series (CExWPS) distributions proposed by [25, 48, 17, 19, 18] and [12] respectively. In the same approach, we have the exponentiated exponential Poisson (EEP) by [42], exponentiated exponential binomial(GEB) by [6], generalized exponential power series (GEPS) by [28], binomial exponential-2 (BE2) by [5], linear failure rate-power series (LFRPS) by [29], Weibull power series(WPS) by [34], extended Weibull power series (EWPS) by [45], exponentiated Weibull-logarithmic (EWL) [27], exponentiated Weibull Poisson (EWP) by [30], Additive Weibull-Geometric by [15], exponentiated Weibull geometric (EWG) by [31], exponentiated Weibull power series (EWPS) by [32], Lindley-Poisson (LP) by [16], Generalized Gompertz-power series (GGPS) by [47], exponentiated BurrXII Poisson (EBXIIP) by [14], modified Weibull geometric (MWG) by [49], Dagum-Poisson (DP) by [40], Poisson-Lomax (PL) by [3], complementary exponentiated BurrXII Poisson (CEBXIIP) by [38], Poisson-odd generalized exponential family (POGE) by [37] and the additive modified Weibull odd Log-logistic-G Poisson family by [33] etc.

The rest of the paper is arranged as follows, the properties of the proposed distribution are derived and discussed in section 2. In section 3, estimation of the unknown parameters by maximum likelihood and simulation results are presented. An application of the HLP is given in section 4. Sections 5 conclude the paper.

2. HLP and Properties

The probability density function and cumulative distribution function of the half Logistic distribution with scale parameter $\alpha > 0$ are defined by

$$g(x) = \frac{2\alpha e^{-\alpha y}}{(1 + e^{-\alpha y})^2} \tag{1}$$

and

$$G(x) = \frac{1 - e^{-\alpha y}}{1 + e^{-\alpha y}}. \tag{2}$$

respectively.

The proposed model, that is the half Logistic Poisson (HLP) distribution is obtained using the procedure followed by [26] and [8], the process is summarized by the following *proposition*.

Proposition 2.1. *Let* $X = \min\{Y_1, Y_2, \cdots, Y_k\}$*, where* $Y_1, Y_2, \cdots, Y_k$*, are independent and identically distributed half Logistic random variables with (1). Let* K *be a Poisson random variable truncated at zero with probability mass function given by* $P(k; \lambda) = \lambda^k((\exp(\lambda) - 1)\ k!)^{-1}$*,* $\lambda > 0$ *and* $k \in \mathbb{N}$*. Then, the probability density function of X (the half Logistic Poisson) is obtained via.*

The probability density function, cumulative distribution function, survival and hazard rate functions of the half Logistic Poisson with parameters $\alpha > 0$ and $\lambda \in \mathbb{R} - \{0\}$ are defined by

$$f(x, \alpha, \lambda) = \frac{2\alpha\lambda e^{-\alpha x}}{(1 - e^{-\lambda})(1 + e^{-\alpha x})^2} e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)}, \tag{3}$$

$$F(x) = \frac{1 - e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)}}{1 - e^{-\lambda}}, \tag{4}$$

$$s(x) = \frac{e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)} - e^{-\lambda}}{1 - e^{-\lambda}}, \tag{5}$$

$$h(x) = \frac{2\alpha\lambda e^{-\alpha x} e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)}}{(1 + e^{-\alpha x})^2 \left(e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)} - e^{-\lambda}\right)}, \tag{6}$$

respectively. The limiting distribution of the HLP distribution given by (4) when $\lambda \to 0^+$ is $\lim_{\lambda \to 0^+} F(x) = \left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)$. Figure 1 shows the plots of the pdf and hrf of the half logistic Poisson distribution.

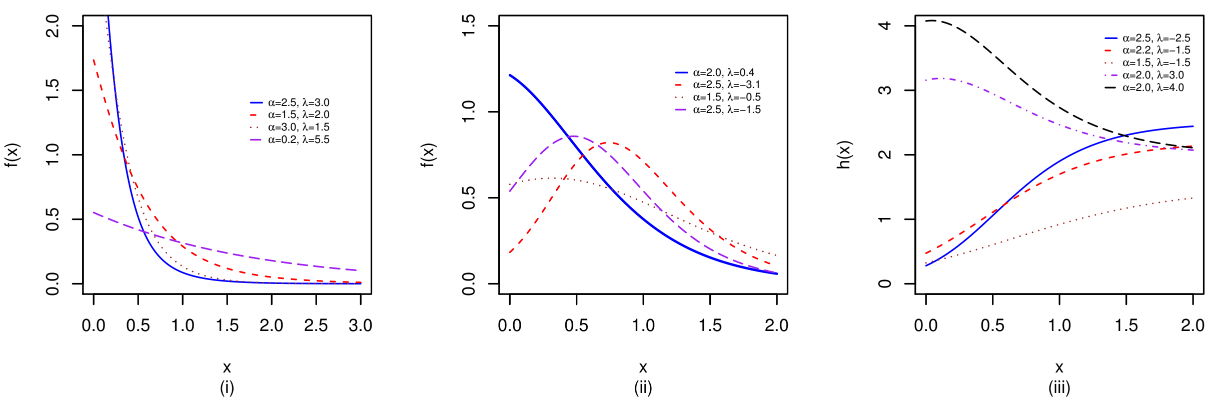

FIGURE 1. Plots of the (i), (ii) pdf and (iii) hrf of the HLP distribution for some selected values of parameters.

The quantile function of the HLP distribution can be used to generate a random data distributed according to HLP(α, λ) when $p \sim U(0, 1)$, where $U(0, 1)$ is the uniform distribution. The quantile of X can be presented as

$$\varrho_{\alpha,\lambda}(p) = \frac{-1}{\alpha} \log \left(\frac{1 - \left(\frac{\log(1-p(1-e^{-\lambda}))}{-\lambda}\right)}{1 + \left(\frac{\log(1-p(1-e^{-\lambda}))}{-\lambda}\right)} \right). \tag{7}$$

The median of the HLP distribution can be obtained when $p = 0.5$ in (7). Figure 2 shows that the median is decreasing function in both α and $\lambda > 0$.

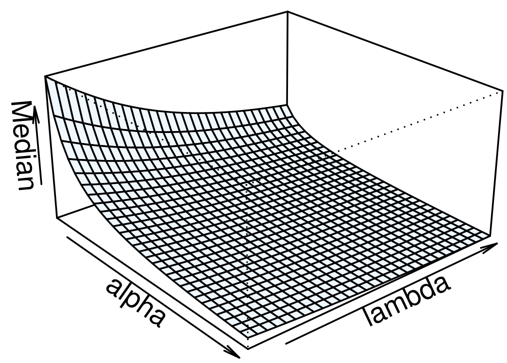

FIGURE 2. Plots of the median of the HLP distribution for $\lambda > 0$

2.1. **Moments.** The features and characteristics of a distribution can be studied through its moments such as the mean, variance and moment generating function, etc. If X has the HLP (α, λ) distribution, then the r^{th} moment of X ($\mu_r = E[X^r]$) can be computed from $E[X^r] = \int_0^\infty x^r f(x)dx$ as follows

$$E[X^r] = \int_0^\infty \frac{2\alpha\lambda x^r\, e^{-\alpha x}}{(1-e^{-\lambda})(1+e^{-\alpha x})^2} e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)} dx, \tag{8}$$

by letting $u = 1 - e^{-\alpha x}$, then exponential expansion, generalized binomial expansion and some algebraic manipulations we get

$$E[X^r] = \frac{2\lambda}{\alpha^r(1-e^{-\lambda})} \sum_{i=1}^{\infty}\sum_{w=0}^{\infty} \frac{(-1)^{i+r}\lambda^i}{i!} \left(\begin{array}{c} -(2+i) \\ w \end{array} \right) \frac{\partial^r}{\partial c^r} B(i+1, w+1), \tag{9}$$

where $B(.,.)$ is the complete beta function defined by $B(a,b) = \int_0^1 u^{a-1}(1-u)^{b-1}du$ and $c = w+1$. Therefore, the mean (μ_1) can be obtain when $r = 1$ in (9) and variance (σ^2) can be computed from $\sigma^2 = E(X^2) - (E(X))^2$. Figure 3 shows that the mean and variance of HLP are decreasing functions in both α and $\lambda > 0$

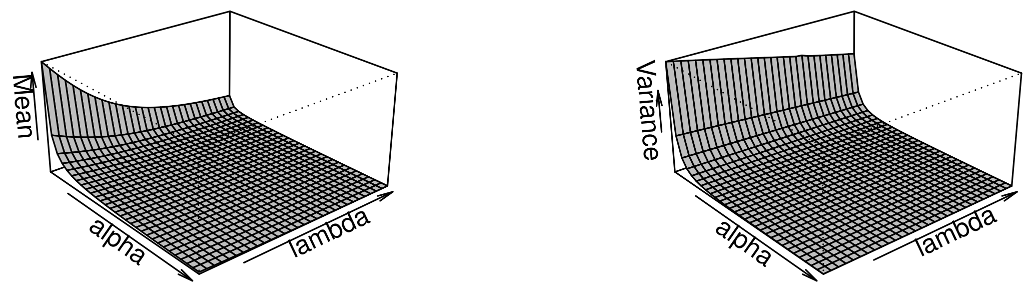

FIGURE 3. Plots of the mean and variance of the HLP distribution for $\lambda > 0$

The moment generating function of the HLP distribution is obtained directly using $M_X(t) = E(e^{tX})$ which can be expanded to

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X^r), \tag{10}$$

hence, by putting (9) in (10) we have

$$M_X(t) = \frac{2\lambda}{\alpha^r(1-e^{-\lambda})} \sum_{r=0}^{\infty}\sum_{i=1}^{\infty}\sum_{w=0}^{\infty} \frac{(-1)^{i+r}\, t^r\, \lambda^i}{i!\, r!} \begin{pmatrix} -(2+i) \\ w \end{pmatrix} \frac{\partial^r}{\partial c^r} B(i+1, w+1). \tag{11}$$

2.2. **Order statistics.** Let $X_1, X_2, X_3, \cdots, X_n$ be a random sample from HLP distribution, let $X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}$, be the order statistics obtained from this random sample, then for $j = 1, 2, 3, \cdots, n$, the corresponding pdf, say $f_{j:n}(x)$ is obtained from

$$f_{x_{j}:n}(x; \alpha, \lambda) = \frac{n!}{(j-1)!(n-j)!} f(x)(F(x))^{j-1}(1-F(x))^{n-j},$$

where $f(x)$ and $F(x)$ are the pdf and cdf of the HLP distribution, we have

$$f_{x_{j}:n}(x; \alpha, \lambda) = \sum_{l=0}^{n-j} \frac{n!\,(-1)^l}{(j-1)!(n-j-l)!\, l!} f(x)(F(x))^{j+l-1},$$

using the binomial expansion of F^{j+l-1} and after some algebraic manipulations we have

$$f_{x_{j}:n}(x; \alpha, \lambda) = \sum_{l=0}^{n-j}\sum_{k=0}^{j+l-1} \zeta_{j,k,l,n}(\lambda)\, f(x; \alpha, \lambda(k+1)), \tag{12}$$

where

$$\zeta_{j,k,l,n}(\lambda)=\binom{j+l-1}{k}\frac{n!(-1)^{k+l}\left(1-e^{-\lambda(k+1)}\right)}{(1-e^{-\lambda})(k+1)(j-1)!(n-j-l)!l!}, \tag{13}$$

and $f(x;\alpha,\lambda(k+1))$ is the probability density function of the half logistic Poisson distribution with parameter α and $\lambda(k+1)$.
The r^{th}-moment of j^{th} order statistic of the HLP distribution is computed using (12) as

$$E(X^r)=\int_0^\infty x^r\, f_{x_j:n}(x;\alpha,\lambda)dx \tag{14}$$

$$=\sum_{l=0}^{n-j}\sum_{k=0}^{j+l-1}\zeta_{j,k,l,n}(\lambda)\int_0^\infty x^r f(x;\alpha,\lambda(k+1))dx \tag{15}$$

by considering (9) we have,

$$E(X^r)=\sum_{l=0}^{n-j}\sum_{k=0}^{j+l-1}\sum_{l=1}^{\infty}\sum_{w=0}^{\infty}\zeta^*_{j,k,l,n,r,w}(\alpha,\lambda)\frac{\partial^r}{\partial c^r}B(i+1,w+1), \tag{16}$$

where $\zeta^*_{j,k,l,n,r,w}(\alpha,\lambda)=\binom{j+l-1}{k}\binom{-(2+i)}{w}\frac{2(-1)^{i+k+l+r}n!\lambda^i(k+1)^i}{\alpha^r(1-e^{-\lambda})(j-1)!(n-j-l)!l!i!}$.

2.3. **Shannon entropy.** An entropy of a randomvariable X is a measure of variation of the uncertainty. The Shannon entropy of a random variable X with density function given by (3) can be defined as $E[-\log f(x)]$, we first consider the following *lemma* and *proposition* which are very essential for the computation of the Shannon entropy of the HLP distribution.

Lemma 2.2. *For $a_1>0$ and $a_2>0$, let*

$$\psi(a_1,a_2)=\int_0^\infty\frac{(1-e^{-\alpha x})^{a_1}e^{-\alpha x}}{(1+e^{-\alpha x})^{a_2}}e^{-\lambda\left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)}dx, \tag{17}$$

then,

$$\psi(a_1,a_2)=\frac{2\lambda}{(1-e^{-\lambda})}\sum_{i=1}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^i\lambda^i}{i!}\begin{pmatrix}-(a_2+i)\\ w\end{pmatrix}B(a_1+i,w+1), \tag{18}$$

where $B(.,.)$ is a complete beta function.

proof. Follow similar in computing (9).

Proposition 2.3. *For a random variable X with pdf given by (3), then,*

$$E\left(\log\left(1+e^{-\alpha X}\right)\right)=\frac{2\alpha\lambda}{(1-e^{-\lambda})}\frac{\partial}{\partial t}\psi(0,2-t)|_{t=0}, \tag{19}$$

$$E\left(\frac{1-e^{-\alpha X}}{1+e^{-\alpha X}}\right)=\frac{2\alpha\lambda}{(1-e^{-\lambda})}\psi(1,3). \tag{20}$$

proof. By using the lemma 2.2.

Thus, we can compute the Shannon entropy of the HLP distribution using (19) and (20) as

$$
\begin{aligned}
E\left[-\log f(X)\right] = \log\left(\frac{1-e^{-\lambda}}{2\,\alpha\,\lambda}\right) + \alpha E(X) + \frac{4\alpha\lambda}{(1-e^{-\lambda})}\frac{\partial}{\partial t}\phi(0,2-t)\mid_{t=0} \\
+\frac{2\alpha\lambda^2}{(1-e^{-\lambda})}\phi(1,3),
\end{aligned} \tag{21}
$$

where $E(X)$ can be calculated using (9) when $r = 1$.

3. Estimation

The maximum likelihood of α and λ can be obtain simultaneously by the numerical solutions of (23) and (24) when set equal to zero using mathematical software such as R and *Mathematica*. The total log likelihood function of the half-logistic Poisson distribution is given by

$$
\begin{aligned}
\log l(\theta) = n\log 2 + n\log\alpha + n\log\lambda - \alpha\sum_{i=1}^{n} x_i - \lambda\sum_{i=1}^{n}\left(\frac{1-e^{-\alpha x_i}}{1+e^{-\alpha x_i}}\right) \\
- n\log(1-e^{-\lambda}) - 2\sum_{i=1}^{n}\log(1-e^{-\alpha x_i}).
\end{aligned} \tag{22}
$$

where $\theta = (\alpha, \lambda)^T$, the first partial derivative of $\log\ell(\theta)$ that is $\partial\log\ell(\theta)/\partial\alpha$ and $\partial\log\ell(\theta)/\partial\lambda$ are computed as

$$
\frac{\partial(\log l(\theta))}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} x_i - 2\lambda\sum_{i=1}^{n}\frac{x_i e^{-\alpha x}}{(1+e^{-\alpha x_i})^2} - 2\sum_{i=1}^{n}\frac{x_i e^{-\alpha x_i}}{1+e^{-\alpha x_i}}, \tag{23}
$$

$$
\frac{\partial(\log l(\theta))}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}\left(\frac{1-e^{-\alpha x_i}}{1+e^{-\alpha x_i}}\right) + \frac{n e^{-\lambda}}{(1-e^{-\lambda})}. \tag{24}
$$

For a very large sample we apply the usual approximation that the MLEs of the HLP can be approximated as bivariate normal with mean zero and variance covariance matrix $I^{-1}(\theta)$, where $I(\theta)$ is the expected information matrix. Alternatively we can use $J(\theta)$ the 2×2 information matrix defined by $J(\theta) = -\left(\frac{\partial^2(\log\ell(\theta))}{\partial\theta\partial\theta^T}\right)$. The approximate of the MLEs of θ, the $\hat{\theta}$, can be approximated as $N_2(0, J(\hat{\theta})^{-1})$ under the regularity conditions stated in [13]. The asymptotic distribution of $\sqrt{n}(\hat{\theta}-\theta)$ is $N_2(0, J(\hat{\theta})^{-1})$, where $J(\hat{\theta})$ is the unit information matrix evaluated at $\hat{\theta}$, which can be used to construct the approximate confidence interval for each parameter. A $100(1-\epsilon)\%$ asymptotic confidence interval for each parameter θ_r is given by $ACI_r = (\hat{\theta}_r - Z_{\frac{\epsilon}{2}}\sqrt{\hat{I}^{rr}}, \hat{\theta}_r + Z_{\frac{\epsilon}{2}}\sqrt{\hat{I}^{rr}})$, where I^{rr} is the (r, r) diagonal element of

$I_n(\theta)^{-1}$ for $r = 1, 2$ and $Z_{\frac{\epsilon}{2}}$ is the quantile $(1 - \frac{\epsilon}{2})$ of the standard normal distribution. The elements of $J(\theta)$ are given below

$$\frac{\partial^2(\log l(\theta))}{\partial\lambda^2} = -\frac{n}{\lambda^2} - \frac{n\, e^{-\lambda}}{(1-e^{-\lambda})^2}, \tag{25}$$

$$\frac{\partial^2(\log l(\theta))}{\partial\lambda\partial\alpha} = -2\sum_{i=1}^{n} \frac{x_i\, e^{-\alpha x_i}}{(1+e^{-\alpha x_i})^2}, \tag{26}$$

$$\begin{aligned}\frac{\partial^2(\log l(\theta))}{\partial\alpha^2} &= -\frac{n}{\alpha^2} - 2\lambda\sum_{i=1}^{n} \frac{x_i^2\, e^{-\alpha x_i}}{(1+e^{-\alpha x_i})^2} - 4\lambda\sum_{i=1}^{n} \frac{x_i^2\, e^{-2\alpha x_i}}{(1+e^{-\alpha x_i})^3} \\ &\quad - 2\sum_{i=1}^{n} \frac{x_i^2\, e^{-\alpha x_i}}{(1+e^{-\alpha x_i})} - 2\sum_{i=1}^{n} \frac{x_i^2\, e^{-2\alpha x_i}}{(1+e^{-\alpha x_i})^2}.\end{aligned} \tag{27}$$

3.1. **Simulation study.** In this part, we evaluate the performance of the maximum likelihood estimates using simulation study; we generate 10,000 samples from the HLP (α, λ) distribution, each of sample sizes n=50, 100 and 200, for some selected values of $\alpha > 0$ and $\lambda \in \mathbb{R} - \{0\}$. The result of the simulations are displayed in Table 2 below. The result shows that the method of maximum likelihood performed consistently and the standard deviations of the maximum likelihood estimates decrease as we increase the sample size.

TABLE 1. MLEs and Standard deviations for some values of parameters.

Sample size	Actual values		Estimated values		Standard deviations	
n	α	λ	$\hat{\alpha}$	$\hat{\lambda}$	$sd(\hat{\alpha})$	$sd(\hat{\lambda})$
50	1.0	1.0	1.0303	1.0817	0.2771	1.1304
	1.5	1.3	1.5794	1.3164	0.4627	1.2191
	1.2	1.0	1.2285	1.1003	0.3332	1.1579
	1.1	1.8	1.2243	1.7129	0.4027	1.3270
	0.2	2.1	0.2385	1.8289	0.1023	1.3820
	0.5	0.3	0.4859	0.5607	0.0991	0.8544
	0.1	0.8	0.1017	0.9216	0.0257	1.0653
	0.5	0.5	0.5515	0.6305	0.1568	0.9587
	0.1	-0.5	0.1054	-0.7347	0.0175	0.7079
	0.5	-1.0	0.4436	-0.7204	0.113	0.9159
	1.1	-1.1	1.1338	-1.2317	0.1847	0.8120
	1.2	-1.3	1.2346	-1.4407	0.1949	0.8379
	1.5	-1.1	1.5493	-1.2503	0.2522	0.8147

TABLE 2. MLEs and Standard deviations for some values of parameters.

Sample size	Actual values		Estimated values		Standard deviations	
n	α	λ	$\hat{\alpha}$	$\hat{\lambda}$	$sd(\hat{\alpha})$	$sd(\hat{\lambda})$
100	1.0	1.0	1.0109	1.0906	0.2253	1.0369
	1.5	1.3	1.5379	1.3654	0.3850	1.1191
	1.2	1.0	1.2156	1.0847	0.2707	1.0140
	1.1	1.8	1.1482	1.8562	0.3343	1.2707
	0.2	2.1	0.2156	2.0782	0.0687	1.3306
	0.5	0.3	0.4893	0.4611	0.0748	0.6537
	0.1	0.8	0.1011	0.8669	0.0206	0.9265
	0.5	0.5	0.5472	0.5487	0.1332	0.7600
	0.1	-0.5	0.1027	-0.6136	0.0124	0.5135
	0.5	-1.0	0.4293	-0.5828	0.0982	0.7428
	1.1	-1.1	1.1148	-1.1566	0.1288	0.5869
	1.2	-1.3	1.2146	-1.3570	0.1392	0.5954
	1.5	-1.1	1.5202	-1.1580	0.1802	0.5897
200	1.0	1.0	1.0091	1.0278	0.1683	0.7218
	1.5	1.3	1.5165	1.3612	0.3014	0.9175
	1.2	1.0	1.2096	1.0350	0.2029	0.7548
	1.1	1.8	1.1152	1.9107	0.2668	1.1235
	0.2	2.1	0.2047	2.2208	0.0539	1.2364
	0.5	0.3	0.4949	0.3773	0.0528	0.4141
	0.1	0.8	0.1004	0.8382	0.0154	0.6604
	0.5	0.5	0.5479	0.4577	0.1127	0.5434
	0.1	-0.5	0.1012	-0.5506	0.0092	0.3919
	0.5	-0.1	0.4193	-0.4844	0.0908	0.6261
	1.1	-1.1	1.1078	-1.1301	0.0948	0.4241
	1.2	-1.3	1.2070	-1.3291	0.0986	0.4232
	1.5	-1.1	1.5093	-1.1280	0.1286	0.4242

4. Real data illustration

In this section, we provide application of the HLP distribution to a real data for illustration. We consider the AIC (Akaike information criterion), BIC (Bayesian information criteria) and KS (Kolmogorov Smirnov) test statistic for comparison between the HLP distribution and some other existing distributions such as the Olapade generalize half logistic (OGHL) distribution by [39] with cdf given as $F(x) = 1-\left(2^{\theta}(1+e^{x/\alpha})^{-\theta}\right)$, Power half logistic (PwHL) by [23] with cdf $F(x) = 1-(2/(1+\exp(\theta x^{\alpha})))$, generalized half logistic (GHL) with cdf $F(x) = (1-\exp(-\alpha x))^{\theta}(1+\exp(-\alpha x))^{-\theta}$, GHL distribution appears in the study of its estimation procedures by [22, 4, 21, 44] and [43] among others, where some of its important

properties such as the r^{th}-moments, probability weighted moments, Shannon and Renyi entropies can be obtained from [11]. The generalized exponential (GE) with cdf $F(x) = (1-\exp(-\alpha x))^{\theta}$ by [35], BurrXII (BXII) by [7] with $F(x) = 1-(1+x^{\alpha})^{-\theta}$, Chen distribution (Chen) by [9] with $F(x) = 1 - \exp(\theta(1 - \exp(x^{\alpha})))$, exponential (E) with $F(x) = 1 - \exp(-\alpha x)$, and half logistic (HL)distribution given by (2). The model with the smallest value of AIC, BIC and KS provide best fit than the other models.
The data set is provided by [41] and recently analyzed by [20] is the number of successive failures obtained from the air conditioning system of each member in a fleet of 13 Boeing 720 jet airplanes, the data set are: 194, 413, 90, 74, 55, 23, 97, 50, 359, 50, 130, 487, 57, 102, 15, 14, 10, 57, 320, 261, 51, 44, 9, 254, 493, 33, 18, 209, 41, 58, 60, 48, 56, 87, 11, 102, 12, 5, 14, 14, 29, 37, 186, 29, 104, 7, 4 ,72, 270, 283, 7 ,61, 100, 61, 502, 220, 120, 141, 22, 603, 35, 98 ,54, 100, 11, 181, 65 ,49 ,12 ,239, 14 ,18 ,39, 3 ,12, 5 ,32, 9, 438, 43, 134, 184, 20, 386, 182, 71, 80, 188, 230, 152, 5 ,36, 79, 59, 33, 246, 1 ,79, 3 ,27, 201, 84 ,27 ,156, 21 ,16, 88, 130, 14, 118, 44 ,15 ,42, 106 ,46 ,230 ,26, 59 ,153, 104, 20, 206, 5 ,66 ,34, 29, 26, 35 ,5 ,82, 31, 118 ,326 ,12, 54, 36 ,34, 18, 25, 120, 31 ,22 ,18, 216, 139, 67, 310, 3 ,46 ,210, 57, 76 ,14 ,111, 97, 62, 39, 30, 7 ,44, 11, 63, 23 ,22 ,23, 14, 18, 13, 34, 16, 18, 130, 90, 163 ,208, 1 ,24, 70, 16, 101, 52, 208, 95, 62, 11, 191 ,14 ,71.

The computed values of the MLEs, AIC, BIC and KS with its P-value of each model for the given datasets are listed in Table 3, as you can see, the results shows that HLP distribution represent the data better than the other competing distributions. Figur 4 shows the plots of the (i) histogram with fitted HLP and (ii) empirical cdf with the fitted HLP distribution cdf, Figur 5 display the quantile-quantile plot (iii) and hazard rate function of HLP distribution (iv) for the given data set.

Table 3. MLEs, $\ell(\Theta)$, AIC, BIC, KS and p-value for the given data set

Model	α	λ	θ	$\ell(\Theta)$	AIC	BIC	K-S	p-value
HLP	0.0069	3.3077	-	-1036.261	2076.522	2082.995	0.0572	0.5708
OGHL	1.1419	-	0.0125	-1037.644	2079.288	2085.761	0.0862	0.1227
GHL	0.0120	-	0.7248	-1044.156	2092.312	2098.785	0.1017	0.0409
PwHL	0.7680	-	0.0475	- 1039.484	2082.968	2089.441	0.0628	0.4483
GE	0.0102	-	0.9100	- 1037.750	2079.500	2085.973	0.0728	0.2718
BXII	1.0801	-	0.2317	- 1180.456	2364.912	2371.385	0.3681	0.0000
Chen	0.2582	-	0.0396	-1053.409	2110.818	2117.291	0.1013	0.0422
HL	0.0145	-	-	-1051.020	2104.040	2107.270	0.1538	2.9e-4
E	0.0109	-	-	- 1038.248	2078.496	2081.732	0.0843	0.1385

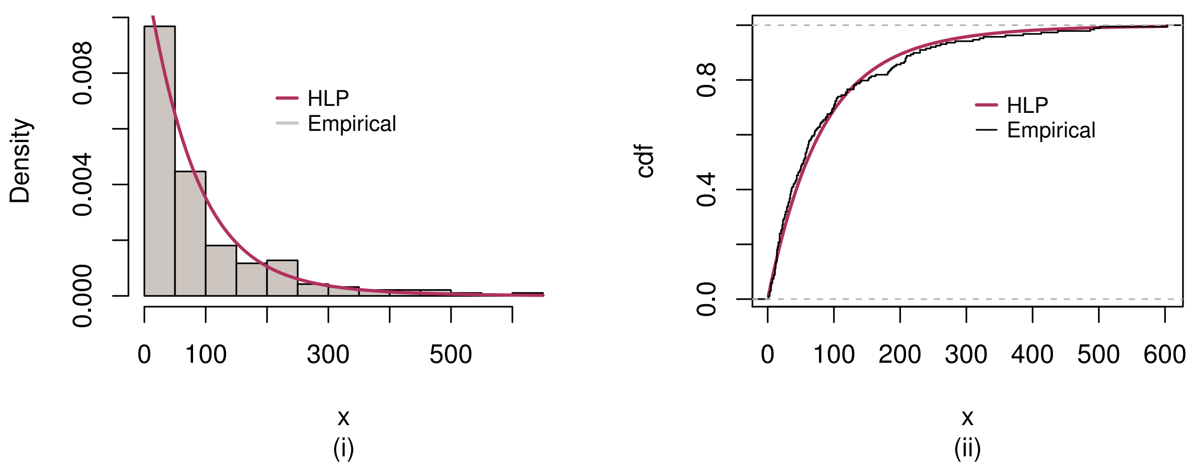

FIGURE 4. Plots of (i) histogram with fitted HLP and (ii) empirical cdf with fitted HLP cdf.

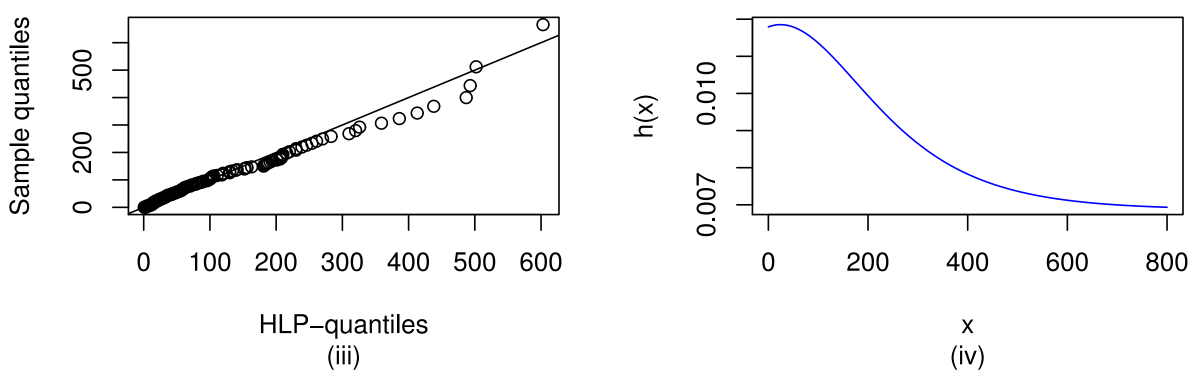

FIGURE 5. (iii) Quantile-quantile plot and (iv) hazard rate function of HLP.

5. CONCLUSIONS

We proposed and study a new two-parameter lifetime distribution called the half-logistic Poisson (HLP) distribution. We provide several properties of the new distribution in which we derive an explicit formula for the r^{th} moment, moment generating function, order statistics, r^{th} moment of order statistic and Shannon entropy. We estimate the two unknown parameters of the distribution by the method of maximum likelihood and assessed by simulation studies. The usefulness of the HLP distribution is illustrated by means of real data set in which HLP provide better fit than some other popular models.

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