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GRAPH MODEL FOR BIO DIVERSITY WITH AN ESTIMATE FOR FITNESS SLOPE OF INDIAN TIGERS

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Abstract

Suppose that G=(V, E) is a (p, q) graph with p=|v(G)| and q=|E(G)|.Let $e_1,e_2,\ldots,e_q$ be the q edges of G. We say that G has α^* representation if f: V(G)→ R^q in such a way that in the r-th component of f(v) = (e_1,…, e_q), we have the entry 1 if e_r has v as one of its end and 0 otherwise. In this paper we showed that every graph G has α^*-representation and discuss such representation for friendship networks and determined the fitness slope of Indian tigers using Leslie matix.

1.Introduction

We restrict our attention to only those graphs that are simple and undirected and for standard notations and terminology we refer to [1].

Graphical representations are quite common to understand the relationship existing among various elements of any network. It may be a communication network or a sociological network or a biological network. Moreover, the availability of plenty of mathematical tools enable us to analyze any system depending on its representation through graph models. It is quite tempting to restrict attention to two or three dimensions while dealing with a network of enormous size. However, it would be very interesting if we could give graph models that fits a given real life situation in any higher r-dimension where r>3.

Graphs are involved to mathematically model a real life problem to obtain valuable insights concerning the base of the problem. Further the computation of the structured parameters like diameter, eccentricity etc of a graph leads to vital solutions of the problem itself. For instance, the graph connectivity parameter denoted k (pronounced kappa) is qualitative in nature and it is known that a graph is connected if and only if there exists a path between any two vertices. In [2] the authors have coined the term dot-product representation of graph G, if f: V(G) → R^n with a constant $t_0>0$ is such that for each v ∈ V(G), f allots a n-dimensional vector $(x_1, x_2,\ldots, x_n)^T$ and for $v_1, v_2 \in v(G)$, $(v_1,v_2) \in E(G)$ if and only if $f(v_1)\cdot f(v_2)$ is at least t_0.

Key words and phrases: Graph, Friendship Network, Fitness Slope, Indian Tiger.

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2. α*- representation

Suppose that G=(V, E) is a (p, q) graph with p=|v(G)| and q=|E(G)|.Let $e_1,e_2,\ldots,e_q$ be the q edges of G. We say that G has α^* representation if $f:V(G)\rightarrow R^q$ in such a way that in the r-th component of $f(v)=(e_1,\ldots,e_q)$, we have the entry 1 if e_r has v as one of its end and 0 otherwise. So $(v_1, v_2) \in E(G)$ means the r-th component of both v_1 and v_2 are 1 and hence $f(v_1)\cdot f(v_2)$ is at least 1.The absence of a link between v_1 and v_2 in G means that $f(v_1)$ and $f(v_2)$ do not agree with each other in any of its components, that is $f(v_1)\cdot f(v_2)=0\cdot 0 = 0 <1$. Hence, any graph admits a α^*-representation.

Note. To explain α^* -representation, consider the graph G_1 shown in Fig.1. Set f_1: $V(G_1)\rightarrow R$ with $f(u_1)=(1/3)$.$f(u_2)= f(u_3)= f(u_4)=(3)$,$f(u_5)=(1/3)$. Clearly f_1 is a 1-dimensional α^*-representation. Set f_2: $v(G)\rightarrow R^7$ with $f_2(u_1) = (1,1,0,0,0,1,0)^T$, $f_2(u_2)=(0,1,1,0,1,0,0)^T$, $f_2(u_3)=(0,0,1,1,0,1,0)^T$, $f_2(u_4)=(0,0,0,0,1,1,1)^T$, $f_2(u_5)=(1,0,1,0,0,0,1)^T$. Then f_2 is α^*-representation of dimension 7.

Let us set $\rho(G)$ as the least dimension of the vectors needed to give α^*-representation for a graph G. Observe that $\rho(G)$ is at most q(G). In [3] it is said that the determinant of $\rho(G)$ for a general G is NR hard.

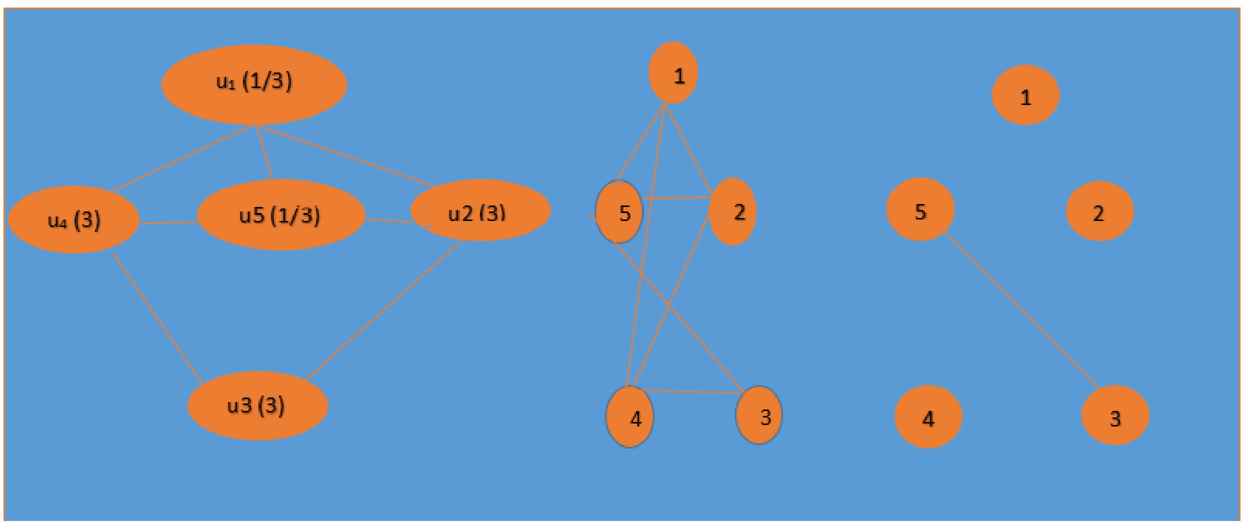


Fig.1 The Graph G_1 **Fig.2 The Graph G_2** **Fig.3 The Friend Network G_3**

3. α*-representation for Friendship Networks

A careful analysis of networks and its representations play a crucial part in social science as one way of data presentation concerning tricky or fuzzy individual relationships. Networks in general are visualized in graph form. Several techniques such as algorithms for positional analysis finding huge cliques, probabilistic dyad models, regression and correlation, matrix methods, multidimensional scaling, etc. are increasingly used in the analysis of social networks.

Statistical approach in analysis of networks has a prominent place in the form of hypothesis, prediction testing and description about relations between various components of the network. For instance, inclusiveness in relationship network is defined as the percentage of non-isolated points. Density is the number of edges as a percentage of total edges when all vertices are connected. Total edges, based on probability, is n!/(n-2)!2!. That is the number of edges connecting 8 vertices considered two at a time is is 8!/(8-2)!2! = 40320/(720 × 2) = 28. So if the friendship network has 21 edges out of 28 edges then the density works out to be 0.75.

If five persons are watched on their wish list for Drink A, Drink B, Drink C, Drink D, Drink E. One can assign values 1, 1/2, -1/2, -1, 0 depending on their wish list observations.

Type of Drink	Person 1	Person 2	Person 3	Person 4	Person 5
A	1	-1/2	-1	-1	0
B	1/2	1	0	-1	1
C	-1	0	1	-1/2	1/2
D	0	1	-1	1/2	-1
E	-1	1/2	1	-1/2	0

If person 3 and person 4 have common wish list for Drink A and Drink E but they differ in their taste for other Drinks. So let us set a value -1 as threshold. Then their wish list leads to the graph G_2 in Fig.2. which is a α^*-representation for a non-friendship network. As the value of a threshold should be positive, let us set it to be t_0=1/10.Then we get a **α^***-representation for a Friendship network G_3 as in Fig.3. Note that it is again a poor Friendship network. For more on this one can refer to [4,5,6].

4. α^*-representation for Fitness slope

A study on how the principle of natural selection has played its part in facilitating the normal flow and alteration of genes of all species in this universe with time and guided in the process of molding, selecting for and getting annihilated or becoming extinct are both interesting and worth doing to unravel the mysteries. It is well known that genetic variability is inevitable to all the species and in the absence of which evolution will not take place. David Mummy in [17] has clearly demonstrated through numerical simulation how a species population change on an adaptive landscape. To comprehend how it happens it is necessary to determine at a given point on the surface the differential derivative also called the selection gradient and apply the same to the Gaussian function to elaborate on the adaptive landscape. He also showed how the population scaled the hump, defying the force of the gradient vector. The mechanism he developed through statistical treatment provide much needed light regarding the pattern of evolutionary change.

As α^*-representations involve vectors, deem the fitness slopes of different kinds of birds, with dimension 4, permitting us to operate on models belonging to higher dimension. The STNS (Secondary Theorem of natural selection) developed in [6] points to trait change "y" of evolutionary nature for each generation as a function of a) the strength of natural selection of trait and b) the genetic heritability of y the trait .

Fitness of an individual can be quantified by its contribution to growth rate s (Malthusian population). That is "s" is the rate with which a given generation communicates to its next level in the population. STNS states that $dy/dt = \sigma^2 ds/dy$. Here σ^2 stands for the genetic heritability and ds/dy is the selection strength. In [7] STNS was considered for the multivariate instance. This considered several traits like fertility, mass of the body and brain. Assuming that $(y_1,\ldots,y_n)$ denote these traits one can suppose that $\bar{y}=(\bar{y}_1,\ldots\ldots, \bar{y}_n)^T$ and $\nabla s = (\partial s/\partial y_1,\ldots, \partial s/\partial y_n)^T$. It is a fact that $\Delta\bar{y} = M\nabla s$, where M is the additive genetic covariance matrix of traits.The slope ∇s is called as the fitness slope while analysing the traits life history.

Fitness, in a biological system that are evolutionary in nature of a given species is the chance or probability that it will live and reproduce.Few traits when considered from its life history reveal that fertility and survival at its juvenile stage affects fitness.consider a simple 2×2 Leslie matrix which comes in handy to mathematically describe or model fitness.Assume that p_1 is the chance of survival at juvenile stage, p_2 is the chance of turning as adult, k is the reproduction rate at the start in conformity with fertility time step, p_3 is the chance of survivsl as adult.Then the Leslie matrix L is, $L=\begin{bmatrix} p_1(1-p_2) & k \\ p_1p_2 & p_3 \end{bmatrix}$.

In order to prevent extinction of large mammals the Indian Government is earmarking and developing protected areas. [8, 9]. Huge carnivores play a crucial role in recovery efforts to maintain biodiversity [10]. Due to low population and elusiveness it is very hard to monitoring their number [11]. A case study on Tiger also called Panthera tigris reveals the truth that its presence range has dwindled almost by 93%, and its numbers have reduced to a few thousands in less than 200 years. In [12] the authors found that the analysis of facts regarding population recovery may be slower than expected due to the recent initiative of making their number a double in the next ten years. The variation in population, pattern of change in age-sex proportion ratios, rate of survival, rate of growth and adult recruitment pattern (with greater than or equal to months), subadult (in the age group of 18 to 36 months), and cubs (with less than or equal to 15 months) were probed and analyzed with the help of camera trap and radio telemetry techniques in [13]. A main reason attributed to the dwindling of tiger population is poaching and loss of habitat and adequate availability of prey. The authors in [14] used cumulative incidence functions to quantify cause-specific mortality rates of tigers. The authors in [15] adopted a blend of techniques such as radio telemetry, camera traps, direct observations, and photo documentation on tigers between 2006 and 2014 in Ranthambhore to assess demographic parameters. They deduced tiger density through systematic camera trap sampling using spatially explicit capture-recapture framework and compared model inferred density with near actual density. In [16] reproductive characteristics of tigers (Panthera tigris) were investigated to comprehend population viability. They probed the reproductive parameters of female Bengal tigers in a dry, tropical, deciduous habitat in Ranthambhore Tiger Reserve in Western India, between April 2005 to March 2010.

The following statistics were taken from [8-10, 12]

Data 1. Amur Tigers[14]
Study period: 1992 to 2012
Total Tigers studied: 57 tigers
Male: 27
Females: 30
Days monitored: An average of 747 days (range 26–4718 days).
Average annual survival rate: 0.75 (all tigers combined)
Causes of mortality:
Primary: Poaching (2005-2015) (account for a loss of 17–19%)
Others: 1. Distemper (Canine distemper virus (CDV))- 5%.

Data 2. Natural/unknown causes.
2. Tigers[12]
Study period: 2006 to 2012
Area covered: 624–1026 km2
Camera traps: 137–200
Total Tigers studied: 200
Male: Not available
Females: Not available
Days monitored: 21,359 trap days
Annual density: 1.25 to 2.01 tigers/100 km2
Abundance: 35 to 58 tigers
Survival rate: 79.6% to 95.5%

Data 3. Indian Tigers[13]
Study period: May 2006 to April 2013
Location: Pench Tiger Reserve (Pench), Madhya Pradesh of Central India
Total Tigers studied: 66
Male: 22
Female: 44
Male: Female ratio: Biased
Survival Rate:
Mean annual growth rate ($\pm$SE): 1.15 ($\pm$ 0.11)
Mean recruitment ($\pm$SE) : 9.2 $\pm$ 2.2.
All tigers: $n = 66$
Cubs: $n = 39$
Survival Rates: 86% (Transient stage: $\geq$18–36 month)
14% (Adult stage: $>$36 month)
Average age of reproduction (Females: $n = 3$): 35 $\pm$ 2.08 months.
Mean growth rate: 15%

Data 4. Indian tigers[16]
Study period: April 2005 to March 2010.
Location: Ranthambhore Tiger Reserve (RTR), India
Total Tigers studied:
Female: 13
Average Age of reproduction: 3.3 years
Cubs born during the study period: 6.2±0.82 per year
:66% of the births occurred between October and December.
Sex ratio (32 cubs): M:F 29:1 F.
Survival rate of cubs: 3 years old.

Data 5. Indian Tigers[12-16]
Location: Ranthambhore, India
Area studied:
Male: female ratio: 0.76 (SE 0.07),
Cub: adult tigress ratio at 0.48 (SE 0.12).
Average litter size: 2.24 (SE 0.14).
Male recruitment from cub to sub-adult stage: 77.8%, SE 2.2
Females recruitment from cub to sub-adult stage: 62.5%, SE 2.4
Male recruitment rate as breeding adults from the sub-adult stage:72.6%, SE 2.0 Females recruitment rate as breeding adults from the sub-adult stage: 86.7%, SE 1.3
Annual survival rates:
Cubs: 85.4%, $CI_{95\%}$80.3–90.5%)
Juvenile: 97.0%, $CI_{95\%}$ 95.4–98.7%)
Sub-adult: 96.4%, $CI_{95\%}$ 94.0–98.9%)
Adult male: 84.8%, $CI_{95\%}$ 80.6–89.2%
Female: 88.7%, $CI_{95\%}$ 85.3–92.2%
Mortality
Human-caused mortality: 47% in cubs and 38% in adults.
Mean dispersal age: 33.9 months (SE 0.8)
Males: 61 Km, SE 2
Females: 12 Km, SE 1.3

Consider an Indian tiger (biological name: Panthera tigris). From the above data analysis we find that it has a chance of survival at juvenile stage is p_1=0.97, a chance of survival at adult stage, $p_3 = 0.96$, a chance of turning into adult, $p_2 = 0.85$ and the reproduction rate at the start with fetility time step k =2.9.Then L-matrix for Pesser domestics is : L=$\begin{bmatrix} 0.97(1-0.85) & 2.9 \\ (0.97)(0.85) & 0.96 \end{bmatrix}$ According to Leslie, ln(λ_1) = growth rate of Panthera tigris, where λ_1 is the largest characteristic root of L. So the fitness slope of this tiger described by the Leslie matix is $\nabla s = (\partial \ln(\lambda_1)/\partial p_1, \partial \ln(\lambda_1)/\partial p_2, \partial \ln(\lambda_1)/\partial k, \partial \ln(\lambda_1)/\partial p_3)^T$. To find the slope we calculate the characteristic roots of L by using $|L - \lambda I| = 0$. That is $|L - \lambda I|$=$\left[\begin{pmatrix} p_1(1-p_2 & k \\ p_1 p_2 & p_3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = =\left[\begin{pmatrix} p_1(1-p_2)-\lambda & k \\ p_1 p_3 & p_3-\lambda \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = 0$. So

$(p_1(1-p_2)-\lambda)(p_3-\lambda)-kp_1p_3 = 0$. That is $(p_1-p_1p_2-\lambda)(p_3-\lambda)-kp_1p_3 = 0$. $p_1p_3-p_1p_2p_3-\lambda p_3-\lambda p_1+\lambda p_1p_2+\lambda^2-kp_1p_3 = 0$. $\lambda^2-(p_1+p_3-p_1p_2)T+p_1p_3(1-p_2-k) = 0$.

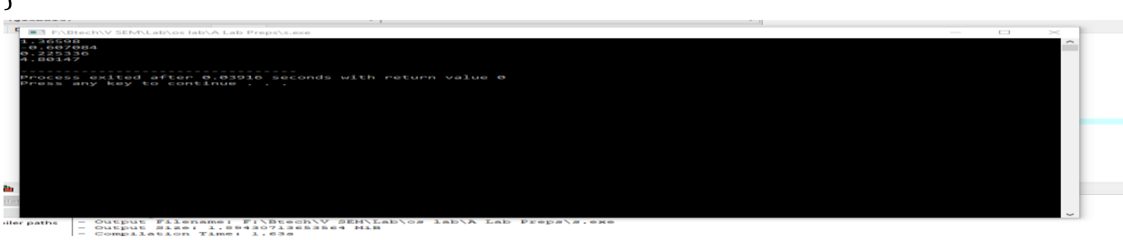
Hence, $\lambda_1 = ((p_1-p_1p_2+p_3)+\sqrt{p_1^2 - 2p_1^2p_2 + p_1^2 P_2^2 - 2p_1p_3 + 2p_1p_2p_3 + p_3^2 + 4kp_1p_3})/2$. Let $P = p_1-p_1p_2+p_3$; $Q=\sqrt{p_1^2 - 2p_1^2p_2 + p_1^2 p_2^2 - 2p_1p_3 + 2p_1p_2p_3 + p_3^2 + 4kp_1p_3}$; $\lambda_1=(P+Q)/2$; $\ln(\lambda_1)=\ln(P+Q/2)$. Then

$\nabla s = ((1/2\lambda_1)[(1-p_2)+(1/2Q)(2p_1(1-2p_2+p_2^2)-2p_3(1-p_2-4k)]; (1/2\lambda_1) [-p1+(1/2\sqrt{Q})(-2p_1(p_1-p_1p_2-p_3))]; (1/\lambda_1)[(1/\sqrt{Q})p_1p_3]; (1/2\lambda_1) [1+(1/2\sqrt{Q})(p_1(p_2-2+4k)+2p_3)])^T$

A C++ program for computing the ∇s is given below with the output. For our chosen data $\nabla s = (1.36598; -0.607084; 0.225336; 4.80147)^T$

A C++ Program for computing ∇s

```
#include<iostream>
#include<math.h>
using namespace std;
int main()
{
        float p1,p2,p3,p,q,k,l,med;
        p1=0.97;
        p2=0.85;
        p3=0.96;
        k=2.9;
        p=p1-p1*p2+p3;
        med=p1*p1-2*p1*p1*p2+p1*p1*p2*p2-2*p1*p3+2*p1*p2*p3+p3*p3+4*k*p1*p3;
        q=sqrt(med);
        l=(p+q)/2;
        float op1=((1-p2)+(2*p1*(1-2*p2+p2*p2)-2*p3*(1-p2-4*k))/(2*sqrt(q)))/(2*l);
        float op2=(-p1+(-2*p1*(p1-p1*p2-p3))/(2*sqrt(q)))/2*l;
        float op3=(p1*p3/sqrt(q))/l;
        float op4=(1+(p1*(p2-2+4*k)+2*p3)/(2*sqrt(q)))/2*l;
        cout<<op1<<"\n"<<op2<<"\n"<<op3<<"\n"<<op4<<"\n";
        return 0;
}
```



Conclusion

We introduced the concept of α^*-representation of a simple graph and discussed about such a representation for social networks. Then we determined ∇s the fitness slope of Indian tigers using the data collected from the literature. We found from the various components of ∇s the orientation of the slope vector (the angle between the primary axis and the vector) is crucial feature to deduce which variable cause more effect on the fitness of the tiger. It reveals at what rate the fitness of the tiger is increasing due to the increase in the number of variables.These fitness slopes can be used to introduce α^*-representation of dimension 4. It is our strong conviction that analysis of the graphical models of networks is an affluent source for various challenging tasks in the future.

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