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GOAL PROGRAMMING PROBLEMS FOR OPTIMAL SELECTION OF RAW FOOD ITEMS IN DIET WITH ENERGY AS OBJECTIVE FUNCTION

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Abstract: In the modern era, optimization modelling plays a vital role in handling food and diet related nutritional management. The application of linear programming problems assisted the purpose of dealing the diet plans with optimized food items. The present study aims to develop Linear Programming Problem (LPP) and Goal Programming Problems (GPP) of various energy levels for selecting optimal combinations of raw food items with desired nutrient requirements. This paper demonstrates that LPP and GPP approaches produce an optimal and feasible solution in order to solve the diet problem for ordinary human survival and derive new insights in considering affordable raw food items to meet sufficient nutritional intake for the benefit of the population in India.

1. Introduction:

Nutrition promotes overall health and well-being. It is the process of nourishing the body through food and diet which constitute a basic part of our existence. Nutrients are found in food which are critical for human growth. Acquiring essential nutrients in adequate amounts and proportion is the indication of good nutrition. Rendering to the changing trends in nutrition and dietetics, the progress has not been uniformed across people with different standards of living. The expenditure on nutritious foods such as fruits, vegetables and animal source foods is typically high, which is not affordable for people with low economic status.

India has shown limited progress towards achieving the diet-related targets, which reflects that it has the greatest number of malnourished children in its population. Poor dietary habits have been identified as one of the main causes of malnutrition. The relatively high cost of nutritious foods can affect nutritional outcomes, while the price elasticities differ across countries and foods. According to the Ministry of Health and Family Welfare, approximately 14% of India's population is undernourished. Nutrient intake was markedly lower in the rural sector of some of the poor performing states to uphold the nutritional

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status. The summary of the 68th round of the National Sample Survey (NSS) on Nutritional Intake in India, the average dietary energy intake per person per day was 2233 calories for rural India, whereas the minimum calories requirement of men and women is 2400 and 2160, while the maximum requirement is 2700 and 3000, respectively.

The main goal of this study is to prepare a diet by selecting a set of available and affordable foods that are within reach of people living in rural areas and meet their daily nutritional requirements. This paper confers the Required Nutrient Intake (RNI) with ordinary raw food items for producing various energy levels. In order to develop a healthy and balanced menu by making effective use of available food items, the applications of the optimization techniques in operations research play a vital role in reaching certain objectives and subject to considering the constraints of the decision variables. Linear Programming problem is used for optimum utilization of resources to achieve the best possible nutritional intake. On another hand, goal programming problem (GPP) is viewed as the generalization of Linear Programming and also comes under the category of multi criteria decision making to minimize the deviations of under-achievements and over-achievements in nutritional requirements with the specified energy levels. LPP and GPP have the capability to develop a suitable model to meet the nutritional requirements and energy levels with limited raw food items.

2. Review of Literature:

Many researchers have developed various linear and goal programming problems to optimize food items among worldwide countries in view of providing sufficient nutrients to the population. Some of them presented GPP to achieve nutritional balance in selected diets by considering 150 raw food materials to satisfy daily nutritional requirements of Thais with complex inter-relationships of the constraints (Anderson and Earle, 1983). Also, some authors proposed GP nutrition model to minimize the deviations from nutrients, energy value and food cost in order to meet daily nutrient needs of the reference woman and reference man of households in Bosnia and Herzegovina, and developed an extended by proposing a linear programming diet model to maximize energy density for choosing food items (containing Macro and Micro-nutrients) consumed in Bosnia and Herzegovina as recommended by WHO in order to reduce food cost (Pašić et al., 2012, 2013). Another work in the same country was done by proposing linear and goal programming optimization models for analysing the food basket (consists of nearly 158 general consumption of food items) in Bosnia and Herzegovina in order to meet various nutritional requirements of WHO and World Bank recommendations. The parameters such as price and nutrient requirement are linearly related to food weight, where LP models deal with minimal value and structure of the food basket of the average person and GP models with minimal deviations from nutrient needs if the budget is fixed (Arnaut-Berilo et al., 2017). In parallel, some researchers have developed a nutrition optimization model to satisfy the daily nutrient needs of adolescents through preemptive goal programming. To minimize the sum of percentage of nutrient deviations according to its priorities of twenty most frequently consumed foods from Indonesian Recommended Dietary Allowances and the available budget as goal and system constraints (Fitra Anugrah et al., 2018).

Several works reviewed in depth investigations on branches of multi-criteria decision modelling such as Lexicographic and Weighted Goal Programming methods in various fields. Some authors clearly identified the further scope of GP models by incorporating different types of normalization procedures and introducing utility theories to develop an intelligent algorithm in the future and focused another review on significant developments of theoretical GP models in the area of intelligent modelling and solution analysis. They also discussed techniques like Pareto efficiency, normalization, and non-standard utility function modeling. Further, they examined the connection between GP and multi-objective programming techniques and utility theory and discussed their ranks as well (Tamiz et al., 1995, 1998). Furthermore, recent researchers reviewed recent improvements of new advancements for solving Linear Goal Programming (LGP) for the last several years (Orumie and Ebong, 2014).

Some authors proposed an efficient method for solving lexicographic GP problems and its formulation with different variable sizes to be a better model than the existing models (Orumie and Ebong, 2013). Other researchers compared Weighted Goal Programming (WGP) and LP models for the targeted DASH diet's tolerable intake levels for various calorie levels with 1500 mg of sodium. WGP model is more efficient in minimizing the deviation of tolerable target levels at desired cost than LP model (Iwuji and Agwu, 2017).

Among all the studies concerned the development of linear and goal programming problems to optimize the raw food items to meet the sufficient demands of macro and micro nutrients, which are necessary for the survivability of a common man. The policy makers of governments and nutritional experts should think in the direction of giving suitable energy levels to the populations which is based on their economic conditions. Our study made an attempt to consider the least possible and affordable raw food items in accordance with sufficient nutrient requirements based on producing various energy levels of the Indian population.

3. Collection of data:

The data of raw food items each quantified at 100 gm and the nutrient information of the same is collected from Indian Food Composition Tables (IFCT) by Longvah et al., 2017. Raw food items (in g) of daily servings which are commonly available and often consumed by all types of economic categories of the Indian population are considered as decision variables. There are 20 different types of raw food in this case, including rice, wheat flour, Black gram, Green gram, Red gram, Amarnath Leaves, Spinach, Brinjal, Tomato, Apple, Banana, Sunflower Oil, carrot, potato, onion, Buffalo Milk, Egg, Chicken, Tuna, and Vanjaram. Nutrients such as Protein, Fat, Fibre, Carbohydrate, Vitamin B2, Vitamin B6, Iron, Zinc, Potassium, Sodium, Carotenoids, Calcium, Saturated Fat included in the study. Dietary intake allowances of nutrients for food items are determined according to the ICMR-National Institute of Nutrition (NIN) located in India. Energy is considered under nutrients and as a goal constraint and other nutrients as system constraints for accurate optimization.

4. Formulation of Mathematical model on nutrient optimization:

The initial model is formulated with LPP by considering energy in kilo calories (Kcal) as an objective function and nutrients as constraints. To minimize the deviations (over-achievement and under-achievement) of various nutrients, GP problem is considered for simultaneous achievement of nutrient-oriented goals. Weights of nutrients are added according to the percentage normalization method through WGP. Further goal priorities of nutrient deviations are achieved through Priority Goal Programming. Finally, the model included weights with priorities of nutrient goals by Priority Goal Programming with weights to minimize the deviations under the method of weight normalization.

4.1 LPP for Energy Minimization:

The model is devoted in minimizing the energy of different food items as an objective function with constraints subjected to the required nutrient intake levels (upper and lower) of food items considered under study. The decision variables are the food items with initial values included in the model. To optimize the overall calorie consumption, fixed the upper and lower limits of required nutrients. The mathematical formulation details are given below.

$$Min\, Z = \sum_{j=1}^{n} E_{jF}$$

Subject to Constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_{jF} \leq b_{iNu} \quad \ldots (1)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_{jF} \geq b_{iNl}$$

$$x_{jF}, b_{iNu}, b_{iNl} \geq 0$$

where

E_{jF} = Energy values of j^{th} food item

n= Number of food items (n=20)

m = Number of nutrients considered in the study (m=14)

a_{ij} = Amount of i^{th} nutrient in j^{th} food items

x_{jF} = decision variables that represent the type of food items

b_{iNu}= Recommended upper level of i^{th} nutrient intake

b_{iNl}= Recommended lower level of i^{th} nutrient intake

4.2 GPP for Minimization of Deviations of Nutrients:

The section deals with GPP for minimizing the deviations of various nutrients. In this model, the objective function is considered as minimizing the sum of deviations of each nutrient of raw food items. The energy constraint is equivalent to the arbitrary calories required approximately for a day, which is termed as goal constraint and other nutrient constraints values correspond to the Required Nutrient Intake levels which are termed as system constraints. Thus, the following mathematical formulation is adopted.

$$Min\, Z = \sum_{i=1}^{m} (d_i^- + d_i^+)$$

$$Subject\ to\ Constraints$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_{jF} + d_i^- - d_i^+ = b_{iNr}\ (System\ Constraints) \quad \ldots (2)$$

$$\sum_{j=1}^{n} E_{jF} + d_i^- - d_i^+ = b_{iNr}\ (Goal\ Constraint)$$

$$x_{jF}, b_{iNr}, d_i^-, d_i^+ \geq 0$$

where

n= Number of food items (n=20)

m= Number of nutrients considered in the study (m=14)

a_{ij} = Amount of i^{th} nutrient in j^{th} food items

x_{jF} = decision variables that represent the type of food items

b_{iNr}= Recommended i^{th} nutrient intake

d_i^+= Positive deviation of nutrients

d_i^- = Negative deviation of nutrients

4.3 Priority Goal Programming for Minimization of Deviations of Nutrients:

The main objective function of the priority goal programming is the minimization of deviational variables placed at a higher priority level. The defining constraints with decision variables and unwanted deviations to be minimized in the process. To solve the programming problem, system constraints are equalled to the required nutrient levels under study for other nutrients and goal constraint (energy) target changes with required calories.

Priorities (P_i):

Constraints and objectives are expressed at distinct priorities. At each priority, an optimization problem is solved either to maximize or minimize the stated objective at that priority. In this case, we define 14 goal constraints divided into 5 priorities. Priorities are taken as per their importance and essentiality for human survival. The objectives to be achieved are as follows:

Priority 1 (P1)

Goal 1 (Energy): Minimize the under-achievement and overachievement of energy (Kcal)

Priority 2 (P2)

Goal 2 (Protein): Minimize under-achievement and over-achievement of protein (g)

Goal 3 (Carbohydrate): Minimize under-achievement and over-achievement of carbohydrate (g)

Goal 4 (Fat): Minimize under-achievement and over-achievement of fat (g)

Goal 5 (Saturated fat): Minimize under-achievement and over-achievement of saturated fat (g)

Priority 3 (P3)

Goal 6 (Fiber): Minimize under-achievement and over-achievement of fiber (g)

Goal 7 (Vitamin B2): Minimize under-achievement and over-achievement of vitamin B2 (mg)

Goal 8 (Vitamin B6): Minimize under-achievement and over-achievement of vitamin B6 (mg)

Priority 4 (P4)

Goal 9 (Iron): Minimize under-achievement and over-achievement of iron (mg)

Goal 10 (Zinc): Minimize under-achievement and over-achievement of zinc (mg)

Goal 11 (Calcium): Minimize under-achievement and over-achievement of calcium (mg)

Priority 5(P5)

Goal 12 (Potassium): Minimize under-achievement and over-achievement of potassium (mg)

Goal 13 (Sodium): Minimize under-achievement and over-achievement of sodium (mg),

Goal 14 (Carotenoids): Minimize under-achievement and over-achievement of carotenoids (mg)

$$Min\,Z = \sum_{i=1}^{m} P_i(d_i^- + d_i^+)$$

$$Subject\,to\,Constraints$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{jF} + d_i^- - d_i^+ = b_{iNr}\,(System\,Constraints) \qquad \ldots (3)$$

$$\sum_{j=1}^{n} E_{jF} + d_i^- - d_i^+ = b_{iNr}\,(Goal\,Constraint)$$

$$x_{jF}, b_{iNr}, d_i^-, d_i^+ \geq 0$$

4.4 WGP for Minimization of Deviations of Nutrients:

WGP is the minimisation of deviational variables by placing them in a weighted, normalised single achievement function. Percentage normalisation is the scaling method here, so there is no difference between the constraints and the target values considered in goal programming with priorities and goal programming with weights.

$$Min\,Z = \sum_{i=1}^{m} (d_i^- + d_i^+) / b_{iNr}$$

$$Subject\,to\,Constraints$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} x_{jF} + d_i^- - d_i^+ = b_{iNr}\,(System\,Constraints) \quad \text{... (4)}$$

$$\sum_{j=1}^{n} E_{jF} + d_i^- - d_i^+ = b_{iNr}\,(Goal\,Constraint)$$

$$x_{jF}, b_{iNr}, d_i^-, d_i^+ \geq 0$$

5. Results and Discussion:

5.1 LPP for Energy Optimization (2878 kcal):

Table-1: Optimal quantity of food items by LPP

Food Item (100g)	LPP	Food Item (100g)	LPP
Rice	0	Banana	397.9
Wheat Flour	161.9	Sunflower Oil	201.2
Black Gram	0	Carrot	0
Green Gram	345.9	Potato	0
Red Gram	0	Onion	0
Amarnath Leaves	80.1	Buffalo Milk	34.2
Spinach	327.3	Egg	199.1
Brinjal	0	Chicken	11.5
Tomato	0	Tuna	0
Apple	0	Vanjaram	150.4

Table-1 represents the raw food item quantities for the energy levels computed through LPP. The objective function in LPP has optimized the energy levels to 2878 kcal and produced only 10 raw food items in the inclusion of diet for a day. The output shows the optimal quantities of various food items and their corresponding nutrients are satisfied for required energy levels.

5.2 Goal Programming Problems for various Optimal Energy Levels:

The portion focuses on optimal quantities of raw food items with goal programming problems (GPP, GPP with priorities, and GPP with weights) with various energy levels such as 2525 Kcal, 2700 Kcal, 2878 Kcal, 3000 Kcal and 3159 Kcal as arbitrary values which one can need for sufficient food intake with economic categories as shown in Table-2, 3 and 4.

Table-2: Optimal Food Quantities of Raw Food Items for various Energy levels by GPP

Food Item	2525 Kcal	2700 Kcal	2878 Kcal	3000 Kcal	3159 Kcal
Rice	0	30.3	37.7	48.2	80.7
Wheat Flour	109.0	120.2	121.9	155	150.9
Black Gram	0	0	0	0	13.0
Green Gram	343.5	354.9	307.2	228.3	188.9
Red Gram	50.2	23.9	48.8	73.1	81.9
Amarnath Leaves	92.1	0	0	0	0
Spinach	219.4	310.3	248.5	178.5	248.5
Brinjal	0	0	0	0	0
Tomato	68.4	290.6	193.7	0	0
Apple	0	0	0	28.8	272.5
Banana	0	224	0	136.9	0
Sunflower Oil	336.0	0	0	0	0
Carrot	0	0	211.9	287.0	215.7
Potato	0	74.4	101.4	121.3	106.2
Onion	56.4	0	0	0	0
Buffalo Milk	0	70.9	137.6	172.5	170.3
Egg	179.1	130.4	40.6	0	0
Chicken	39.0	69.2	143.7	175.4	169.1
Tuna	0	0	20.7	77.8	104.6
Vanjaram	143.1	141.5	132.9	109.1	98.3

Table-3: Optimal Food Quantities of Raw Food Items for various Energy levels by GPP with priorities

Food Item	2525 Kcal	2700 Kcal	2878 Kcal	3000 Kcal	3159 Kcal
Rice	0	112.9	187.7	231.7	234.9
Wheat Flour	96.4	38.0	0	0	0
Black Gram	0	145.5	203.4	215.1	215.8
Green Gram	351.1	233.2	167.7	142.0	140.1
Red Gram	38.3	0	0	0	0
Amarnath Leaves	102.0	94.1	88.1	39.5	35.9
Spinach	230.9	250.5	237.2	207.2	205.2
Brinjal	0	21.5	83.7	146.5	151.2
Tomato	0	0	258.1	341.4	361.8

Food Item	2525 Kcal	2700 Kcal	2878 Kcal	3000 Kcal	3159 Kcal
Apple	0	59.0	103.3	6.00	0
Banana	348.6	470.0	413.5	324.0	317.0
Sunflower Oil	0	0	0	0	69.8
Carrot	0	0	0	0	0
Potato	53.1	59.4	74.0	105.0	107.2
Onion	81.6	0	0	0	0
Buffalo Milk	38.2	41.0	42.9	52.3	52.8
Egg	174.0	157.7	151.1	142.1	141.3
Chicken	45.3	057.3	60.5	63.6	64.0
Tuna	0	0	0	0	0
Vanjaram	141.2	140.8	141.0	141.1	141.1

Table-4: Optimal Food Quantities of Raw Food Items for various Energy levels by GPP with weights

Food Item	2525 Kcal	2700 Kcal	2878 Kcal	3000 Kcal	3159 Kcal
Rice	44.7	182.3	161.9	178.8	178.8
Wheat Flour	71.1	0	0	0	0
Black Gram	62.0	276.7	182.0	200.9	200.8
Green Gram	88.5	85.8	198.2	173.0	173.0
Red Gram	66.6	11.3	0	0	0
Amarnath Leaves	131.4	151.3	134.8	98.0	97.9
Spinach	161.1	130.5	249.1	243.5	243.7
Brinjal	0	0	0	71.0	71.2
Tomato	0	0	0	0	0
Apple	0	30.1	194.6	123.6	124.0
Banana	377	604.0	414.1	431.4	0
Sunflower Oil	0	0	18.9	67.8	67.7
Carrot	7.5	0	0	0	0
Potato	56.0	65.2	50.1	67.7	67.7
Onion	0	0	0	0	0
Buffalo Milk	76.7	114.5	42.0	40.8	40.7
Egg	195.9	177.1	149.4	152.9	152.8
Chicken	0	0	60.6	60.1	60.2
Tuna	0	39.6	0	0	0
Vanjaram	141.2	124.0	141.0	140.9	0

The results in tables-2, 3 and 4 explain the combinations of food items with minimal percentage of deviations or with no deviations of under-achievement and over-achievement of each nutritional requirement. The energy level of 2525 Kcal, GPP, GPP with priorities and GPP with weights have obtained the values for 11, 12 and 13 food items, respectively. The results have improved in increasing 12, 14, and 12 raw food items to the next level of energy (2700 Kcal). A slight increase in raw food items in the energy level (2878 Kcal) with 14, 14 and 13 raw food items and a slight variation in energy (3000 Kcal) with 13, 14 and 14 items. Finally, energy (3159 Kcal) is derived from 13, 14 and 12 raw food items with optimized values. From the overall results, it is depicted that the GPP with different energy levels yields only some of the quantities of food items and also lower nutrient impact in summarizing the combination of food intake. In terms of meeting the sufficient nutrient requirement with raw food items, the model GPP with weights achieves moderate results. The GPP with priorities outperforms the GPP and GPP with weights in terms of quantity of food items. From the initial energy level (2525 Kcal) to the higher energy value (3159 Kcal), the study reveals that optimized raw food items with different quantities and meeting requirements of sufficient nutrients is the most anticipated combination of food items for the population with lower and moderate-income levels. The representation of optimal raw food items of various energy levels with different goal programming techniques is shown in Figure-1.

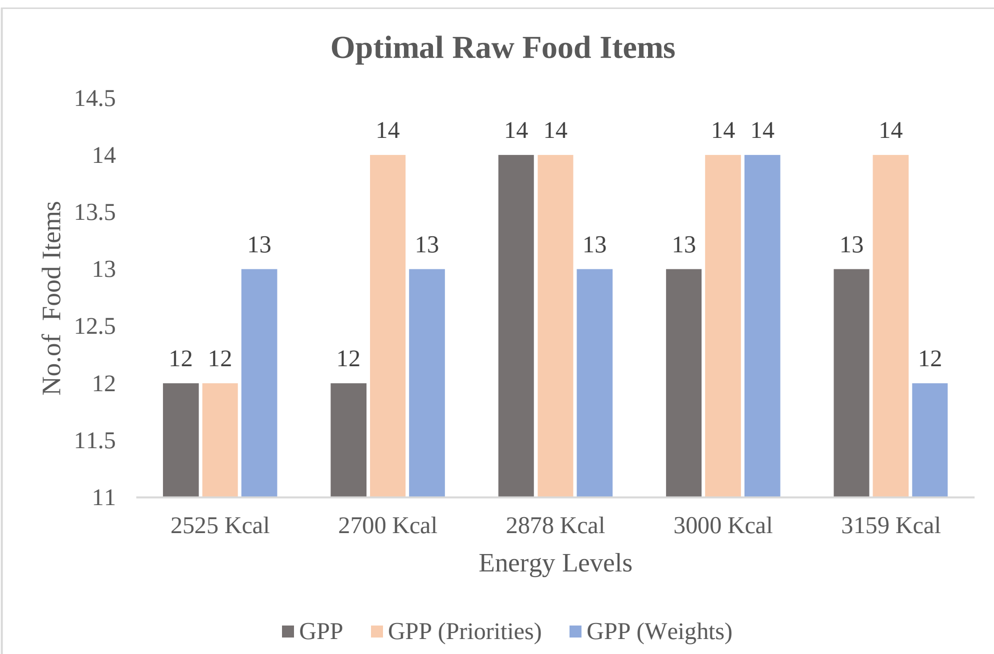

Figure-1: Histogram presenting the Optimal Raw Food Items of Energy levels by GPPs

6. Conclusion:

The study's goal is to consider energy as an objective function and other nutrients as constraints with raw food item decision variables in optimal nutrition management. It is motivated by the fact that many works reported in the literature focused only on minimizing cost barriers rather than introducing energy as an objective function. Optimal energy levels through LPP enlightened the direction in selecting a range from 2525 Kcal to 3159 Kcal that is affordable for all economic categories, particularly the rural population in India. In the case of energy levels (2878 Kcal), GPP produced much better results than LPP. The results of GPPs show that there is a significant difference between the optimal raw food item values obtained from GPP, GPP with priorities, GPP with weights for the energy levels. The objectives satisfied optimal raw food items and their corresponding nutrients that are sufficient for the survival of the common population in rural villages of India with low to moderate economic conditions.

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