ASIAN JOURNAL OF MATHEMATICS AND APPLICATIONS
Volume 2019, Article ID ama0504, 12 pages
ISSN 2307-7743
http://scienceasia.asia

LOT SIZE ON ATI / MAAOQ PROTECTED ACCEPTANCE SAMPLING

RAMKUMAR T BALAN*, ERICK NGANZI

Dept. of Statistics, CNMS, UDOM, Tanzania

*Correspondence: rktmidhuna@gmail.com

Abstract: Formulation of lot size in a production process should be scientific, based on available infrastructure framed in the manufacturing unit and the designed quality of the product. Fixing Average Total Inspection admissible to a production unit, the plausible sampling plans and the feasible size of the lot can be determined. In some cases combinations of sampling plans were available and the required sampling plan is decided with respect to the OC curve on design. Tables, OC curves and examples were presented. The lotsize effect on sampling scheme is discussed.

Introduction

The lot size is a prefixed quantity in most of the production process and it is approximated to 100s or 1000s or its multiples. But there is no technical background to make sense for this number except the convenience. The sampling plans were developed either for assumed infinite lot size or for fixed convenient lot size (Hald 1950). In the former case the sample size for finite lot is developed by conversion formula suggested Hamakar (1959), Soundararajan (1969) and Ramkumar (1993). Also lot size evaluation is described by R Radhakrisnan and Vasathamani (2012) based on 6 sigma limits. Balamurali and Kalyanasundaram(2000) had described variable lot size attribute sampling plan indexed by AQL and AOQL .Also Chung –Ho- Chem and Cha-Yoo-Chow(2004) developed variable lot size sampling plan for continuous production based on minimum ATI. But very few studies were still in library explaining the design of lot size. This study is indexing the parameters (axis) of inflection point of OC curve (MAPD, MAAOQ) and ATI by first developing a sampling plan minimising ATI.

Wayne A Taylor (2015) explained the effect of lot size express the practice of fixing lot size in MIL105E plans with respect to AQL. He exhibits that when lot size is large sample size is proportionally large and high protection is guaranteed for products. Better protection for larger lot is essential as the cost of rejection of good lots and cost of acceptance of bad lots are higher in the large lots.

This paper tries to develop a scientific method to find the lot size considering the quality of the product and infrastructure availabilities of the manufacturing unit. The cost of production consists of fixed costs and varying cost. Inspection requirements of manufacturing is contained in fixed cost while day to day inspection come under

Key words and phrases: Lot Size; ATI / MAAOQ; sampling scheme.

varying cost. Any company on design itself fixes the cost oriented with inspection facilities and manpower for inspection. Generally they will have a clear notion of approximate number of units to be inspected and amenities to be arranged. So these assumed units must be inspected lead to the acceptance of the lot or rejection and hence screening process with rectification is a pre-planned parameter for all manufacturing units. They could not squeeze or enlarge the scientific inspection procedure as and when the outcome of acceptance or rejection is happened.

Average total inspection (ATI) is an efficient parameter indicating the cost of inspection of manufactured units on rejection as well as acceptance. Generally when the sample size of inspection is finalised whole units of the sample is inspected irrespective of outcome. It is good to maintain the process average of defectives in the lot using sampling technique.

When the lot of size N is rejected on a sample of n units, the remaining (N-n) units are screened and the defectives were replaced by non-defectives and after that the lot is sent for shipment. So a manufacturing unit in general had to inspect average of this sizes of the units and it is derived as $ATI = n+(N-n)\ (1- P_a(p))$, where $P_a(p)$ represents probability of acceptance of a lot of quality p of proportion defectives. So the cost of product will be depending on ATI and every company will prefix this level of ATI at the time of production itself. So here ATI is taken as one parameter to develop the lot size.

The process is controlled by suitable control chart and the process average is a very good quality to design the sampling plan as well as the lot size. According to customers they are highly relying on control chart and believe that a process under control is automatically acknowledging only a proportion of defective of that level. Here process average is considered as second parameter to distinguish sampling plan and lot size. The process average is evidently considered as an incoming quality of the product and for saturated quality in terms of outgoing product one more quality is entertained.

Average outgoing quality is the quality level of the outgoing product on average which mentions the level of the product in general expected at the customers window. Only part of product in the lot is inspected when the lot is accepted by sampling technique so that the defined quality cannot be assured to each and every item outgoing. Then the product quality is estimated with respect to average outgoing quality. It is the product of proportion of defectives in the lot with probability of acceptance of the lot with such proportion defective and magnified by proportion of remaining items in the lot (except samples).

$AOQ= p.P_a(p)\ .(N-n)/N$. But in product lot size N is large and sample size is small so that $(N-n)/N$ can be approximated to 1 so that $AOQ \approx p.P_a(p)$. The shape of the AOQ curve is convex starting from a low value and reaching maximum and then decreases even though the shape need not be symmetrical. Thus mathematically and graphically proved that AOQ reaches a maximum, which is the worst proportion of defectives that can be found in the outgoing lot of product and it is called AOQL. But there is no significance yet found for the proportion of defectives in the lot at AOQL and its determination is also difficult and approximate. Also there are arguements by statistician that the maximum is not realised in any of the practical sampling plans. (Anascombe , Pandey , Ramkumar). The value of AOQ is low when the proportion of defectives is too small in the lot and it increases and reaches maximum as p increase but decreases again as Pa(p) is small at higher values of p. Thus there is effect of p and Pa(p) together creates the position of AOQ and it is not reasonable to consider the proportion of defectives beyond MAPD to locate maximum as unwanted proportions are resulting to a maximum AOQ. It is defined as the inflection point of OC curve from which point the probability of acceptance of the lot is suddenly and steeply declined. So the engineers, vendors and consumers were highly accepting this incoming quality. So the outgoing quality at this point is also important and accepted as the maximum allowable outgoing proportion of defective. If the outgoing quality decreases beyond this quality it is not logically valid even though mathematically true. Because the erosion of quality from MAPD is forcing to reduce the outgoing quality and it is not acceptable in the consumer's point of view. MAPD had very high significance approved by laymen to statistician, engineers and vendors due its position (c/n) and its importance in OC curve. Till the quality MAPD, OC curves shapes high acceptance of the product and beyond MAPD , the acceptance is steeply declined. Due to this notion it is not advisable to incorporate proportion of defective beyond MAPD to get a maximum level. Ramkumar (1993) had established a quality which is the outgoing quality at MAPD called MAAOQ=p*Pa(p*). This quality is mainly based on OC curve and defined on n and c so that more accurate and logical. This paper so fixes third parameter MAAOQ at a predefined required level as to find suitable sampling plan and lot size.

It is clear that the manufacturing companies are maintaining an utmost ATI inspection system in terms of sample size inspection and if needed the remaining units of the lot by screening , replacing bad by good items.

Methodology

Let the process is in control with process average $\bar{p}$ and outgoing quality of the lot under rectified inspection is fixed at a predefined level P_M (MAAOQ) .MAPD (p*) is the proportion of incoming product quality at this level. Then the average total inspection ATI is determined at $\bar{p}$.

Let n is the sample size and N the lot size and $P_a(p)$ is the probability of acceptance of the lot with proportion defective p. Then by definition ATI at the process average is

$ATI = I = n+(N-n)\ (1-P_a(\bar{p}))$

I P_M =n P_M +(N-n) (1-$P_a(\bar{p})$) (N P_M -n P_M) (1-$P_a(\bar{p})$)

$$Z = \emptyset(c)+(N P_M -\emptyset(c)) (1-P_a(\bar{p})) \quad \text{where } \emptyset(c) = \frac{\sum_{r=0}^{c} e^{-c} c^{r+1}}{r!}$$

$$I = \frac{\emptyset(c)}{P_M}+(N- \frac{\emptyset(c)}{P_M}) (1-P_a(\bar{p}))$$

$$(I - \frac{\emptyset(c)}{P_M}) = (N- \frac{\emptyset(c)}{P_M}) (1-P_a(\bar{p}))$$

$$(N- \frac{\emptyset(c)}{P_M}) =(\frac{(I - \frac{\emptyset(c)}{P_M})}{(1-Pa(\bar{p}))}) = \frac{(I - \frac{\emptyset(c)}{P_M})}{(1-\frac{\sum_{r=0}^{c} e^{-n\bar{p}}(n\bar{p})^r}{r!})}$$

$$(N- \frac{\emptyset(c)}{P_M}) =(\frac{(I - \frac{\emptyset(c)}{P_M})}{(1-Pa(\bar{p}))}) = \frac{(I - \frac{\emptyset(c)}{P_M})}{(1-\frac{\sum_{r=0}^{c} e^{-n\left(\left(\frac{\bar{p}}{P_M}\right)(P_M)\right)}\left\{n\left(\frac{\bar{p}}{P_M}\right)(P_M)\right\}^r}{r!}}$$

$$(N- \frac{\emptyset(c)}{P_M}) =(\frac{(I - \frac{\emptyset(c)}{P_M})}{(1-Pa(\bar{p}))}) = \frac{(I - \frac{\emptyset(c)}{P_M})}{(1-\frac{\sum_{r=0}^{c} e^{-\emptyset(c)(K)}\{\emptyset(c)(K)\}^r}{r!}} \quad \text{where } K=\frac{\bar{p}}{P_M}$$

$$N= \frac{\emptyset(c)}{P_M} + \frac{(I - \frac{\emptyset(c)}{P_M})}{(1-\frac{\sum_{r=0}^{c} e^{-\emptyset(c)(K)}\{\emptyset(c)(K)\}^r}{r!})} \quad ....(1)$$

The value of N corresponding to P_M and $\bar{p}$ is found out by substituting $K=\frac{\bar{p}}{P_M}$ for each value of c for prefixed ATI =I, Also the sample size

$n=\frac{\emptyset(c)}{P_M}$ is determined......................(2)

Thus for fixed ATI , MAAOQ and process average , combinations of (n,c) and corresponding lot size can be determined. Using the OC curve or minimum sample size or convenient lot size the sampling plan can be finalised.

For example ATI= 200, MAAOQ=.04 and process average =.025

The possible sampling plans were (19,1) , (34,2) , (49,3) , (63,4), (77,5), (91,6), (105,7), (119,8) ,

The corresponding lot size were (1700), (2176), (2951), (3991) , (5291) , (6779) , (8229) , (9089)

From the observed sampling plans, one is selected and corresponding lot size can also be fixed. Let the selected plan is (19,1) then the outgoing items were supplied in a lot of size (1700).

Example:2 Find the lot size for ATI =100 and K=.4 with P_M=.05

From Table: 2(b) and Table:1, possible c=1,2,3 , and with sample sizes18,34,49 respectively and the lot sizes are 2405,4144,7855.

Example:3 Find sampling plan for $\bar{p}$ =.025 , and K=0.5 and ATI=50

Then P_M=.05 . From Table:2(c) The acceptance numbers were c=1,2,3 with lot size 679,758,686 respectively. Thus optimum (minimum lot size is 679 with a sampling plan (15,1) (27,2) and maximum sampling plan (39,3).(See the OC curve)

This system can be used for modifications for the company as a switching rule. If the product reaches consistent acceptance at first sampling plan, they can improve the sampling plan to second level and so on . For example the company can choose a tightened OC curve P2(p) with (39,3) in the early production having a ATI =50 and and lot size 686 units. When the quality is consistent they can choose OC curve P1(p) with sampling plan(27,2) and lot size 758 . again stability realises choose OC curve P (p) and corresponding plan is (15,1) with lot size 679 . Thus modification in plan is possible and corresponding lot sizes can be designed

Now fixing the sampling plan one can change lot size as ATI is changed . For example take ATI =50 and sampling plan is (34,2) , the lot size is 758. Let the ATI =100 , with the same plan (k=.5,P_M=.05) . then lot size is 2352 and for ATI=200 lot size=5541.Thus when the production is consistent , keeping same quality and sampling plan the lot size can be multiplied by facilitating new average total inspection.

OC curve

The OC curve will indicate the probability of acceptance of the product with an incoming quality p* at inflection of the curve. Thus using OC curve the company can fix the required sampling plan. From the figure and table it was found that MAPD is slightly increased on increasing the acceptance number. So it was good to start with first sampling plan and further improvements can be made on reaching quality production.

Construction and use of Tables

Table :1 shows the sample sizes of sampling plan for fixed c and P_M ,it is calculated from Equation 2 where $\emptyset(c) = \frac{\sum_{r=0}^{c} e^{-c} c^{r+1}}{r!}$ by substituting values for c=1,2,....10. It is found that MAPD=c/n improves as the sample size and acceptance number increases. Table : 2 is giving the lot sizes for specified ATI and MAAOQ and K=$\frac{\bar{p}}{P_M}$. One can find N by substituting the above values in equation (1) for c=1,2,.10. the value of N is fixed at a maximum of 10000 units in this study.

Table: 1 The sample size for MAAOQ=0.01, 02, 03, 04,05,0.1 at c=1-10.

c\\P_M	0.01	0.02	0.03	0.04	0.05	0.1
1	74	37	25	18	15	7
2	135	68	45	34	27	14
3	194	97	65	49	39	19
4	251	126	84	63	50	25
5	307	154	103	77	62	31
6	363	182	121	91	73	36
7	419	210	140	105	84	42
8	474	237	158	119	95	47
9	528	264	176	132	106	53
10	583	292	194	146	117	58

Table:2 a) Lot size for k=.3 ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=0.3				PM			
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		665	1234	1519	1690	2033
2				638	1993	2506	4432
3					531	3731	
1	100	1329	3039	3609	3894	4065	4407
2			3986	6696	8052	8865	
3			1061				
1	200	6078	7788	8358	8643	8814	9157
2		7971					
3		2122					

Table:2 b) Lot size for k=.4, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=0.4					PM		
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		407	739	905	1004	1203
2				321	947	1321	2072
3					225	1398	3743
4							6642
1	100	815	1808	2140	2306	2405	2604
2			1893	3143	3768	4144	4894
3			452	4359	6313	7855	9830
4				4386	9948		
1	200	3617	4611	4943	5108	5208	5407
2		3785	7523	8787	9413	9788	
3		903					

Table:2 c) Lot size for K=.5, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=0.5					PM		
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		285	503	613	679	809
2				201	549	758	1176
3					133	686	1792
4							2683
5							3836
6							5044
7							5559
8							3331
1	100	571	1226	1444	1554	1619	1750
2			1098	1795	2143	2352	2770
3			267	2109	3031	3584	4690
4				1812	4033	5365	8030
5					4636		
6					3419		
1	200	2452	3107	3326	3435	3501	3632
2		2197	4287	4983	5332	5541	5958
3		532	6062	7606	8827	9380	
4			8065				
5			9272				
6			6839				
1	500	8096	8751	8970	9079	9145	9276

Table:2 d) Lot size for k=.6, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=0.6					PM		
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		217	373	451	498	591
2				144	362	493	755
3					96	401	
1	100	435	902	1058	1136	1183	1276
2			725	1162	1380	1510	1772
3			191	1210	1718	2024	2635
4				933	2015	2664	3962
5					2047	3350	5957
6					1355	3876	8917
7						3766	
8						2021	
1	200	1805	2273	2428	2506	2553	2646
2		1451	2759	3195	3414	3544	3806
3		383	3437	4455	464	5269	5880
4			4030	6193	7275	7924	9221
5			4094	8438			
6			2710				
1	500	5915	6382	6538	6616	6662	6756
2		7552	8861	9297	9515	9646	9908

Table:2 e) Lot size for k=.8, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=0.8					PM		
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		148	240	285	313	368
2				95	202	265	392
3					68	192	441
4							487
5							515
6							503
7							418
8							211
1	100	297	571	663	709	736	791
2			404	616	722	785	913
3			137	551	758	882	1131
4				384	753	974	1417
5					657	1030	1776
6					401	1006	2216
7						83	2747
8						423	3376
9							4106
10							4929
1	200	1143	1418	1510	1555	1583	1638
2		808	1444	1656	1763	1626	1953
3		274	1516	1930	2137	2261	2510
4			1506	2244	2613	2834	3277
5			1314	2558	3179	3552	428
6			802	2819	3828	4433	5643
7				2948	4539	5494	7404
8				2816	5276	6753	9706
9				2221	5966	8212	
10				854	6481	9857	
1	500	3682	3957	4048	4094	4122	4176
2		3929	4565	4778	4884	4947	5075
3		4412	5653	6067	6274	6398	6647
4		4873	7086	7824	8193	8414	8857
5		5151	8881				
6		5031					
7		4187					
8		2117					
1	1000	7914	8188	8280	8326	8353	8408
2		9131	9767	9979			

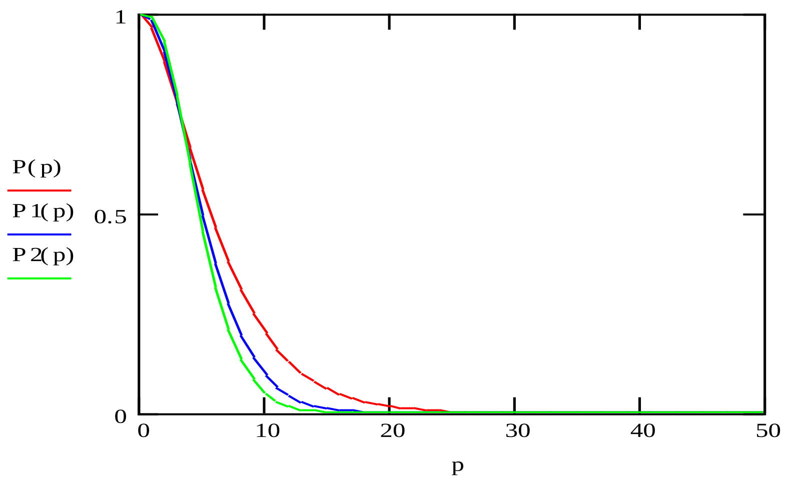
Table:2 f) Lot size for k=1.0, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05,0.1

k=1.0					PM		
c	ATI	0.01	0.02	0.03	0.04	0.05	0.1
1	50		115	175	206	224	260
2				76	137	174	248
3					59	123	250
4							249
5							239
6							214
7							169
8							96
1	100	230	412	472	503	521	557
2			275	398	459	496	570
3			119	330	436	500	627
4				229	397	498	700
5					326	478	781
6					209	429	868
7						339	959
8						193	1050
9							1138
10							1216
1	200	824	1006	1067	1097	1115	1152
2		551	919	1042	1103	1140	1214
3		238	873	1086	1192	1255	1382
4			795	1131	1299	1400	1602
5			653	1159	1411	1563	1866
6			418	1151	1578	1737	2177
7				1092	1608	1918	2538
8				958	1672	2100	2956
9				723	1693	2275	3439
10				351	1652	2433	3994
1	500	2607	2789	2849	2880	2898	2934
2		2483	2851	2973	3035	3071	3145
3		2502	3138	3350	3455	3519	3646
4		2492	3501	3837	4005	4106	4308
5		2391	3908	4413	4666	4818	5121
6		2146	4344	5076	5442	5662	6102
7		1697	4796	5829	6346	6656	7276
8		968	5250	6678	7391	7819	8676
9			5688	7627	8597	9179	
10			6082	8683	9984		
1	1000	5577	5759	5820	5850	5868	5905
2		5701	6069	6192	6253	6290	6364
3		6275	6911	7123	7229	7292	7419
4		7002	8010	8346	8514	8615	8817
5		7816	9332	9838			
6		8687					
7		9592					

Table:2 g) Lot size for k=1.2, ATI=50,100,200 for P_M=.01,.02,.03,.04,.05.

k=1.2					PM	
c	ATI	0.01	0.02	0.03	0.04	0.05
1	50		96	139	161	174
2				67	106	129
3					55	92
1	100	192	322	365	387	400
2			212	291	330	354
3			111	235	297	334
4				169	260	314
5				80	212	288
6					150	251
7						202
8						137
1	200	644	774	817	839	852
2		425	661	740	779	803
3		222	594	718	780	818
4			520	702	792	847
5			425	677	803	879
6			300	638	807	909
7			139	581	784	935
8				501	750	954
9				393	697	965
10				251		
1	500	2001	2130	2173	2195	2208
2		1776	2008	2086	2126	2149
3		1673	2046	2170	2232	2269
4		1574	2118	2299	2390	2444
5		1446	2198	2450	2575	2651
6		1259	2273	2611	2780	2886
7		1012	2387	2780	3001	3134
8		666	2414	2954	3237	3407
9		267	2413	3129	3487	3702
10				3305	3750	4018

Figure: 1 OC curves for Sampling plans (15,1) (27,2) (39,3)



Reference

[1] Anascombe, F.J. (1958) Rectifying inspection of continuous output, Journal of Royal Statistical Society,53, p702

[2] Pandey,R.J. (1988) Three decision ASR plan providing average quality protectionin terms of IAOQ, Sankhya , B series, 42, pp 235-244.

[3] Ramkumar(1993).Un published MPhil thesis on single sampling plan indexed by MAPD

[4] Radhakrishnan R, Vasanthamani.(2012) P. Determination of lot size in the construction of Six Sigma based Double Sampling Plans ,Global Journal of Technology and Optimisation.

[5] Soundararajan.V. (1967). Construction of Single Sampling Plan for fixed sample size. Proceedings of IVth all India conference of Quality Control, ISI Calcutta 1-37.

[6] Hald.A. (1981) Statistical theory of sampling inspection by attributes, Academic Press, New York

[7] Hamakar, H. C. (1950) The theory of lot by lot inspection by attributes.Review of International Statistical Institute Vol.18,pp.179-196.

[8] Wayne A Taylor (2015) The effect of lot size, Applied Statistics for Engineers and Quality in FDA regulated Industries.