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THE DISTRIBUTION OF PRIME AND COMPOSITE NUMBERS BY ITS UNIT DIGIT

DAGNACHEW JENBER

College of Natural and Social Sciences, Mathematics Division, Addis Ababa Science and Technology University, Addis Ababa, Ethiopia

*Email: djdm_101979@yahoo.com

Abstract. This paper studies the nature of prime and composite numbers that the way they distribute using their unit digits. Specifically, it studies the nature of numbers with unit digit 1, 3, 7 and 9 only. There are polynomial sequence functions associated with numbers with unit digit 1, 3, 7 and 9. I hope programmers will use these sequence functions to code an algorithm which is much more important than the previous algorithms to determine the factor of particular composite numbers as well as tracking new prime numbers with in short period of computers time.

1. Introduction

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself[1],[4]. A natural number greater than 1 that is not a prime number is called a composite number[1],[3],[4]. For example, 5 is prime because 1 and 5 are its only positive integer factors, whereas 6 is composite because it has the divisors 2 and 3 in addition to 1 and 6. The fundamental theorem of arithmetic establishes the central role of primes in number theory: any integer greater than 1 can be expressed as a product of primes that is unique up to ordering[1],[11]. The uniqueness in this theorem requires excluding 1 as a prime because one can include arbitrarily many instances of 1 in any factorization, e.g., $3, 1*3, 1*1*3$, etc. are all valid factorizations of 3. The property of being prime (or not) is called primality[12],[13], [9],[5]. A simple but slow method of verifying the primality of a given number n is known as trial division[7],[10]. It consists of testing whether n is a multiple of any integer between 2 and $\sqrt{n}$. Algorithms much more efficient than trial division have been devised to test the primality of large numbers[7].

Key words and phrases. prime and composite numbers; sequence functions of one and two variables.

There is no known useful formula that sets apart all of the prime numbers from composites. However, the distribution of primes, that is to say, the statistical behaviour of primes in the large, can be modelled. The first result in that direction is the prime number theorem, proven at the end of the 19^{th} century, which says that the probability that a given, randomly chosen number n is prime is inversely proportional to its number of digits, or to the logarithm of n. Many questions regarding prime numbers remain open, such as Goldbach's conjecture (that every even integer greater than 2 can be expressed as the sum of two primes), and the twin prime conjecture (that there are infinitely many pairs of primes whose difference is 2)[8]. Such questions spurred the development of various branches of number theory, focusing on analytic or algebraic aspects of numbers. Primes are used in several routines in information technology, such as public-key cryptography, which makes use of properties such as the difficulty of factoring large numbers into their prime factors. Prime numbers give rise to various generalizations in other mathematical domains, mainly algebra, such as prime elements and prime ideals.

I have published one paper titled by the distribution of prime numbers and finding the factor of composite numbers without searching [2]. This paper is more specific than the previous. From my previous paper the formula $f(p) = 105+2p$ for $p \neq 3n, 5n, 7n$[13] is most important and fundamental for this paper.
This paper contains formulas that they enables us to set apart all primes from composites until the desired one (lists down all composites and primes until the desired one), find the factor of composite numbers.

2. A number with unit digit 1

2.1. A number with unit digit 1 happens at a number with unit digit 3

Theorem 1. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 1 for all natural numbers p with unit digit 3, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 3, then $p = 10n-7$, $n \in \mathbb{N}$. Therefore

$$f(p) = 105 + 2p = 105 + 2(10n - 7) = 105 + 20n - 14 = 20n + 91 \Rightarrow f(p) = 20n + 91, n \in \mathbb{N}$$

represents a natural number with unit digit 1. □

Theorem 2. *The sequence function $f(p) = 20p + 91$ represents a natural number with unit digit 1 which are different from $3n$ and $7n$ for all $p \neq 3n - 2, 7n$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 7) = 20p + 91 \neq 3n, 7n$ for all $(10p - 7) \neq 3n, 7n \Rightarrow p \neq \frac{3n+7}{10}, \frac{7n+7}{10}$.

If $p \neq \frac{3n+7}{10}$, then p is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+7}{10} = \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p \neq 3n - 2$. Therefore $f(p) = 20p + 91 \neq 3n$ for all $p \neq 3n - 2$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+7}{10}$, then p is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+7}{10} = \frac{7(10n-1)+7}{10} = 7n \Rightarrow p \neq 7n$. Therefore $f(p) = 20p + 91 \neq 7n$ for all $p \neq 7n$, $n \in \mathbb{N}$. □

Theorem 3. *If $g(p) = 20p + 91$ for $p \neq 3n - 2, 7n$, then the sequence function $g(f_1(p_1, p_2))$ represents a composite natural number with unit digit 1 with factors $10p_1 - 9$ and $10p_2 - 9$ and $\frac{g(f_1)-105}{2}$ = a natural number with unit digit 3, where $f_1(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(9p_1 + 9p_2 + 1)$ for all $p_1 \neq 3n, 7n - 4, 2n$ and $p_2 \neq 3n, 7n - 4, 2n - 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 91 \Rightarrow g(f_1) = 20f_1 + 91 = 20(5p_1p_2 - \frac{1}{2}(9p_1 + 9p_2 + 1)) + 91 = (10p_1 - 9)(10p_2 - 9)$.
Since $(10p_1 - 9)$ and $(10p_2 - 9)$ represents a natural number with unit digit 1, then $g(f_1(p_1, p_2)) = (10p_1 - 9)(10p_2 - 9)$ represents a composite natural number with unit digit 1 with factors $(10p_1 - 9)$ and $(10p_2 - 9)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 9) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$, then $(10p_1 - 9)(10p_2 - 9) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
Therefore $g(f_1(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p_1 \neq 7n - 4$.
Therefore $g(f_1(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n - 4$.
Since $f_1(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(9p_1 + 9p_2 + 1)$, then f_1 to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(9p_1 + 9p_2 + 1)$ must be even.
Hence $g(f_1(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1 \neq 3n, 7n - 4, 2n$ and $p_2 \neq 3n, 7n - 4, 2n - 1$.
□

Theorem 4. *If $g(p) = 20p + 91$ for $p \neq 3n - 2, 7n$, then the sequence function $g(f_2(p_1, p_2))$ represents a composite natural number with unit digit 1 with factors $10p_1 - 3$ and $10p_2 - 7$ and $\frac{g(f_2)-105}{2}$ = a natural number with unit digit 3, where $f_2(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 3p_2 + 7)$ for all $p_1 \neq 3n, 7n - 6, 2n$ and $p_2 \neq 3n - 2, 7n, 2n + 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 91 \Rightarrow g(f_2) = 20f_2 + 91 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 3p_2 + 7)) + 91 = (10p_1 - 3)(10p_2 - 7)$.
Since $(10p_1-3)$ and $(10p_2-7)$ represents a natural number with unit digit 7 and 3 respectively, then
$g(f_2(p_1, p_2)) = (10p_1 - 3)(10p_2 - 7)$ represents a composite natural number with unit digit 1 with factors $(10p_1 - 3)$ and $(10p_2 - 7)$.
If $(10p_1 - 3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2 - 7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1 - 3)(10p_2 - 7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p_2 \neq 3n - 2$.
Therefore $g(f_2(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n - 2$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n - 6 \Rightarrow p_1 \neq 7n - 6$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_2(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n - 6, p_2 \neq 7n$.
Since $f_2(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 3p_2 + 7)$, then f_2 to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(7p_1 + 3p_2 + 7)$ must be even.
Hence $g(f_2(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1 \neq 3n, 7n - 6, 2n$ and $p_2 \neq 3n - 2, 7n, 2n + 1$. □

Theorem 5. *If $g(p) = 20p + 91$ for $p \neq 3n - 2, 7n$, then the sequence function $g(f_3(p_1, p_2))$ represents a composite natural number with unit digit 1 with factors $10p_1 - 1$ and $10p_2 - 1$ and $\frac{g(f_3)-105}{2}$ = a natural number with unit digit 3, where $f_3(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + p_2 + 9)$ for all $p_1 \neq 3n - 2, 7n - 2, 2n$ and $p_2 \neq 3n - 2, 7n - 2, 2n + 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 91 \Rightarrow g(f_3) = 20f_3 + 91 = 20(5p_1p_2 - \frac{1}{2}(p_1 + p_2 + 9)) + 91 = (10p_1 - 1)(10p_2 - 1)$.
Since $(10p_1 - 1)$ and $(10p_2 - 1)$ represents a natural number with unit digit 9, then
$g(f_3(p_1, p_2)) = (10p_1 - 1)(10p_2 - 1)$ represents a composite natural number with unit digit 1 with factors $(10p_1 - 1)$ and $(10p_2 - 1)$.
If $(10p_1 - 1) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$ and $(10p_2 - 1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1 - 1)(10p_2 - 1) \neq 3n, 7n$.

If $p_1 \neq \frac{3n+1}{10}$, then p_1 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+1}{10} \Rightarrow p_1 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_1 \neq 3n-2$.
Therefore $g(f_3(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+1}{10}$, then p_1 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+1}{10} \Rightarrow p_1 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_1 \neq 7n-2$.
Therefore $g(f_3(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n-2$.
Since $f_3(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + p_2 + 9)$, then f_3 to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(p_1 + p_2 + 9)$ must be even.
Hence $g(f_3(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1 \neq 3n-2, 7n-2, 2n$ and $p_2 \neq 3n-2, 7n-2, 2n+1$. □

Theorem 6. *If $p \notin \{3n-2\} \cup \{7n\} \cup \{f_1\} \cup \{f_2\} \cup \{f_3\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 91$ represents a prime number with unit digit 1 greater than or equal to 131, where $f_1(p_1, p_2)$, $f_2(p_1, p_2)$ and $f_3(p_1, p_2)$ are defined from the above Theorem 3, Theorem 4 and Theorem 5 respectively.*

Proof. Clearly followed from Theorem 3, 4 and 5. □

2.2. A number with unit digit 1 happens at a number with unit digit 8

Theorem 7. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 1 for all natural numbers p with unit digit 8, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 8, then $p = 10n-2$, $n \in \mathbb{N}$. Therefore

$$f(p) = 105 + 2p = 105 + 2(10n-2) = 105 + 20n - 4 = 20n + 91 \Rightarrow f(p) = 20n + 101, n \in \mathbb{N}$$

represents a natural number with unit digit 1. □

Theorem 8. *The sequence function $f(p) = 20p + 101$ represents a natural number with unit digit 1 which are different from $3n$ and $7n$ for all $p \neq 3n-1, 7n-4$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p-2) = 20p + 101 \neq 3n, 7n$ for all $(10p-2) \neq 3n, 7n \Rightarrow p \neq \frac{3n+2}{10}, \frac{7n+2}{10}$.
If $p \neq \frac{3n+2}{10}$, then p is to be natural number $n = 6, 16, 26, 36, ... = 10n-4$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+2}{10} = \frac{3(10n-4)+2}{10} = 3n-1 \Rightarrow p \neq 3n-1$. Therefore $f(p) = 20p + 101 \neq 3n$ for all $p \neq 3n-1$, $n \in \mathbb{N}$.

If $p \neq \frac{7n+2}{10}$, then p is to be natural number $n = 4, 14, 24, 34, ... = 10n-6$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+2}{10} = \frac{7(10n-6)+2}{10} = 7n-4 \Rightarrow p \neq 7n-4$. Therefore $f(p) = 20p+101 \neq 7n$ for all $p \neq 7n-4$, $n \in \mathbb{N}$.

□

Theorem 9. *If $g(p) = 20p+101$ for $p \neq 3n-1, 7n-4$, then the sequence function $g(f_5(p_1, p_2))$ represents a composite natural number with unit digit 1 with factors $10p_1 - 9$ and $10p_2 - 9$ and $\frac{g(f_5)-105}{2}$ = a natural number with unit digit 8, where $f_5(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(9p_1+9p_2+2)$ for all $p_1, p_2 \neq 3n, 7n-4, 2n-1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 101 \Rightarrow g(f_5) = 20f_5 + 101 = 20(5p_1p_2 - \frac{1}{2}(9p_1 + 9p_2 + 2)) + 101 = (10p_1 - 9)(10p_2 - 9)$.
Since $(10p_1 - 9)$ and $(10p_2 - 9)$ represents a natural number with unit digit 1, then
$g(f_5(p_1, p_2)) = (10p_1 - 9)(10p_2 - 9)$ represents a composite natural number with unit digit 1 with factors $(10p_1 - 9)$ and $(10p_2 - 9)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 9) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$, then $(10p_1 - 9)(10p_2 - 9) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
Therefore $g(f_5(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n-4 \Rightarrow p_1 \neq 7n-4$.
Therefore $g(f_5(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n-4$.
Since $f_5(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(9p_1 + 9p_2 + 2)$, then f_5 to be natural number p_1 and p_2 should be even/odd natural number because the expression $(9p_1 + 9p_2 + 2)$ must be even.
Hence $g(f_5(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1, p_2 \neq 3n, 7n-4, 2n-1$. □

Theorem 10. *If $g(p) = 20p + 101$ for $p \neq 3n-1, 7n-4$, then the sequence function $g(f_6(p_1, p_2))$ represents a composite natural number with unit digit 1 with factors $10p_1 - 3$ and $10p_2 - 7$ and $\frac{g(f_6)-105}{2}$ = a natural number with unit digit 8, where $f_6(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 3p_2 + 8)$ for all $p_1 \neq 3n, 7n-6, 2n+1$ and $p_2 \neq 3n-2, 7n, 2n+1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 101 \Rightarrow g(f_6) = 20f_6 + 101 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 3p_2 + 8)) + 101 = (10p_1 - 3)(10p_2 - 7)$.
Since $(10p_1-3)$ and $(10p_2-7)$ represents a natural number with unit digit 7 and 3 respectively, then

$g(f_6(p_1,p_2)) = (10p_1-3)(10p_2-7)$ represents a composite natural number with unit digit 1 with factors $(10p_1-3)$ and $(10p_2-7)$.
If $(10p_1-3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2-7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1-3)(10p_2-7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_6(p_1,p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_1 \neq 7n-6$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_6(p_1,p_2)) \neq 7n$ for all $p_1 \neq 7n-6, p_2 \neq 7n$.
Since $f_6(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(7p_1+3p_2+8)$, then f_6 to be natural number p_1 and p_2 should be even/odd natural number because the expression $(7p_1+3p_2+8)$ must be even.
Hence $g(f_6(p_1,p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1 \neq 3n, 7n-6, 2n+1$ and $p_2 \neq 3n-2, 7n, 2n+1$.

□

Theorem 11. *If* $g(p) = 20p + 101$ *for* $p \neq 3n-1, 7n-4$*, then the sequence function* $g(f_7(p_1,p_2))$ *represents a composite natural number with unit digit* 1 *with factors* $10p_1-1$ *and* $10p_2-1$ *and* $\frac{g(f_7)-105}{2}$ = *a natural number with unit digit* 8*, where* $f_7(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(p_1+p_2+10)$ *for all* $p_1, p_2 \neq 3n-2, 7n-2, 2n+1$*, where* $n \in \mathbb{N}$.

Proof. $g(p) = 20p+101 \Rightarrow g(f_7) = 20f_7+101 = 20(5p_1p_2 - \frac{1}{2}(p_1+p_2+10)) + 101 = (10p_1-1)(10p_2-1)$.
Since $(10p_1-1)$ and $(10p_2-1)$ represents a natural number with unit digit 9, then $g(f_7(p_1,p_2)) = (10p_1-1)(10p_2-1)$ represents a composite natural number with unit digit 1 with factors $(10p_1-1)$ and $(10p_2-1)$.
If $(10p_1-1) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$ and $(10p_2-1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1-1)(10p_2-1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+1}{10}$, then p_1 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+1}{10} \Rightarrow p_1 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_1 \neq 3n-2$.
Therefore $g(f_7(p_1,p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n-2$.

If $p_1 \neq \frac{7n+1}{10}$, then p_1 is to be natural number $n = 7, 17, 27, 37, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+1}{10} \Rightarrow p_1 \neq \frac{7(10n-3)+1}{10} = 7n - 2 \Rightarrow p_1 \neq 7n - 2.$
Therefore $g(f_7(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n - 2$.
Since $f_7(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + p_2 + 10)$, then f_7 to be natural number p_1 and p_2 should be even/odd natural number because the expression $(p_1 + p_2 + 10)$ must be even.
Hence $g(f_7(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 1 for all $p_1, p_2 \neq 3n - 2, 7n - 2, 2n + 1$.

□

Theorem 12. *If* $p \notin \{3n-1\} \cup \{7n-4\} \cup \{f_5\} \cup \{f_6\} \cup \{f_7\}$, *where* $n \in \aleph$ *then* $g(p) = 20p+101$ *represents a prime number with unit digit* 1 *greater than or equal to* 181, *where* $f_5(p_1, p_2)$, $f_6(p_1, p_2)$ *and* $f_7(p_1, p_2)$ *are defined from the above Theorem* 9, *Theorem* 10 *and Theorem* 11 *respectively.*

Proof. Clearly followed from Theorem 9, 10 and 11. □

Theorem 13. *From the above Theorem* 6 *and Theorem* 12 *the functions* $g(p) = 20p + 91$ *and* $g(p) = 20p + 101$ *respectively represents all prime numbers with unit digit* 1 *greater than or equal to* 131.

Proof. Clearly followed from Theorem 6 and 12. □

3. A number with unit digit 3

3.1. A number with unit digit 3 happens at a number with unit digit 4

Theorem 14. *The sequence function* $f(p) = 105 + 2p$ *represents a natural number with unit digit* 3 *for all natural numbers* p *with unit digit* 4, *where* $n \in \mathbb{N}$.

Proof. Since p is every natural number with unit digit 4, then $p = 10n - 6$, $n \in \mathbb{N}$. Therefore

$$f(p) = 105 + 2p = 105 + 2(10n - 6) = 105 + 20n - 12 = 20n + 93 \Rightarrow f(p) = 20n + 93, n \in \mathbb{N}$$

represents a natural number with unit digit 3. □

Theorem 15. *The sequence function* $f(p) = 20p + 93$ *represents a natural number with unit digit* 3 *which are different from* $3n$ *and* $7n$ *for all* $p \neq 3n, 7n - 5$, *where* $n \in \mathbb{N}$.

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 6) = 20p + 93 \neq 3n, 7n$ for all $(10p - 6) \neq 3n, 7n \Rightarrow p \neq \frac{3n+6}{10}, \frac{7n+6}{10}$.
If $p \neq \frac{3n+6}{10}$, then p is to be natural number $n = 8, 18, 28, 38, ... = 10n - 2$, $n \in \mathbb{N}$. This

implies that
$p \neq \frac{3n+6}{10} = \frac{3(10n-2)+6}{10} = 3n \Rightarrow p \neq 3n$. Therefore $f(p) = 20p + 93 \neq 3n$ for all $p \neq 3n$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+6}{10}$, then p is to be natural number $n = 2, 12, 22, 32, ... = 10n - 8$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+6}{10} = \frac{7(10n-8)+6}{10} = 7n - 5 \Rightarrow p \neq 7n - 5$. Therefore $f(p) = 20p + 93 \neq 7n$ for all $p \neq 7n - 5$, $n \in \mathbb{N}$.

□

Theorem 16. *If $g(p) = 20p+93$ for $p \neq 3n, 7n-5$, then the sequence function $g(f_9(p_1, p_2))$ represents a composite natural number with unit digit 3 with factors $10p_1 - 9$ and $10p_2 - 7$ and $\frac{g(f_9)-105}{2}$ = a natural number with unit digit 4, where $f_9(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 3)$ for all $p_1 \neq 3n, 7n-4, 2n-1$, $p_2 \neq 3n-2, 7n, 2n$ where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 93 \Rightarrow g(f_9) = 20f_9 + 93 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 3)) + 93 = (10p_1 - 9)(10p_2 - 7)$.
Since $(10p_1-9)$ and $(10p_2-7)$ represents a natural number with unit digit 1 and 3 respectively, then
$g(f_9(p_1, p_2)) = (10p_1 - 9)(10p_2 - 7)$ represents a composite natural number with unit digit 3 with factors $(10p_1 - 9)$ and $(10p_2 - 7)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1 - 9)(10p_2 - 7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p_2 \neq 3n - 2$.
Therefore $g(f_9(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n - 2$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p_1 \neq 7n - 4$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_9(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n - 4, p_2 \neq 7n$.
Since $f_9(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 3)$, then f_9 to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(7p_1 + 9p_2 + 3)$ must be even.

Hence $g(f_9(p_1,p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 3 for all $p_1 \neq 3n, 7n-4, 2n-1$ and $p_2 \neq 3n-2, 7n, 2n$.

□

Theorem 17. *If $g(p) = 20p+93$ for $p \neq 3n, 7n-5$, then the sequence function $g(f_{10}(p_1,p_2))$ represents a composite natural number with unit digit 3 with factors $10p_1-3$ and $10p_2-1$ and $\frac{g(f_{10})-105}{2}$ = a natural number with unit digit 4, where $f_{10}(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(p_1+3p_2+9)$ for all $p_1 \neq 3n, 7n-6, 2n+1$, $p_2 \neq 3n-2, 7n-2, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p+93 \Rightarrow g(f_{10}) = 20f_{10}+93 = 20(5p_1p_2 - \frac{1}{2}(p_1+3p_2+9)) + 93 = (10p_1-3)(10p_2-1)$.
Since $(10p_1-3)$ and $(10p_2-1)$ represents a natural number with unit digit 7 and 9 respectively, then
$g(f_{10}(p_1,p_2)) = (10p_1-3)(10p_2-1)$ represents a composite natural number with unit digit 3 with factors $(10p_1-3)$ and $(10p_2-1)$.
If $(10p_1-3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2-1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1-3)(10p_2-1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{10}(p_1,p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_1 \neq 7n-6$.
If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_2 \neq 7n-2$.
Therefore $g(f_{10}(p_1,p_2)) \neq 7n$ for all $p_1 \neq 7n-6, p_2 \neq 7n-2$.
Since $f_{10}(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(p_1+3p_2+9)$, then f_{10} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression (p_1+3p_2+9) must be even.
Hence $g(f_{10}(p_1,p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 3 for all $p_1 \neq 3n, 7n-6, 2n+1$ and $p_2 \neq 3n-2, 7n-2, 2n$.

□

Theorem 18. *If $p \notin \{3n\} \cup \{7n-5\} \cup \{f_9\} \cup \{f_{10}\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 93$ represents a prime number with unit digit 3 greater than or equal to 113, where $f_9(p_1, p_2)$ and $f_{10}(p_1, p_2)$ are defined from the above Theorem 16 and Theorem 17 respectively.*

Proof. Clearly followed from Theorem 16 and 17. □

3.2. A number with unit digit 3 happens at a number with unit digit 9

Theorem 19. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 3 for all natural numbers p with unit digit 9, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 9, then $p = 10n - 1$, $n \in \mathbb{N}$. Therefore

$f(p) = 105 + 2p = 105 + 2(10n - 1) = 105 + 20n - 2 = 20n + 103 \Rightarrow f(p) = 20n + 103, n \in \mathbb{N}$

represents a natural number with unit digit 3.

□

Theorem 20. *The sequence function $f(p) = 20p + 103$ represents a natural number with unit digit 3 which are different from $3n$ and $7n$ for all $p \neq 3n - 2, 7n - 2$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 1) = 20p + 103 \neq 3n, 7n$ for all $(10p - 1) \neq 3n, 7n \Rightarrow p \neq \frac{3n+1}{10}, \frac{7n+1}{10}$.
If $p \neq \frac{3n+1}{10}$, then p is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+1}{10} = \frac{3(10n-7)+1}{10} = 3n - 2 \Rightarrow p \neq 3n - 2$. Therefore $f(p) = 20p + 103 \neq 3n$ for all $p \neq 3n - 2$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+1}{10}$, then p is to be natural number $n = 7, 17, 27, 37, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+1}{10} = \frac{7(10n-3)+1}{10} = 7n - 2 \Rightarrow p \neq 7n - 2$. Therefore $f(p) = 20p + 93 \neq 7n$ for all $p \neq 7n - 2$, $n \in \mathbb{N}$. □

Theorem 21. *If $g(p) = 20p + 103$ for $p \neq 3n - 2, 7n - 2$, then the sequence function $g(f_{12}(p_1, p_2))$ represents a composite natural number with unit digit 3 with factors $10p_1 - 9$ and $10p_2 - 7$ and $\frac{g(f_{12})-105}{2}$ = a natural number with unit digit 9, where $f_{12}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 4)$ for all $p_1 \neq 3n, 7n - 4, 2n - 1$, $p_2 \neq 3n - 2, 7n, 2n + 1$ where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 103 \Rightarrow g(f_{12}) = 20f_{12} + 103 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 4)) + 103 = (10p_1 - 9)(10p_2 - 7)$.
Since $(10p_1 - 9)$ and $(10p_2 - 7)$ represents a natural number with unit digit 1 and 3 respectively, then
$g(f_{12}(p_1, p_2)) = (10p_1 - 9)(10p_2 - 7)$ represents a composite natural number with unit digit

3 with factors $(10p_1 - 9)$ and $(10p_2 - 7)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1 - 9)(10p_2 - 7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p_2 \neq 3n - 2$.
Therefore $g(f_{12}(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n - 2$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p_1 \neq 7n - 4$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_{12}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n - 4, p_2 \neq 7n$.
Since $f_{12}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 9p_2 + 4)$, then f_{12} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(7p_1 + 9p_2 + 4)$ must be even.
Hence $g(f_{12}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 3 for all $p_1 \neq 3n, 7n - 4, 2n - 1$ and $p_2 \neq 3n - 2, 7n, 2n + 1$.

□

Theorem 22. *If $g(p) = 20p + 103$ for $p \neq 3n - 2, 7n - 2$, then the sequence function $g(f_{13}(p_1, p_2))$ represents a composite natural number with unit digit 3 with factors $10p_1 - 3$ and $10p_2 - 1$ and $\frac{g(f_{13})-105}{2}$ = a natural number with unit digit 9, where $f_{13}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 3p_2 + 10)$ for all $p_1 \neq 3n, 7n - 6, 2n + 1$, $p_2 \neq 3n - 2, 7n - 2, 2n + 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 103 \Rightarrow g(f_{13}) = 20f_{10} + 93 = 20(5p_1p_2 - \frac{1}{2}(p_1 + 3p_2 + 10)) + 103 = (10p_1 - 3)(10p_2 - 1)$.
Since $(10p_1-3)$ and $(10p_2-1)$ represents a natural number with unit digit 7 and 9 respectively, then
$g(f_{13}(p_1, p_2)) = (10p_1 - 3)(10p_2 - 1)$ represents a composite natural number with unit digit 3 with factors $(10p_1 - 3)$ and $(10p_2 - 1)$.
If $(10p_1 - 3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2 - 1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1 - 3)(10p_2 - 1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \aleph$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies

that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{13}(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_1 \neq 7n-6$.
If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_2 \neq 7n-2$.
Therefore $g(f_{13}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n-6, p_2 \neq 7n-2$.
Since $f_{13}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 3p_2 + 10)$, then f_{13} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(p_1 + 3p_2 + 10)$ must be even.
Hence $g(f_{13}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 3 for all $p_1 \neq 3n, 7n-6, 2n+1$ and $p_2 \neq 3n-2, 7n-2, 2n+1$.

□

Theorem 23. *If $p \notin \{3n-2\} \cup \{7n-2\} \cup \{f_{12}\} \cup \{f_{13}\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 103$ represents a prime number with unit digit 3 greater than or equal to 163, where $f_{12}(p_1, p_2)$ and $f_{13}(p_1, p_2)$ are defined from the above Theorem 21 and Theorem 22 respectively.*

Proof. Clearly followed from Theorem 21 and 22. □

Theorem 24. *From the above Theorem 18 and Theorem 23 the functions $g(p) = 20p + 93$ and $g(p) = 20p + 103$ respectively represents all prime numbers with unit digit 3 greater than or equal to 113.*

Proof. Clearly followed from Theorem 18 and 23. □

4. A number with unit digit 7

4.1. A number with unit digit 7 happens at a number with unit digit 1

Theorem 25. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 7 for all natural numbers p with unit digit 1, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 1, then $p = 10n - 9$, $n \in \mathbb{N}$. Therefore

$f(p) = 105 + 2p = 105 + 2(10n-9) = 105 + 20n - 18 = 20n + 87 \Rightarrow f(p) = 20n + 87, n \in \mathbb{N}$

represents a natural number with unit digit 7.

□

Theorem 26. *The sequence function $f(p) = 20p + 87$ represents a natural number with unit digit 7 which are different from $3n$ and $7n$ for all $p \neq 3n, 7n - 4$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 9) = 20p + 87 \neq 3n, 7n$ for all $(10p - 9) \neq 3n, 7n \Rightarrow p \neq \frac{3n+9}{10}, \frac{7n+9}{10}$.
If $p \neq \frac{3n+9}{10}$, then p is to be natural number $n = 7, 17, 27, 37, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+9}{10} = \frac{3(10n-3)+9}{10} = 3n \Rightarrow p \neq 3n$. Therefore $f(p) = 20p + 87 \neq 3n$ for all $p \neq 3n$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+9}{10}$, then p is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+9}{10} = \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p \neq 7n - 4$. Therefore $f(p) = 20p + 87 \neq 7n$ for all $p \neq 7n - 4$, $n \in \mathbb{N}$.

□

Theorem 27. *If $g(p) = 20p + 87$ for $p \neq 3n, 7n-4$, then the sequence function $g(f_{15}(p_1, p_2))$ represents a composite natural number with unit digit 7 with factors $10p_1 - 9$ and $10p_2 - 3$ and $\frac{g(f_{15})-105}{2}$ = a natural number with unit digit 1, where $f_{15}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 9p_2 + 6)$ for all $p_1 \neq 3n, 7n - 4, 2n - 1$, $p_2 \neq 3n, 7n - 6, 2n + 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 87 \Rightarrow g(f_{15}) = 20f_{15} + 87 = 20(5p_1p_2 - \frac{1}{2}(3p_1 + 9p_2 + 6)) + 87 = (10p_1 - 9)(10p_2 - 3)$.
Since $(10p_1-9)$ and $(10p_2-3)$ represents a natural number with unit digit 1 and 7 respectively, then
$g(f_{15}(p_1, p_2)) = (10p_1 - 9)(10p_2 - 3)$ represents a composite natural number with unit digit 7 with factors $(10p_1 - 9)$ and $(10p_2 - 3)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 3) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$, then $(10p_1 - 9)(10p_2 - 3) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+3}{10}$, then p_2 is to be natural number $n = 9, 19, 29, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+3}{10} \Rightarrow p_2 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_2 \neq 3n$.
Therefore $g(f_{15}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p_1 \neq 7n - 4$.
If $p_2 \neq \frac{7n+3}{10}$, then p_2 is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that

$p_2 \neq \frac{7n+3}{10} \Rightarrow p_2 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_2 \neq 7n-6.$
Therefore $g(f_{15}(p_1,p_2)) \neq 7n$ for all $p_1 \neq 7n-4, p_2 \neq 7n-6$.
Since $f_{15}(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(3p_1+9p_2+6)$, then f_{15} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(3p_1+9p_2+6)$ must be even.
Hence $g(f_{15}(p_1,p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 7 for all $p_1 \neq 3n, 7n-4, 2n-1$ and $p_2 \neq 3n, 7n-6, 2n+1$.

□

Theorem 28. *If $g(p) = 20p+87$ for $p \neq 3n, 7n-4$, then the sequence function $g(f_{16}(p_1,p_2))$ represents a composite natural number with unit digit 7 with factors $10p_1-7$ and $10p_2-1$ and $\frac{g(f_{16})-105}{2}$ = a natural number with unit digit 1, where $f_{16}(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(p_1+7p_2+8)$ for all $p_1 \neq 3n-2, 7n, 2n+1$, $p_2 \neq 3n-2, 7n-2, 2n+1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p+87 \Rightarrow g(f_{16}) = 20f_{16}+87 = 20(5p_1p_2 - \frac{1}{2}(p_1+7p_2+8)) + 87 = (10p_1-7)(10p_2-1)$.
Since $(10p_1-7)$ and $(10p_2-1)$ represents a natural number with unit digit 3 and 9 respectively, then
$g(f_{16}(p_1,p_2)) = (10p_1-7)(10p_2-1)$ represents a composite natural number with unit digit 7 with factors $(10p_1-7)$ and $(10p_2-1)$.
If $(10p_1-7) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$ and $(10p_2-1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1-7)(10p_2-1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+7}{10}$, then p_1 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+7}{10} \Rightarrow p_1 \neq \frac{3(10n-9)+7}{10} = 3n-2 \Rightarrow p_1 \neq 3n-2.$
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2.$
Therefore $g(f_{16}(p_1,p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+7}{10}$, then p_1 is to be natural number $n = 9, 19, 29, 39, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+7}{10} \Rightarrow p_1 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_1 \neq 7n.$
If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_2 \neq 7n-2.$
Therefore $g(f_{16}(p_1,p_2)) \neq 7n$ for all $p_1 \neq 7n, p_2 \neq 7n-2$.
Since $f_{16}(p_1,p_2) = 5p_1p_2 - \frac{1}{2}(p_1+7p_2+8)$, then f_{16} to be natural number p_1 and p_2 should be even/odd natural number because the expression (p_1+7p_2+8) must be even.
Hence $g(f_{16}(p_1,p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 7 for all $p_1 \neq 3n-2, 7n, 2n+1$ and $p_2 \neq 3n-2, 7n-2, 2n+1$.

□

Theorem 29. *If $p \notin \{3n\} \cup \{7n-4\} \cup \{f_{15}\} \cup \{f_{16}\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 87$ represents a prime number with unit digit 7 greater than or equal to 107, where $f_{15}(p_1, p_2)$ and $f_{16}(p_1, p_2)$ are defined from the above Theorem 27 and Theorem 28 respectively.*

Proof. Clearly followed from Theorem 27 and 28. □

4.2. A number with unit digit 7 happens at a number with unit digit 6

Theorem 30. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 7 for all natural numbers p with unit digit 6, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 6, then $p = 10n - 4$, $n \in \mathbb{N}$. Therefore

$f(p) = 105 + 2p = 105 + 2(10n - 4) = 105 + 20n - 8 = 20n + 97 \Rightarrow f(p) = 20n + 97, n \in \mathbb{N}$

represents a natural number with unit digit 7.

□

Theorem 31. *The sequence function $f(p) = 20p + 97$ represents a natural number with unit digit 7 which are different from $3n$ and $7n$ for all $p \neq 3n - 2, 7n - 1$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 4) = 20p + 97 \neq 3n, 7n$ for all $(10p - 4) \neq 3n, 7n \Rightarrow p \neq \frac{3n+4}{10}, \frac{7n+4}{10}$.
If $p \neq \frac{3n+4}{10}$, then p is to be natural number $n = 2, 12, 22, 32, ... = 10n - 8$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+4}{10} = \frac{3(10n-8)+4}{10} = 3n - 2 \Rightarrow p \neq 3n - 2$. Therefore $f(p) = 20p + 97 \neq 3n$ for all $p \neq 3n - 2$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+4}{10}$, then p is to be natural number $n = 8, 18, 28, 38, ... = 10n - 2$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+4}{10} = \frac{7(10n-2)+4}{10} = 7n - 1 \Rightarrow p \neq 7n - 1$. Therefore $f(p) = 20p + 97 \neq 7n$ for all $p \neq 7n - 1$, $n \in \mathbb{N}$.

□

Theorem 32. *If $g(p) = 20p + 97$ for $p \neq 3n - 2, 7n - 1$, then the sequence function $g(f_{18}(p_1, p_2))$ represents a composite natural number with unit digit 7 with factors $10p_1 - 9$ and $10p_2 - 3$ and $\frac{g(f_{18})-105}{2}$ = a natural number with unit digit 6, where $f_{18}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 9p_2 + 7)$ for all $p_1 \neq 3n, 7n - 4, 2n - 1$, $p_2 \neq 3n, 7n - 6, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 97 \Rightarrow g(f_{18}) = 20f_{18} + 97 = 20(5p_1p_2 - \frac{1}{2}(3p_1 + 9p_2 + 7)) + 97 = (10p_1 - 9)(10p_2 - 3)$.
Since $(10p_1 - 9)$ and $(10p_2 - 3)$ represents a natural number with unit digit 1 and 7 respectively, then

$g(f_{18}(p_1, p_2)) = (10p_1 - 9)(10p_2 - 3)$ represents a composite natural number with unit digit 7 with factors $(10p_1 - 9)$ and $(10p_2 - 3)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 3) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$, then $(10p_1 - 9)(10p_2 - 3) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+3}{10}$, then p_2 is to be natural number $n = 9, 19, 29, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+3}{10} \Rightarrow p_2 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_2 \neq 3n$.
Therefore $g(f_{18}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n - 7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n - 4 \Rightarrow p_1 \neq 7n - 4$.
If $p_2 \neq \frac{7n+3}{10}$, then p_2 is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+3}{10} \Rightarrow p_2 \neq \frac{7(10n-9)+3}{10} = 7n - 6 \Rightarrow p_2 \neq 7n - 6$.
Therefore $g(f_{18}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n - 4, p_2 \neq 7n - 6$.
Since $f_{18}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 9p_2 + 7)$, then f_{18} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(3p_1 + 9p_2 + 7)$ must be even.
Hence $g(f_{18}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 7 for all $p_1 \neq 3n, 7n - 4, 2n - 1$ and $p_2 \neq 3n, 7n - 6, 2n$.

□

Theorem 33. *If $g(p) = 20p + 97$ for $p \neq 3n - 2, 7n - 1$, then the sequence function $g(f_{19}(p_1, p_2))$ represents a composite natural number with unit digit 7 with factors $10p_1 - 7$ and $10p_2 - 1$ and $\frac{g(f_{19})-105}{2}$ = a natural number with unit digit 6, where $f_{19}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 7p_2 + 9)$ for all $p_1 \neq 3n - 2, 7n, 2n + 1$, $p_2 \neq 3n - 2, 7n - 2, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 97 \Rightarrow g(f_{19}) = 20f_{19} + 97 = 20(5p_1p_2 - \frac{1}{2}(p_1 + 7p_2 + 9)) + 97 = (10p_1 - 7)(10p_2 - 1)$.
Since $(10p_1 - 7)$ and $(10p_2 - 1)$ represents a natural number with unit digit 3 and 9 respectively, then
$g(f_{19}(p_1, p_2)) = (10p_1 - 7)(10p_2 - 1)$ represents a composite natural number with unit digit 7 with factors $(10p_1 - 7)$ and $(10p_2 - 1)$.
If $(10p_1 - 7) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$ and $(10p_2 - 1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1 - 7)(10p_2 - 1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+7}{10}$, then p_1 is to be natural number $n = 1, 11, 21, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that

$p_1 \neq \frac{3n+7}{10} \Rightarrow p_1 \neq \frac{3(10n-9)+7}{10} = 3n-2 \Rightarrow p_1 \neq 3n-2$.
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{19}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+7}{10}$, then p_1 is to be natural number $n = 9, 19, 29, 39, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+7}{10} \Rightarrow p_1 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_1 \neq 7n$.
If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_2 \neq 7n-2$.
Therefore $g(f_{19}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n, p_2 \neq 7n-2$.
Since $f_{19}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 7p_2 + 9)$, then f_{19} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(p_1 + 7p_2 + 9)$ must be even.
Hence $g(f_{19}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 7 for all $p_1 \neq 3n-2, 7n, 2n+1$ and $p_2 \neq 3n-2, 7n-2, 2n$. □

Theorem 34. *If $p \notin \{3n-2\} \cup \{7n-1\} \cup \{f_{18}\} \cup \{f_{19}\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 97$ represents a prime number with unit digit 7 greater than or equal to 137, where $f_{18}(p_1, p_2)$ and $f_{19}(p_1, p_2)$ are defined from the above Theorem 32 and Theorem 33 respectively.*

Proof. Clearly followed from Theorem 32 and 33. □

Theorem 35. *From the above Theorem 29 and Theorem 34 the functions $g(p) = 20p + 87$ and $g(p) = 20p + 97$ respectively represents all prime numbers with unit digit 7 greater than or equal to 107.*

Proof. Clearly followed from Theorem 29 and 34. □

5. A number with unit digit 9

5.1. A number with unit digit 9 happens at a number with unit digit 2

Theorem 36. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 9 for all natural numbers p with unit digit 2, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 2, then $p = 10n-8$, $n \in \mathbb{N}$. Therefore

$$f(p) = 105 + 2p = 105 + 2(10n-8) = 105 + 20n - 16 = 20n + 89 \Rightarrow f(p) = 20n + 89, n \in \mathbb{N}$$

represents a natural number with unit digit 9.

□

Theorem 37. *The sequence function $f(p) = 20p + 89$ represents a natural number with unit digit 9 which are different from $3n$ and $7n$ for all $p \neq 3n-1, 7n-2$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p-8) = 20p + 89 \neq 3n, 7n$ for all $(10p-8) \neq 3n, 7n \Rightarrow p \neq \frac{3n+8}{10}, \frac{7n+8}{10}$.
If $p \neq \frac{3n+8}{10}$, then p is to be natural number $n = 4, 14, 24, 34, ... = 10n-6$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+8}{10} = \frac{3(10n-6)+8}{10} = 3n-1 \Rightarrow p \neq 3n-1$. Therefore $f(p) = 20p+89 \neq 3n$ for all $p \neq 3n-1$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+8}{10}$, then p is to be natural number $n = 6, 16, 26, 36, ... = 10n-4$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+8}{10} = \frac{7(10n-4)+8}{10} = 7n-2 \Rightarrow p \neq 7n-2$. Therefore $f(p) = 20p+89 \neq 7n$ for all $p \neq 7n-2$, $n \in \mathbb{N}$.

□

Theorem 38. *If $g(p) = 20p + 89$ for $p \neq 3n-1, 7n-2$, then the sequence function $g(f_{21}(p_1, p_2))$ represents a composite natural number with unit digit 9 with factors $10p_1 - 9$ and $10p_2 - 1$ and $\frac{g(f_{21})-105}{2}$ = a natural number with unit digit 2, where $f_{21}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 8)$ for all $p_1 \neq 3n, 7n-4, 2n-1$, $p_2 \neq 3n-2, 7n-2, 2n+1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 89 \Rightarrow g(f_{21}) = 20f_{21} + 89 = 20(5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 8)) + 89 = (10p_1 - 9)(10p_2 - 1)$.
Since $(10p_1-9)$ and $(10p_2-1)$ represents a natural number with unit digit 1 and 9 respectively, then
$g(f_{21}(p_1, p_2)) = (10p_1 - 9)(10p_2 - 1)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 9)$ and $(10p_2 - 1)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1 - 9)(10p_2 - 1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{21}(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n-4 \Rightarrow p_1 \neq 7n-4$.

If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n - 2 \Rightarrow p_2 \neq 7n - 2$.
Therefore $g(f_{21}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n - 4, p_2 \neq 7n - 2$.
Since $f_{21}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 8)$, then f_{21} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(p_1 + 9p_2 + 8)$ must be even.
Hence $g(f_{21}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1 \neq 3n, 7n - 4, 2n - 1$ and $p_2 \neq 3n - 2, 7n - 2, 2n + 1$.

□

Theorem 39. *If $g(p) = 20p + 89$ for $p \neq 3n - 1, 7n - 2$, then the sequence function $g(f_{22}(p_1, p_2))$ represents a composite natural number with unit digit 9 with factors $10p_1 - 7$ and $10p_2 - 7$ and $\frac{g(f_{22})-105}{2}$ = a natural number with unit digit 2, where $f_{22}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 4)$ for all $p_1, p_2 \neq 3n - 2, 7n, 2n + 1$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 89 \Rightarrow g(f_{22}) = 20f_{22} + 89 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 4)) + 89 = (10p_1 - 7)(10p_2 - 7)$.
Since $(10p_1 - 7)$ and $(10p_2 - 7)$ represents a natural number with unit digit 3, then $g(f_{22}(p_1, p_2)) = (10p_1 - 7)(10p_2 - 7)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 7)$ and $(10p_2 - 7)$.
If $(10p_1 - 7) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$ and $(10p_2 - 7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1 - 7)(10p_2 - 7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+7}{10}$, then p_1 is to be natural number $n = 1, 11, 21, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+7}{10} \Rightarrow p_1 \neq \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n - 2 \Rightarrow p_2 \neq 3n - 2$.
Therefore $g(f_{22}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n - 2$.
If $p_1 \neq \frac{7n+7}{10}$, then p_1 is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+7}{10} \Rightarrow p_1 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_1 \neq 7n$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_{22}(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n$.
Since $f_{22}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 4)$, then f_{22} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(7p_1 + 7p_2 + 4)$ must be even.
Hence $g(f_{22}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1, p_2 \neq 3n - 2, 7n, 2n + 1$.

□

Theorem 40. *If* $g(p) = 20p + 89$ *for* $p \neq 3n-1, 7n-2$, *then the sequence function* $g(f_{23}(p_1, p_2))$ *represents a composite natural number with unit digit* 9 *with factors* $10p_1 - 3$ *and* $10p_2 - 3$ *and* $\frac{g(f_{23})-105}{2}$ = *a natural number with unit digit* 2, *where* $f_{23}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 8)$ *for all* $p_1, p_2 \neq 3n, 7n-6, 2n+1$, *where* $n \in \mathbb{N}$.

Proof. $g(p) = 20p + 89 \Rightarrow g(f_{23}) = 20f_{23} + 89 = 20(5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 8)) + 89 = (10p_1 - 3)(10p_2 - 3)$.
Since $(10p_1 - 3)$ and $(10p_2 - 3)$ represents a natural number with unit digit 7, then $g(f_{23}(p_1, p_2)) = (10p_1 - 3)(10p_2 - 3)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 3)$ and $(10p_2 - 3)$.
If $(10p_1 - 3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2 - 3) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$, then $(10p_1 - 3)(10p_2 - 3) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+3}{10}$, then p_2 is to be natural number $n = 9, 19, 29, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+3}{10} \Rightarrow p_2 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_2 \neq 3n$.
Therefore $g(f_{23}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n - 6 \Rightarrow p_1 \neq 7n - 6$.
If $p_2 \neq \frac{7n+3}{10}$, then p_2 is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+3}{10} \Rightarrow p_2 \neq \frac{7(10n-9)+3}{10} = 7n - 6 \Rightarrow p_2 \neq 7n - 6$.
Therefore $g(f_{23}(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n - 6$.
Since $f_{23}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 8)$, then f_{23} to be natural number p_1 and p_2 should be even/odd natural number because the expression $(3p_1 + 3p_2 + 8)$ must be even.
Hence $g(f_{23}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1, p_2 \neq 3n, 7n - 6, 2n + 1$.

□

Theorem 41. *If* $p \notin \{3n-1\} \cup \{7n-2\} \cup \{f_{21}\} \cup \{f_{22}\} \cup \{f_{23}\}$, *where* $n \in \mathbb{N}$ *then* $g(p) = 20p + 89$ *represents a prime number with unit digit* 9 *greater than or equal to* 109, *where* $f_{21}(p_1, p_2)$, $f_{22}(p_1, p_2)$ *and* $f_{23}(p_1, p_2)$ *are defined from the above Theorem* 38, *Theorem* 39 *and Theorem* 40 *respectively.*

Proof. Clearly followed from Theorem 38, 39 and 40. □

5.2. A number with unit digit 9 happens at a number with unit digit 7

Theorem 42. *The sequence function $f(p) = 105 + 2p$ represents a natural number with unit digit 9 for all natural numbers p with unit digit 7, where $n \in \mathbb{N}$.*

Proof. Since p is every natural number with unit digit 7, then $p = 10n - 3$, $n \in \mathbb{N}$. Therefore

$f(p) = 105 + 2p = 105 + 2(10n - 3) = 105 + 20n - 6 = 20n + 99 \Rightarrow f(p) = 20n + 99, n \in \mathbb{N}$

represents a natural number with unit digit 9.

□

Theorem 43. *The sequence function $f(p) = 20p + 99$ represents a natural number with unit digit 9 which are different from $3n$ and $7n$ for all $p \neq 3n, 7n - 6$, where $n \in \mathbb{N}$.*

Proof. Since we know that $f(p) = 105 + 2p \neq 3n, 7n$ for all $p \neq 3n, 7n$, then
$f(p) = 105 + 2(10p - 3) = 20p + 99 \neq 3n, 7n$ for all $(10p - 3) \neq 3n, 7n \Rightarrow p \neq \frac{3n+3}{10}, \frac{7n+3}{10}$.
If $p \neq \frac{3n+3}{10}$, then p is to be natural number $n = 9, 19, 29, 39, ... = 10n - 1$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{3n+3}{10} = \frac{3(10n-1)+3}{10} = 3n \Rightarrow p \neq 3n$. Therefore $f(p) = 20p + 99 \neq 3n$ for all $p \neq 3n$, $n \in \mathbb{N}$.
If $p \neq \frac{7n+3}{10}$, then p is to be natural number $n = 1, 11, 21, 31, ... = 10n - 9$, $n \in \mathbb{N}$. This implies that
$p \neq \frac{7n+3}{10} = \frac{7(10n-9)+3}{10} = 7n - 6 \Rightarrow p \neq 7n - 6$. Therefore $f(p) = 20p + 99 \neq 7n$ for all $p \neq 7n - 6$, $n \in \mathbb{N}$.

□

Theorem 44. *If $g(p) = 20p + 99$ for $p \neq 3n, 7n - 6$, then the sequence function $g(f_{25}(p_1, p_2))$ represents a composite natural number with unit digit 9 with factors $10p_1 - 9$ and $10p_2 - 1$ and $\frac{g(f_{25})-105}{2}$ = a natural number with unit digit 7, where $f_{25}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 9)$ for all $p_1 \neq 3n, 7n - 4, 2n - 1$, $p_2 \neq 3n - 2, 7n - 2, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 99 \Rightarrow g(f_{25}) = 20f_{25} + 99 = 20(5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 9)) + 99 = (10p_1 - 9)(10p_2 - 1)$.
Since $(10p_1 - 9)$ and $(10p_2 - 1)$ represents a natural number with unit digit 1 and 9 respectively, then
$g(f_{25}(p_1, p_2)) = (10p_1 - 9)(10p_2 - 1)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 9)$ and $(10p_2 - 1)$.
If $(10p_1 - 9) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+9}{10}, \frac{7n+9}{10}$ and $(10p_2 - 1) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+1}{10}, \frac{7n+1}{10}$, then $(10p_1 - 9)(10p_2 - 1) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+9}{10}$, then p_1 is to be natural number $n = 7, 17, 27, ... = 10n - 3$, $n \in \mathbb{N}$. This implies that

$p_1 \neq \frac{3n+9}{10} \Rightarrow p_1 \neq \frac{3(10n-3)+9}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+1}{10}$, then p_2 is to be natural number $n = 3, 13, 23, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+1}{10} \Rightarrow p_2 \neq \frac{3(10n-7)+1}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{25}(p_1, p_2)) \neq 3n$ for all $p_1 \neq 3n, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+9}{10}$, then p_1 is to be natural number $n = 3, 13, 23, 33, ... = 10n-7$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+9}{10} \Rightarrow p_1 \neq \frac{7(10n-7)+9}{10} = 7n-4 \Rightarrow p_1 \neq 7n-4$.
If $p_2 \neq \frac{7n+1}{10}$, then p_2 is to be natural number $n = 7, 17, 27, 37, ... = 10n-3$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+1}{10} \Rightarrow p_2 \neq \frac{7(10n-3)+1}{10} = 7n-2 \Rightarrow p_2 \neq 7n-2$.
Therefore $g(f_{25}(p_1, p_2)) \neq 7n$ for all $p_1 \neq 7n-4, p_2 \neq 7n-2$.
Since $f_{25}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(p_1 + 9p_2 + 9)$, then f_{25} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(p_1 + 9p_2 + 9)$ must be even.
Hence $g(f_{25}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1 \neq 3n, 7n-4, 2n-1$ and $p_2 \neq 3n-2, 7n-2, 2n$.

□

Theorem 45. *If $g(p) = 20p + 99$ for $p \neq 3n-1, 7n-2$, then the sequence function $g(f_{26}(p_1, p_2))$ represents a composite natural number with unit digit 9 with factors $10p_1 - 7$ and $10p_2 - 7$ and $\frac{g(f_{26})-105}{2}$ = a natural number with unit digit 7, where $f_{26}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 5)$ for all $p_1 \neq 3n-2, 7n, 2n+1$, $p_2 \neq 3n-2, 7n, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 99 \Rightarrow g(f_{26}) = 20f_{26} + 99 = 20(5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 5)) + 99 = (10p_1 - 7)(10p_2 - 7)$.
Since $(10p_1 - 7)$ and $(10p_2 - 7)$ represents a natural number with unit digit 3, then $g(f_{26}(p_1, p_2)) = (10p_1 - 7)(10p_2 - 7)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 7)$ and $(10p_2 - 7)$.
If $(10p_1 - 7) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$ and $(10p_2 - 7) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+7}{10}, \frac{7n+7}{10}$, then $(10p_1 - 7)(10p_2 - 7) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+7}{10}$, then p_1 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+7}{10} \Rightarrow p_1 \neq \frac{3(10n-9)+7}{10} = 3n-2 \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+7}{10}$, then p_2 is to be natural number $n = 1, 11, 21, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+7}{10} \Rightarrow p_2 \neq \frac{3(10n-9)+7}{10} = 3n-2 \Rightarrow p_2 \neq 3n-2$.
Therefore $g(f_{26}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n-2$.
If $p_1 \neq \frac{7n+7}{10}$, then p_1 is to be natural number $n = 9, 19, 29, 39, ... = 10n-1$, $n \in \mathbb{N}$. This implies that

$p_1 \neq \frac{7n+7}{10} \Rightarrow p_1 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_1 \neq 7n$.
If $p_2 \neq \frac{7n+7}{10}$, then p_2 is to be natural number $n = 9, 19, 29, 39, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+7}{10} \Rightarrow p_2 \neq \frac{7(10n-1)+7}{10} = 7n \Rightarrow p_2 \neq 7n$.
Therefore $g(f_{26}(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n$.
Since $f_{26}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(7p_1 + 7p_2 + 5)$, then f_{26} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression $(7p_1 + 7p_2 + 5)$ must be even.
Hence $g(f_{26}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1 \neq 3n-2, 7n, 2n+1$ and $p_2 \neq 3n-2, 7n, 2n$.

□

Theorem 46. *If $g(p) = 20p + 99$ for $p \neq 3n-1, 7n-2$, then the sequence function $g(f_{27}(p_1, p_2))$ represents a composite natural number with unit digit 9 with factors $10p_1 - 3$ and $10p_2 - 3$ and $\frac{g(f_{27})-105}{2}$ = a natural number with unit digit 7, where $f_{27}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 9)$ for all $p_1 \neq 3n, 7n-6, 2n+1$, $p_2 \neq 3n, 7n-6, 2n$, where $n \in \mathbb{N}$.*

Proof. $g(p) = 20p + 99 \Rightarrow g(f_{27}) = 20f_{27} + 99 = 20(5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 9)) + 99 = (10p_1 - 3)(10p_2 - 3)$.
Since $(10p_1 - 3)$ and $(10p_2 - 3)$ represents a natural number with unit digit 7, then $g(f_{27}(p_1, p_2)) = (10p_1 - 3)(10p_2 - 3)$ represents a composite natural number with unit digit 9 with factors $(10p_1 - 3)$ and $(10p_2 - 3)$.
If $(10p_1 - 3) \neq 3n, 7n \Rightarrow p_1 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$ and $(10p_2 - 3) \neq 3n, 7n \Rightarrow p_2 \neq \frac{3n+3}{10}, \frac{7n+3}{10}$, then $(10p_1 - 3)(10p_2 - 3) \neq 3n, 7n$.
If $p_1 \neq \frac{3n+3}{10}$, then p_1 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{3n+3}{10} \Rightarrow p_1 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_1 \neq 3n$.
If $p_2 \neq \frac{3n+3}{10}$, then p_2 is to be natural number $n = 9, 19, 29, ... = 10n-1$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{3n+3}{10} \Rightarrow p_2 \neq \frac{3(10n-1)+3}{10} = 3n \Rightarrow p_2 \neq 3n$.
Therefore $g(f_{27}(p_1, p_2)) \neq 3n$ for all $p_1, p_2 \neq 3n$.
If $p_1 \neq \frac{7n+3}{10}$, then p_1 is to be natural number $n = 1, 11, 21, 31, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_1 \neq \frac{7n+3}{10} \Rightarrow p_1 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_1 \neq 7n-6$.
If $p_2 \neq \frac{7n+3}{10}$, then p_2 is to be natural number $n = 1, 11, 21, 31, ... = 10n-9$, $n \in \mathbb{N}$. This implies that
$p_2 \neq \frac{7n+3}{10} \Rightarrow p_2 \neq \frac{7(10n-9)+3}{10} = 7n-6 \Rightarrow p_2 \neq 7n-6$.
Therefore $g(f_{27}(p_1, p_2)) \neq 7n$ for all $p_1, p_2 \neq 7n-6$.
Since $f_{27}(p_1, p_2) = 5p_1p_2 - \frac{1}{2}(3p_1 + 3p_2 + 9)$, then f_{27} to be natural number p_1 should be even natural number and p_2 should be odd natural number and vice versa because the expression

$(3p_1 + 3p_2 + 9)$ must be even.
Hence $g(f_{27}(p_1, p_2))$ represents a composite natural number which are different from $3n$ and $7n$ with unit digit 9 for all $p_1 \neq 3n, 7n - 6, 2n + 1$ and $p_2 \neq 3n, 7n - 6, 2n$.

□

Theorem 47. *If $p \notin \{3n\} \cup \{7n - 6\} \cup \{f_{25}\} \cup \{f_{26}\} \cup \{f_{27}\}$, where $n \in \mathbb{N}$ then $g(p) = 20p + 99$ represents a prime number with unit digit 9 greater than or equal to 139, where $f_{25}(p_1, p_2)$, $f_{26}(p_1, p_2)$ and $f_{27}(p_1, p_2)$ are defined from the above Theorem 44, Theorem 45 and Theorem 46 respectively.*

Proof. Clearly followed from Theorem 44, 45 and 46. □

Theorem 48. *From the above Theorem 41 and Theorem 47 the functions $g(p) = 20p + 89$ and $g(p) = 20p + 99$ respectively represents all prime numbers with unit digit 9 greater than or equal to 109.*

Proof. clearly followed from Theorem 41 and Theorem 47. □

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