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MODELLING THE DYNAMICS OF ZIKA VIRUS AMONG HUMAN BEINGS, PRIMATES, AND MOSQUITOES POPULATIONS

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Abstract. Due to the fact that ZIKA virus circulate in both primates and human beings populations, the dynamics of the virus can be described better by a mathematical model which include both populations. This work presents a compartmental mathematical model, SEIR-SEI-SEIR, for human beings, mosquitoes, and primates respectively. The model considers sexual transmission for both human beings and primates populations, and vertical transmission for all the three populations. The work examines effects of some model parameters on basic reproductive number, on human beings disease incidence, and on human beings disease prevalence. Our study finally concludes that neither vaccination of human beings alone nor the application of mosquito nets, condoms, and abstinence from sexual activities alone can eradicate the disease. Instead, one can eradicate the disease by applying one, or a combination, of the following possible, but not limited to, sets of strategies: vaccine both primates and human beings; heavily use of mosquito pesticides that can let mosquito's death rate be at least 15 times more than its natural death rate; a combination of the use of mosquito pesticides, and the use of mosquito nets, condoms, or abstinence from sexual activities; a combination of the use of mosquito pesticide and vaccinating human beings/primates.

1. Introduction

ZIKA virus is a mosquito - transmitted flavivirus of the family fraviviridae [13]. It causes fever, Guillain-Barre syndrome (GBS), and Microcephaly - brain defects to a fetus of an infected mother [12]. The congenital neurological disorder due to ZIKA disease poses a significant concern for the nearly 15 million children born in the Americas each year[17]. Moreover, WHO estimated that about four million Americans could be infected by ZIKA in 2016 [10].

ZIKA virus is primarily transmitted through mosquito bites of an Aedes genus [11]. It can also be transmitted through sexual intercourse [9, 15]. Vertical transmission from an infected mother to her fetus during and before delivery is possible too [1]. Infected mosquitoes can also lay eggs which are infected by ZIKA virus [18].

ZIKA virus was first isolated from sentinel rhesus in 1947 in Uganda at Zika forest [14]. In Nigeria, the first human infection was reported in 1954; thereafter, ZIKA outbreak occurred

Key words and phrases. ZIKA virus, Primates, Basic reproductive number, Sexual transmission, and Vertical transmission.

in Yap Island in the Federated States of Micronesia in 2007 [14]. This was followed by a large outbreak in French Polynesia in 2013-2014, then it spread to New Caledonia, the Cook Islands and Eastern Islands and then to Brazil in 2015 [14].

Many studies on ZIKA dynamics have been conducted since the 2015 ZIKA outbreak in Brazil. However, we lack a comprehensive mathematical model which consider dynamics of ZIKA virus among and between human beings, mosquitoes, and primates populations. This work presents a compartmental mathematical model with human beings and primates as the virus hosts, and mosquitoes as vectors. The model includes sexual transmission for both human beings and primates populations, and vertical transmission for all the three populations.

2. Materials and Methods

2.1. Model Development. We regard human beings and primates as the current known hosts of the virus, while mosquitoes (Aedes species) act as transmission vectors. We use the SEIR, SEI, and SEIR compartments for human beings, mosquitoes and primates respectively to model the dynamics of ZIKA virus disease. We neglect the human death due to ZIKA since very few cases have been reported and not yet confirmed by serious examination. We assume that human beings and primates are the only hosts of ZIKA virus. We also assume that all Aedes mosquitoes responsible for ZIKA transmission have similar transmission properties, and therefore, we categorize them in one population. We assume that the epidemic process is deterministic. We ignore migrations in all the three populations by assuming that emigrations counter balance immigrations.

In our model, a susceptible human (S_h) gets the infection in two ways: first, through having sex with an infected human(I_h) and second, through a bite of an infected mosquito (I_m). He/she then progresses to an exposed class at a rate which is proportional to contact rate and level of infections. However, not all contact can lead to transmission of the infection. Transmission of infection depends on susceptibility of the susceptible, infectivity of the infected member, and probability that a contact will transmit infection [5].

In this model, susceptible members leave the susceptible class to exposed class at a rate $\beta_1(\lambda_{hh}I_h/N_1+\lambda_{mh}I_m/N_2)$, where, β_1 is human being susceptibility, λ_{hh} is an adequate human to human contact rate, and λ_{mh} is an adequate contact rate from mosquito to human. The fractions I_h/N_1, or I_m/N_1, represents a possibility that a person or a mosquito that is contacted with a susceptible is infectious [5].

Since ZIKA virus can be transmitted vertically, there is a proportional p_1 that a new born in human population N_1 is born susceptible at a rate γ_1 and a proportional $1-p_1$ that the new born is infected. A susceptible human beings remains in an exposed class for some days after he/she gets the virus. An exposed human (E_h) progresses to the infectious class at a rate α_1 and then recovers naturally at a rate α_2 and attains permanent immunity against reinfections. Since the death due to ZIKA disease is very rarely, almost negligible, the model considers only natural death at a rate μ_1 for human.

A susceptible primate (S_p) gets infection in two ways: first, through having sex with an infected primate (I_p), and second, through a bite of an infected mosquito (I_m). It then progresses to an exposed class at the rate $\beta_3(\lambda_{mp}I_m/N_2+\lambda_{pp}I_p/N_3)$. Due to vertical transmission, there is a proportional p_3 that a new born in primate population N_3 is born susceptible at a rate γ_3, and a proportional $1-p_3$ that the new born is infected. An exposed primate (E_p) progresses from an exposed class to infectious class at a rate α_3. It then recovers naturally at a rate α_5 and attains permanent immunity against reinfection. A primate dies naturally at a rate μ_3. No death of a primate due to ZIKA virus has been reported. A susceptible Aedes

Table 1. Model Variables

Variable	Description
S_h	Susceptible human beings
E_h	Exposed Human beings
I_h	Infected Human beings
R_h	Recovered Human beings
S_m	Susceptible Mosquitoes
E_m	Exposed Mosquitoes
I_m	Infected Mosquitoes
S_p	Susceptible primates
E_p	Exposed primates
I_p	Infected primates
R_p	Recovered primates

mosquito (S_m) gets the infection when it bites either an infected human (I_h) or an infected primate (I_p). It then progresses to an exposed class at the rate $\beta_2(\lambda_{hm}I_h/N_1+\lambda_{pm}I_p/N_3)$. A proportion, p_2 of eggs laid by an infected mosquito are not infected. Meanwhile, the remaining proportion $1-p_2$ of eggs laid by an infected mosquito are infected. Approximately 1 egg out of 290 eggs of infected mosquito are found to be infected with ZIKA virus [18]. Mosquitoes are recruited through birth at a rate γ_2. The susceptible mosquito remains in an exposed class for some days and then progress to infectious class at a rate α_2. It then dies naturally at a rate μ_3 after about 1 month from the day it hatched. Table 1 and Table 2 show model's variables and parameters respectively.

Table 2: Model Parameters

Parameter	**Description**	**Units**
μ_1	Natural death rate of a human being	*death* day^{-1}

Continued on next page

Table 2 – *Continued from previous page*

Parameter	**Description**	**Units**
μ_2	Natural death rate of a mosquito	$death\ day^{-1}$
μ_3	Natural death rate of a primate	$death\ day^{-1}$
γ_1	Birth rate of a human being	$birth\ day^{-1}$
γ_2	Hatching rate of a mosquito	$hatches\ day^{-1}$
γ_3	Birth rate of a human primate	$birth\ day^{-1}$
p_1	Probability that a human being is born infected	unit less
p_2	Probability that a mosquito is hatched out infected	unit less
p_3	Probability that a primate is born infected	unit less
β_1	Human being's susceptibility	unit less
β_2	Mosquito's susceptibility	unit less
β_3	Primate's susceptibility	unit less
α_1	Human being's progressive rate from exposed class to infectious class	$humans\ day^{-1}$
α_2	Human being's recover rate	$humans\ day^{-1}$
α_3	Mosquito's progressive rate from exposed class to infectious class	$mosquitoes\ day^{-1}$
α_4	Primate's progressive rate from exposed class to infectious class	$primates\ day^{-1}$
α_5	Primate's recover rate	$primates\ day^{-1}$
λ_{hh}	Adequate contact rate that lead to infection from human being to human being through sex	$contacts\ day^{-1}$
λ_{pp}	Adequate contact rate that lead to infection from primate to primate through sex	$contacts\ day^{-1}$
λ_{hm}	Adequate contact rate that lead to infection from human to mosquito	$contacts\ day^{-1}$
λ_{mh}	Adequate contact rate that lead to infection from mosquito to human	$contacts\ day^{-1}$
λ_{pm}	Adequate contact rate that lead to infection from primate to mosquito	$contacts\ day^{-1}$
λ_{mp}	Adequate contact rate that lead to infection from mosquito to primate	$contacts\ day^{-1}$

The dynamics of ZIKA described in Section 2.1 can be represented by the compartmental diagram shown in Figure 1. The doted line shows interactions that lead into transmission of the infection, and the arrows shows movements of individuals, mosquitoes, and primates from one compartment to another.

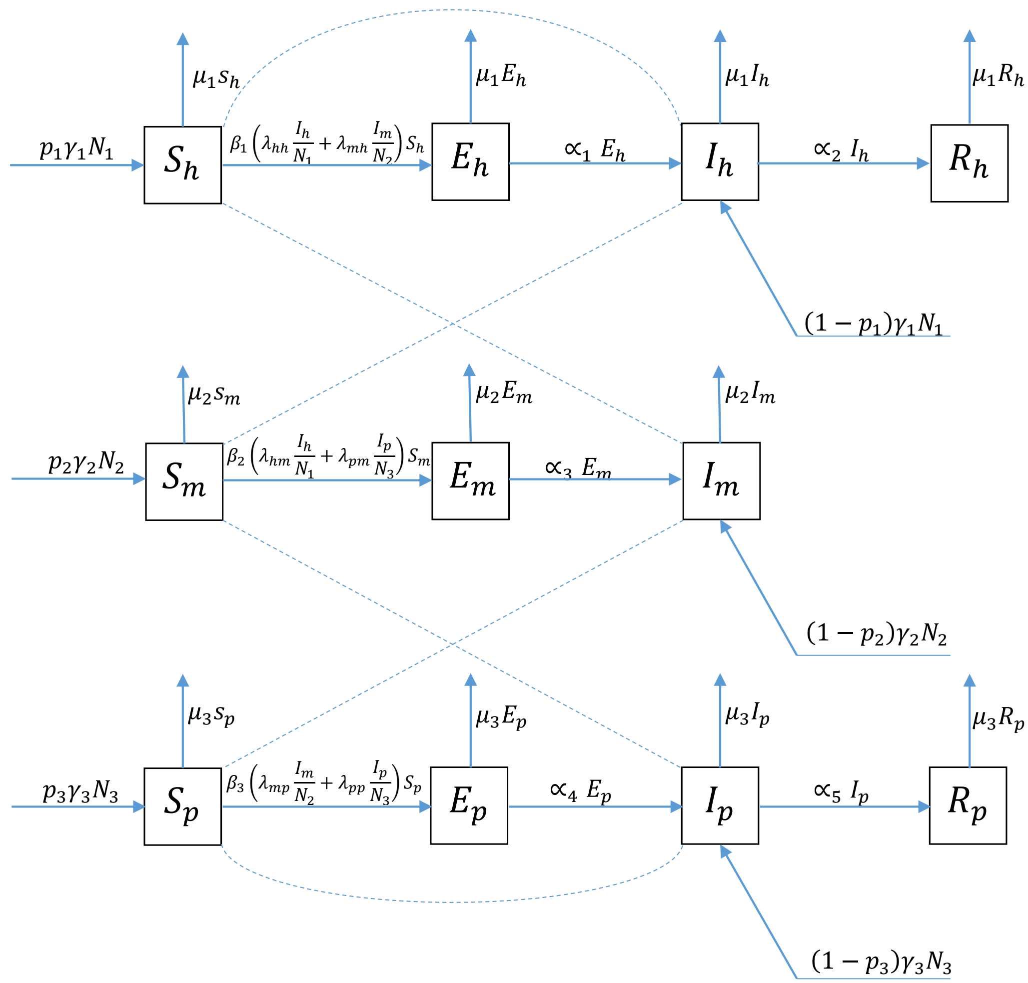


FIGURE 1. Flow diagram of ZIKA infections between human beings, mosquitoes, and primates.

2.1.1. *Model Equations.* The model equations as deduced from the flow diagram in Figure 1. The equations are categorized into three groups: human beings, mosquitoes, and primates populations.

Human beings

$$\frac{dS_h}{dt} = p_1\gamma_1 N_1 - \mu_1 S_h - \beta_1(\lambda_{hh} I_h/N_1 + \lambda_{mh} I_m/N_2)S_h, \tag{1}$$

$$\frac{dE_h}{dt} = \beta_1(\lambda_{hh} I_h/N_1 + \lambda_{mh} I_m/N_2)S_h - \mu_1 E_h - \alpha_1 E_h, \tag{2}$$

$$\frac{dI_h}{dt} = \alpha_1 E_h + (1 - p_1)\gamma_1 N_1 - \mu_1 I_h - \alpha_2 I_h, \tag{3}$$

$$\frac{dR_h}{dt} = \alpha_2 I_h - \mu_1 R_h. \tag{4}$$

Mosquitoes

$$\frac{dS_m}{dt} = p_2\gamma_2 N_2 - \mu_2 S_m - \beta_2(\lambda_{hm} I_h/N_1 + \lambda_{pm} I_p/N_3)S_m, \tag{5}$$

$$\frac{dE_m}{dt} = \beta_2(\lambda_{hm} I_h/N_1 + \lambda_{pm} I_p/N_3)S_m - \mu_2 E_m - \alpha_3 E_m, \tag{6}$$

$$\frac{dI_m}{dt} = \alpha_3 E_m + (1 - p_2)\gamma_2 N_2 - \mu_2 I_m. \tag{7}$$

Primates

$$\frac{dS_p}{dt} = p_3\gamma_3 N_3 - \mu_3 S_p - \beta_3(\lambda_{mp} I_m/N_2 + \lambda_{pp} I_p/N_3)S_p, \tag{8}$$

$$\frac{dE_p}{dt} = \beta_3(\lambda_{mp} I_m/N_2 + \lambda_{pp} I_p/N_3)S_p - \mu_3 E_p - \alpha_4 E_p, \tag{9}$$

$$\frac{dI_p}{dt} = \alpha_4 E_p + (1 - p_3)\gamma_3 N_3 - \mu_3 I_p - \alpha_5 I_p, \tag{10}$$

$$\frac{dR_p}{dt} = \alpha_5 I_p - \mu_3 R_p, \tag{11}$$

where: $N_1 = S_h + E_h + I_h + R_h$, $N_2 = S_m + E_m + I_m$, $N_3 = S_p + E_p + I_p + R_p$ and $(S_h, E_h, I_h, R_h, S_m, E_m, I_m, S_p, E_p, I_p, R_p) \geq 0$.

3. Model Analysis

3.1. Invariant Region. Our system of differential equations for human being, mosquito and primate, all together can be written in the form $dX/dt = A_{(x)}X + F$.
$X = (S_h, E_h, I_h, R_h, S_m, E_m, I_m, S_p, E_p, I_p, R_p)^T$,
$F = (p_1\gamma_1 N_1, 0, (1 - p_1)\gamma_1 N_1, 0, p_2\gamma_2 N_2, 0, (1 - p_2)\gamma_2 N_2, p_3\gamma_3 N_3, 0, (1 - p_3)\gamma_3 N_3, 0)^T$, and

(12)
$$A_{(x)} = \begin{pmatrix} -a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_1\beta_1 & -b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & -c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_2\beta_2 & -e & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_3 & -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_3\beta_3 & -g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_4 & -h & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_5 & -\mu_3 \end{pmatrix},$$

where: $a = \mu_1 + k_1\beta_1, b = \mu_1 + \alpha_1, c = \mu_1 + \alpha_2, d = \mu_2 + k_2\beta_2, e = \mu_2 + \alpha_3, f = \mu_3 + k_3\beta_3, g = \mu_3 + \alpha_4, h = \mu_3 + \alpha_5, k_1 = \lambda_{hh}I_h/N_1 + \lambda_{mh}I_m/N_2, k_2 = \lambda_{hm}I_h/N_1 + \lambda_{pm}I_p/N_3$, and $k_3 = \lambda_{mp}I_m/N_2 + \lambda_{pp}I_p/N_3$.
Since $A_{(x)}$ is a Metzler matrix, i.e. a matrix such that off diagonal terms are nonnegative for all X, and $F \geq 0$, we conclude that the system of differential equations for human beings, mosquitoes, and primates is positive invariant in $\mathbb{R}_+^{11}$ [8].

3.2. Positivity of Solution. So long as we are dealing with populations, all variables (solutions) in the system of the differential equations must be greater or equal to zero (non-negative) for $t \geq 0$. We first let the initial values (i.e. at $t = 0$) of the variables as follows: $(E_h(0), E_m(0), E_p(0), I_h(0), I_m(0), I_p(0), R_h(0), R_p(0)) \geq 0$, $(S_h(0), S_m(0), S_p(0)) > 0$. Consider equation (1): $\dfrac{dS_h}{dt} = p_1\gamma_1 N_1 - \mu_1 S_h - \beta_1(\lambda_{hh}I_h/N_1 + \lambda_{mh}I_m/N_2)S_h$. Then, $\dfrac{dS_h}{dt} \geq -\mu_1 S_h - \beta_1(\lambda_{hh}I_h/N_1 + \lambda_{mh}I_m/N_2)S_h$. Solving the differential inequality we get a solution, $S_h(t) \geq Ce^{-(\mu_1+\beta_1\lambda_{hh}I_h/N_1+\beta_1\lambda_{mh}I_m/N_2)t}$. Put initial conditions: $t = 0$ and $S_h(t) = S_h(0)$, we get $C = S_h(0)$. Thus, we get $S_h(t) \geq S_h(0)e^{-(\mu_1+\beta_1\lambda_{hh}I_h/N_1+\beta_1\lambda_{mh}I_m/N_2)t}$. Since $e^{-(\mu_1+\beta_1\lambda_{hh}I_h/N_1+\beta_1\lambda_{mh}I_m/N_2)t} \geq 0$, then, $S_h(t) \geq 0, \forall t \geq 0$. Using the same approach, we have $S_m(t) \geq 0, S_h(t) \geq 0, S_p(t) \geq 0, E_h(t) \geq 0, E_m(t) \geq 0, E_p(t) \geq 0, I_h(t) \geq 0, I_m(t) \geq 0, I_p(t) \geq 0, R_m(t) \geq 0$, and $R_p(t) \geq 0$ for $\forall t \geq 0$.

3.3. Disease free equilibrium point. We find the disease free equilibrium points for three populations, human beings, mosquitoes, and primates, separately before combining the points to get the equilibrium point for the whole system.
Consider system of differential equations (1) - (4) that represents the dynamics in human beings population. At the equilibrium point $\frac{dS_h}{dt} = \frac{dE_h}{dt} = \frac{dI_h}{dt} = \frac{dR_h}{dt} = 0$. For disease free situation $E_h = I_h = R_h = 0, S_h = N_1$, $p_1 = 1$. Since $\frac{dS_h}{dt} = 0$, then $\frac{dN_1}{dt} = 0$, thus $N_1 = N_1^* =$

constant. From equation (1), we have:

$$S_h = p_1\gamma_1 N_1^*/\mu_1 \tag{13}$$

Substituting $p_1 = 1$ into equation (13), we get $S_h = \frac{\gamma_1 N_1^*}{\mu_1}$. Following the same procedures for mosquitoes and primates populations, we obtain $S_m = \frac{\gamma_2 N_2^*}{\mu_2}$ and $S_p = \frac{\gamma_3 N_3^*}{\mu_3}$. Thus, the disease free equilibrium point for the whole system is $(\frac{\gamma_1 N_1^*}{\mu_1}, 0, 0, 0, \frac{\gamma_2 N_2^*}{\mu_2}, 0, 0, \frac{\gamma_3 N_3^*}{\mu_3}, 0, 0, 0)$.

3.4. Basic reproductive number. Basic reproductive number is the number of secondary infections produced by an infectious, during his/her entire infectious period, when exposed to a purely susceptible population [16]. When the basic reproduction number is greater than one, the disease-free steady state of a dynamics system is unstable, and when it is less than one the infection-free steady state is stable [6].

We use the next generation method to find the basic reproduction number of ZIKA disease. Consider system of differential equations (1) - (11). Let f_i be a vector that consists of new infectious terms of transmitting components and v_i be a vector of the remaining transfer terms.

$$f_i = \begin{pmatrix} \beta_1(\lambda_{hh}I_h/N_1 + \lambda_{mh}I_m/N_2)S_h \\ (1-p_1)\gamma_1 N_1 \\ 0 \\ \beta_2(\lambda_{hm}I_h/N_1 + \lambda_{pm}I_p/N_3)S_m \\ (1-p_2)\gamma_2 N_2 \\ \beta_3(\lambda_{mp}I_m/N_2 + \lambda_{pp}I_p/N_3)S_p \\ (1-p_3)\gamma_3 N_3 \\ 0 \end{pmatrix}, \; v_i = \begin{pmatrix} \mu_1 E_h + \alpha_1 E_h \\ -\alpha_1 E_h + \mu_1 I_h + \alpha_2 I_h \\ -\alpha_2 I_h + \mu_1 R_h \\ \mu_2 E_m + \alpha_3 E_m \\ -\alpha_3 E_m + \mu_2 I_m \\ \mu_3 E_p + \alpha_4 E_p \\ -\alpha_4 E_p + \mu_3 I_p + \alpha_5 I_p \\ -\alpha_5 I_p + \mu_3 R_p \end{pmatrix}.$$

Let F and V be the Jacobean of F_i and V_i respectively, evaluated at the disease free equilibrium point.

$$F = \begin{pmatrix} 0 & \beta_1\lambda_{hh} & 0 & 0 & \frac{\beta_1\lambda_{mh}N_1}{N_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\beta_2\lambda_{hm}N_2}{N_1} & 0 & 0 & 0 & 0 & \frac{\beta_2\lambda_{pm}N_2}{N_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_3\lambda_{mp}N_3}{N_2} & 0 & \beta_3\lambda_{pp} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$V = \begin{pmatrix} \mu_1+\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_1 & \mu_1+\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_2 & \mu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_2+\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_3 & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_3+\alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha_4 & \mu_3+\alpha_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_5 & \mu_3 \end{pmatrix}.$$

The basic reproductive number is the largest Eigenvalue of a matrix FV^{-1} and is given by:

$$R_o = 1/6P - 2/3(\frac{3C - B^2}{P}) - 1/3B. \tag{14}$$

$P = (-108D + 36CB + 12\sqrt{12B^3D - 3B^2C^2 - 54BCD + 12C^3 + 81D^2} - 8B^3)^{1/3}$,
$B = -a - p$, $C = ap - ce - km$, $D = akm + cep$, $a = \frac{\beta_1\lambda_{hh}\alpha_1}{(\mu_1+\alpha_1)(\mu_1+\alpha_2)}$, $p = \frac{\beta_3\lambda_{pp}\alpha_4}{(\mu_3+\alpha_4)(\mu_3+\alpha_5)}$, $c = \frac{\beta_1\lambda_{mh}N_1\alpha_3}{N_2(\mu_2+\alpha_3)(\mu_2)}$, $e = \frac{\beta_2\lambda_{hm}N_2\alpha_1}{N_1(\mu_1+\alpha_1)(\mu_1+\alpha_2)}$, $k = \frac{\beta_2\lambda_{pm}N_2\alpha_4}{N_3(\mu_3+\alpha_4)(\mu_3+\alpha_5)}$, $m = \frac{\beta_3\lambda_{mp}N_3\alpha_3}{N_2(\mu21+\alpha_3)(\mu_2}$.

3.5. Parameter estimation and simulations. Most of parameter values used in this study were obtained from different studies on ZIKA disease. Some parameters were estimated and some were assumed. The reader should also note that all parameters used in this study are subject to change. Mosquito biting rate for example, may vary according to the weather. Human-human sexual contact rate may also vary from one group of people to another.

We consider the adequate contact rate as the product of a contact (or a biting) rate and a probability of an infected one to transmit infection. We estimate that a human being can make an average of 2 sexual contact per week (i.e. 2/7 per day) and primate does make an average of 1 sexual contact per week (i.e. 1/7 per day). We estimate a range, $0.1 - 0.75$ as a possibility of infected person to transmit infections, and we assume the same for infected primate. Thus, we calculate adequate contact rate from human to human, $\lambda_{hh} = 0.6(2/7) = 0.17$ per day, and the adequate contact rate from primate to primate, $\lambda_{pp} = 0.6(1/7) = 0.086$ per day.

Table 3: Model Parameters

Parameter	**Range**	**Value**	**Sources**
μ_1		1/21900	[4]
μ_2		1/14	[4]
μ_3		1/5475	[2]
γ_1		1/21900	$Assumed = \mu_1$
γ_2		1/14	$Assumed = \mu_2$
γ_3		1/5475	$Assumed = \mu_3$

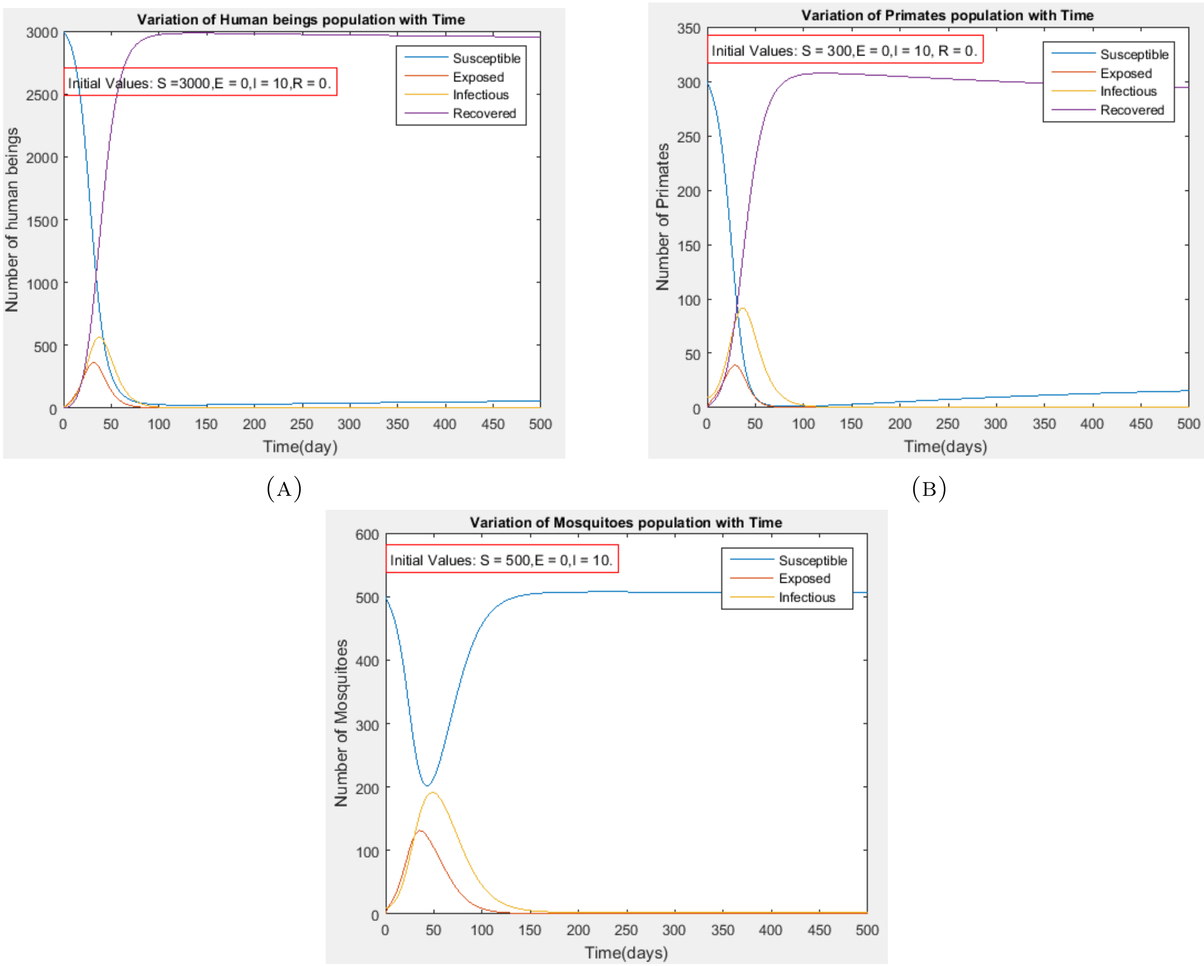
Continued on next page

Table 3 – *Continued from previous page*

Parameter	Range	Value	Sources
p_1	0.8 - 1	0.95	Estimated
p_2		0.997	[18]
p_3	0.8 - 1	0.97	Estimated
β_1	0 - 1	0.85	Estimated
β_2	0 - 1	0.96	Estimated
β_3	0 - 1	0.85	Estimated
α_1	1/7 - 1/2	1/4	[10]
α_2	1/14 - 1/3	1/7	[3]
α_3	1/12 - 1/8	1/8	[10]
α_4	1 /6 - 1/2	1/4	[7]
α_5	at most 1/9	1/12	[7]
λ_{hh}	(0.1 - 0.75)(2/7)	0.17	Estimated
λ_{pp}	(0.1 - 0.75)(1/7)	0.086	Estimated
λ_{hm}	(0.1-0.75)(0.3-1)	0.18	[3]
λ_{mh}	(0.1-0.75)(0.3-1)	0.21	[3]
λ_{pm}		0.30	[2]
λ_{mp}		0.35	[2]

The susceptibilities, β_1, β_2, and β_3 (possibilities of getting infection after being exposed to the adequate contact rate) for human being, mosquito and primate are assumed to be 0.85, 0.96, and 0.85 respectively. The assumptions base on the fact that susceptibility, herein, depends on natural immunity against ZIKA infections, and we roughly estimate that very few (15% of human beings population, 4% of mosquitoes population, 15% percent of primates population) can resist the infection naturally. Thus, 85%, 96%, and 85% are the respective possibilities for human being, mosquito, and primate to catch the infection, when exposed to the adequate contact rate. For the purpose of simulations, we assume that we deal with stable populations that birth rate is approximately equal to death rate as shown in Table 3.

Recovery rates, for both primates and human beings populations, are approximated to be equal to the reciprocal of their respective infectious periods, as it is proved to be an appropriate mathematical assumption in [5]. We hold the same assumption and let the progressive rates from the exposed classes to infectious classes be equal to the reciprocals of the latent periods. We roughly estimate the possibility of a person, and that of a primate, to be born with infection as 0.05 and 0.03 respectively. This is to say, out of 100 live birth, 5 babies are born with ZIKA infection, and 3 primates out of 100 live birth born with the infection. Table 3 shows parameter' ranges, values, and their origins.

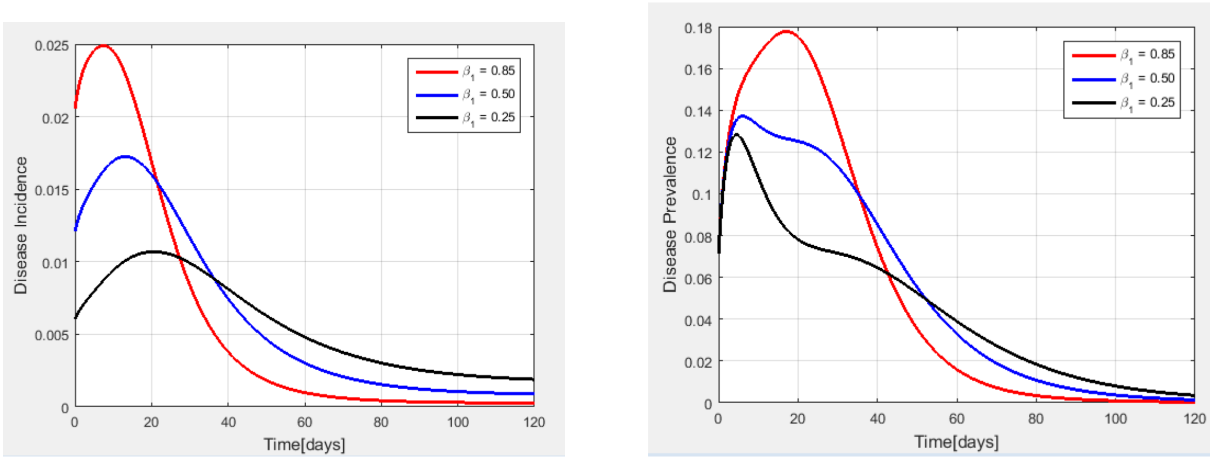


(A) (B)

(C)

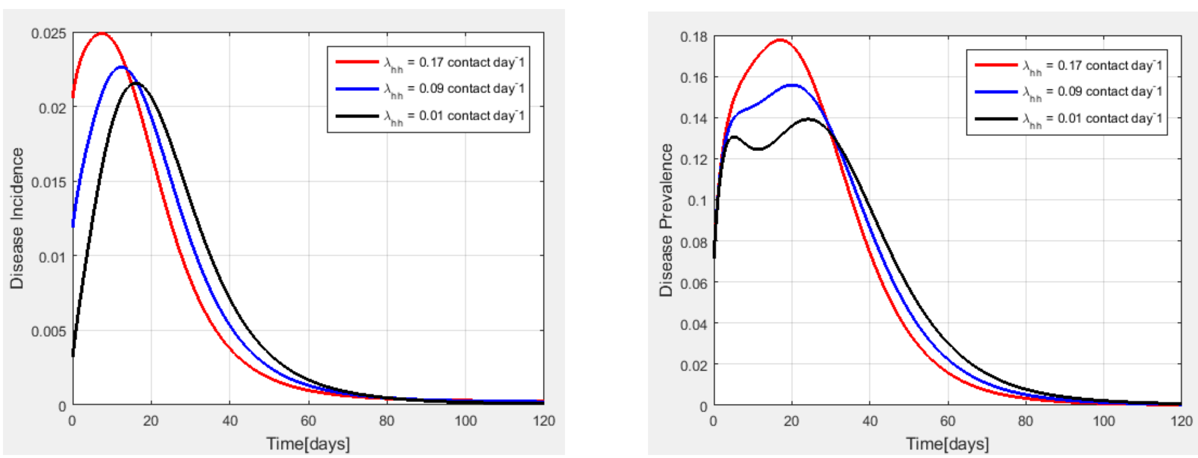
FIGURE 2. Variation of populations with time.

In order to understand the behaviors of variables of the model, we plot the graphs of the variables against time. Figure 2(a) shows the variation of human beings population (in four classes: susceptible, exposed, infectious, and recovery) with time. while the susceptible human beings always decreases from initial value and approach zero asymptotically, the recovered human beings always increase and seems to approach a constant value asymptotically. On the other hand, exposed human beings and infectious human beings initially increase from their starting values, attain their maximum values, and then gradually decrease and approach zero asymptotically. This is so because we set the birth rate equal to death rate, and recovered individuals are not going back to susceptible class since they attain permanent immunity against new infections. Therefore, as time gets larger and larger the susceptibles decreases since new birth counter balance with death. The recovered individuals increase as susceptibles decreases because we regard ZIKA as not a fatal disease, and hence those who leave the susceptible class ultimately end to the recovered class.



(A) (B)

FIGURE 3. The plots of disease incidence and disease prevalence against time for the different values of susceptibility of human beings.



(A) (B)

FIGURE 4. The plots of disease incidence and disease prevalence against time for different values of human to human adequate contact rate through sexual activities.

Figure 2(b) shows the variation of primates population (in four classes: susceptible, exposed, infectious, and recovered) with time. The variation of primates with time exhibits similar behavior as observed in human being population.

Figure 2(c) shows the variation of mosquitoes (in three classes: susceptible, exposed, and infectious) with time. The infectious mosquitoes, and exposed mosquitoes, initially increase to its maximum value and gradually decrease and approach zero asymptotically. On the other hand, the susceptible mosquitoes decreases as the infectious mosquitoes increases, and increases as the number of infectious mosquitoes decreases. This is because,

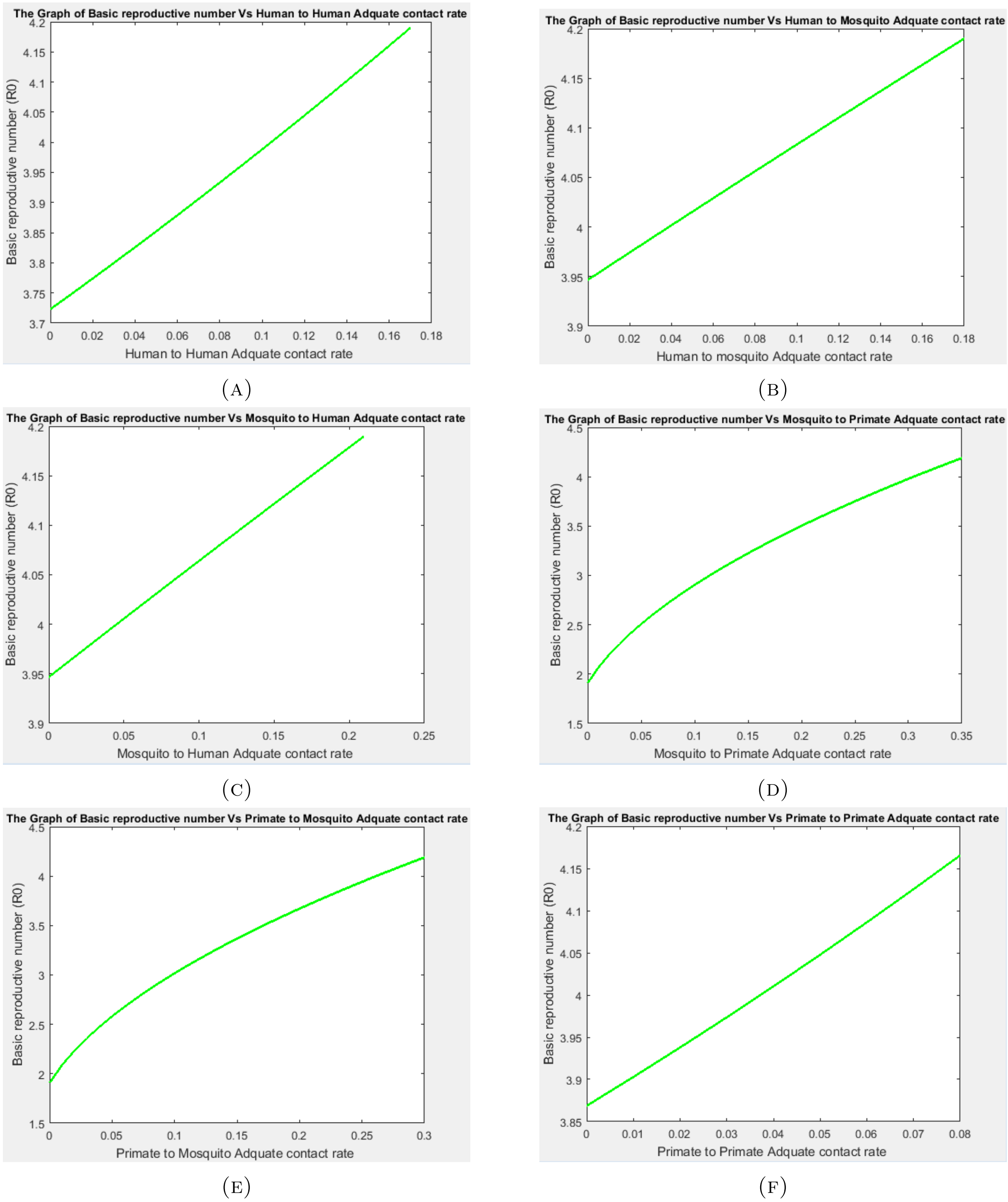
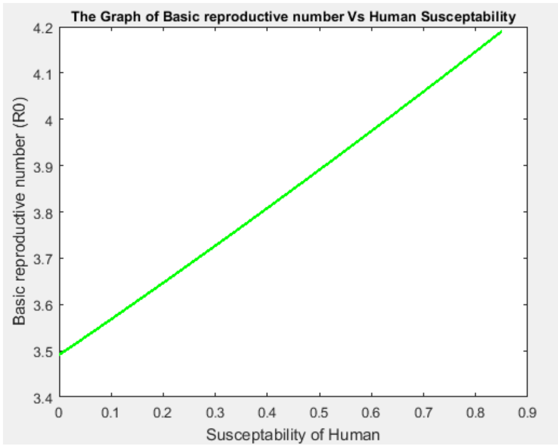
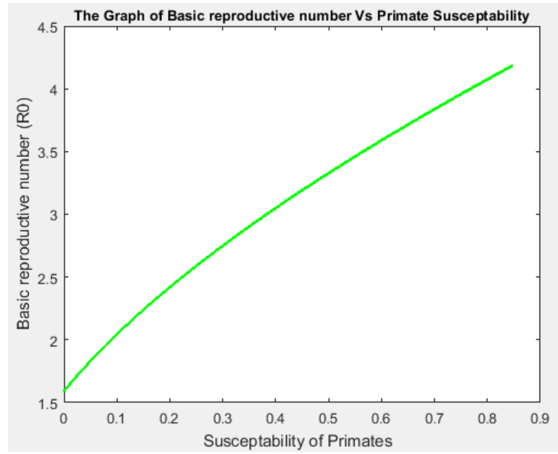


FIGURE 5. Variation of R_0 with adequate contact rates.

in mosquitoes populations there is no recovery class. Infectious mosquitoes will remain infectious throughout their entire life. Hence, all mosquitoes who are leaving the susceptible class will ultimately end to the infectious class.

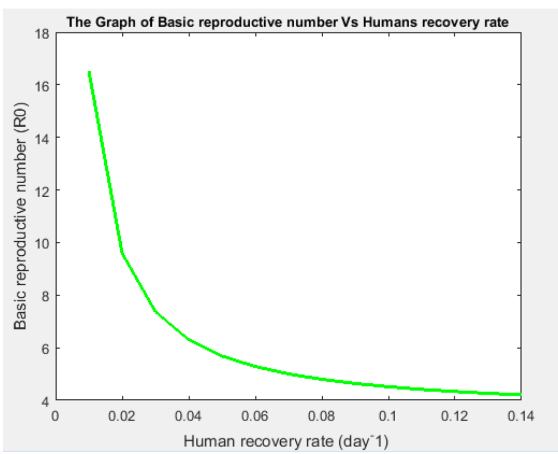


(A)

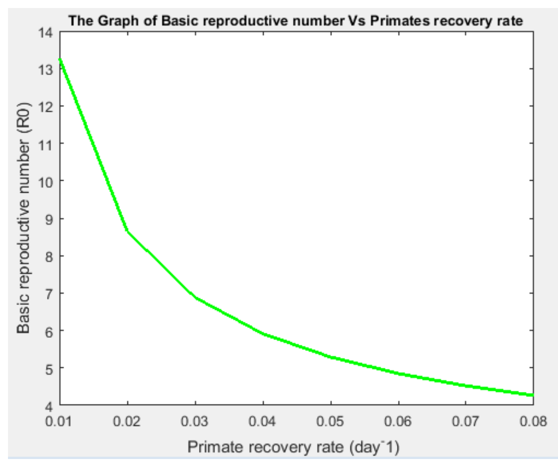


(B)

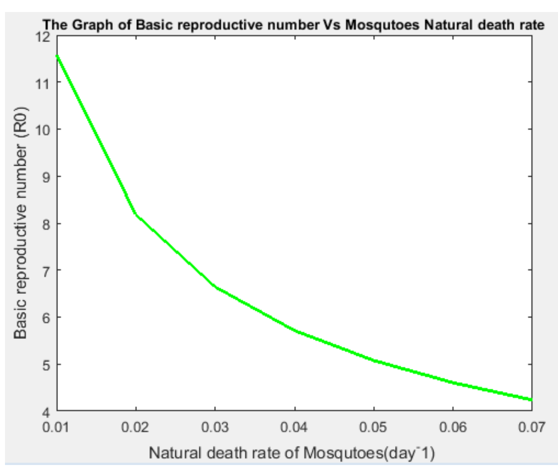
FIGURE 6. Variation of R_o with susceptibilities.



(A)



(B)



(C)

FIGURE 7. Variation of R_o with recovery rates and mosquito birth rate.

It is also very important to understand the effects of varying some parameters on disease incidence, and on disease prevalence. Figure 3(a) shows disease incidence, in human beings population, decreases with a decrease of human susceptibility. Figure 3(b) shows disease prevalence, in human beings population, also decreases with a decrease of human susceptibility. This imply that number new infection, and proportion of people who are infected can be reduced by reducing human susceptibility. Remember, susceptibility can be reduced by vaccinating some proportion of members of a population as described earlier. For example, Figure 3(a) shows that when susceptibility, $\beta_1 = 0.85$, the peak value of new infection rate is 0.025 incidences per day, and when susceptibility, $\beta_1 = 0.25$, the peak value of new infection rate is about 0.006 incidences per day.

Figure 4(a) shows disease incidence, in human beings population, decreases with a decrease of human-human adequate contact rate through sex. Figure 4(b) shows disease prevalence, in human beings population, also decreases with a decrease of human-human adequate contact rate through sex. This suggests that the use of condoms or abstinence from sexual activities can help to reduce both new infection and proportion of people who are infected by ZIKA virus.

To understand more about the impact of varying the parameters on existence of the disease, we plot graphs of basic reproductive number against parameters. Figures 5(a) - 5(f) show that the basic reproductive number increases as the adequate contact rates increase. For example, Figure 5(a) shows that reducing human adequate sexual contact rate (either by using condoms or abstinence from sexual activities) lead to a decrease of the basic reproduction number. However, the plot shows that when the adequate sexual contact rate is zero, the basic reproductive number is still greater than one. This is to say, reducing sexual contacts even up to zero does not result in eradication of the disease.

Moreover, when we tried to vary parameters, by keeping all human adequate contact rates (human to human, human to mosquito, and mosquito to human) zero, and left non-human adequate contact rates and other parameters with their values shown in Table 3, the basic reproductive number was still greater than one. The observation suggests that both human beings and primates populations should be considered if we want to eradicate the disease by reducing adequate contact rate alone - through application of mosquito nets, condoms, and abstinence from sexual activities. But in practice, it is impossible to let primates use mosquito nets and condoms or abstained from sexual activities. Thus, we need to incorporate other control strategies such as vaccination and the use of pesticide that kills mosquitoes rather than relying on mosquito nets, condoms, and abstinence from sexual activities alone.

It should be noted that the above conclusion is true only if the values $\lambda_{pp}, \lambda_{mp}, \lambda_{pm}, \beta_3$, and α_4 are the same or greater than as they are estimated and appeared in Table 3. If we deal with a primate population in which these parameters are much less as appeared in Table 3, for example, we can possibly eradicate ZIKA disease through the application of mosquito nets, condoms, and abstinence from sexual activities alone. For example, if we set $\lambda_{pp} = 0.001, \lambda_{mp} = 0.001, \lambda_{pm} = 0.001, \beta_3 = 0.71$, and $\alpha_4 = 0.001$, and reduce $\lambda_{hh}, \lambda_{mh}$, and λ_{hm} to 0.01, 0.1, and 0.1 respectively, the basic reproductive number reduced to 0.7434 which implies the disease disappearance.

Figure 6 shows the variation of the basic reproductive number with susceptibilities. The basic reproductive number increases as a susceptibility of human or primate increases. As we described earlier that susceptibility is a parameter that reflects the natural immunity against ZIKA infections one can possess i.e. a population with no individual who possesses natural immunity against the disease has 1.0 susceptibility, and a population in which 20% of its members possess the natural immunity has 0.8 susceptibility.

Thus, susceptibility can be reduced by increasing natural immunity against ZIKA infection by possibly, vaccinating some proportion of either human beings population or primates population or both. By doing so the basic reproductive number will decrease. We ignore the discussion of mosquito's susceptibility, thought it also increases with the basic reproductive number because it sound impracticable to think about vaccinating mosquitoes. However one can vaccinate mosquitoes and reduces the basic reproductive number.

This study shows that reducing susceptibility of one population even up to zero does not guarantee the eradication of ZIKA disease. Figure 6(a) and 6(b) shows that the basic reproduction number is greater than one when either human beings susceptibility is zero or primate susceptibility is zero. This implies that when all human beings are either vaccinated or possess natural immunity against the disease, the disease will still circulating within the primates population, and having the possibility of reappear in human being population when significant number of not vaccinated individuals are born.

Figure 7(a) and 7(b) display how the basic reproductive number varies with recovery rates. As the recovery rate of primate or human being increases, the basic reproductive number decreases. Since we made a mathematical assumption that recovery rate is a reciprocal of infectious period, we can reduce the basic reproductive number by reducing the number of infectious period, probably, through medicating the victims so as they can recover much earlier than before.

Figure 7(c) shows the basic reproductive number decreases to zero asymptotically as the mosquitoes natural death rate increases. This suggests that we can also reduce the disease by killing large amount of Aedes mosquitoes responsible for ZIKA transmissions if ethics allow. Moreover, unlike vaccination and application of mosquito nets and condoms, the use of mosquito pesticides alone seems to be able to eradicate the disease if we can use them, heavily, to kill large amount of mosquitoes. By increasing mosquito's birth rate from 1/14 to 15/14, the basic reproduction number decreases from 4.1901 to 0.9571. This implies possibility of eradicating the disease by dealing with mosquitoes alone.

4. Conclusion and Recommendations

This work shows the role played by primates in dynamics of ZIKA disease. It reveals that it is not easy to reduce the basic reproductive number below one if we ignore the dynamics of ZIKA in primates population. It is observed that treating only human beings population cannot let the basic reproduction number become less than one. Treating human beings population alone, may let the disease disappear in the population but still circulating in primates population and ultimately rebound back in the human beings population via mosquitoes.

Our study generally, suggests that any control strategies - vaccination, the use of mosquito nets, pesticides, condoms, and abstinence from sexual activities - can be used to reduce ZIKA infection to some extent since they all have an impact on basic reproductive number.

However, vaccinating human beings alone against ZIKA disease or the application of condoms, mosquito nets, and abstinence from sexual affairs alone can never eradicate the disease. Instead, the disease can be eradicated by application of the following possible (but not limited to) combinations of different control strategies: vaccine both primates and human beings; heavily use of mosquito pesticides that can let mosquito's death rate be at least 15times more than its natural death rate; a combination of the use of mosquito pesticides and the use of nets, condoms, and abstinence from sexual activities; a combination of the use of mosquito pesticide and vaccinating human beings/primates.

This work does not suggest any best set of control strategies for fighting against ZIKA disease. It rather suggests some possible strategies that can be used to fight against the disease. We therefore, recommend an optimal control study, to be done, that will find out an optimal set of control strategies that can eradicate ZIKA disease effectively and efficiently. The study should specify exactly (but not limited to): a threshold proportion of human beings population or/and primate population to be vaccinated with/without combination of other strategies; a threshold mosquito birth rate to be attained with/without combinations of other strategies if viable.

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