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FORECASTING RAINFALL OF A REGION BY USING FUZZY TIME SERIES

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Abstract. The definition of fuzzy time series was proposed by Song and Chissom .They presented the time invariant model and the time variant model for dealing with forecasting problems. In resent year, many method have been proposed to deal with forecasting problems using fuzzy time series .In this paper we present a new method to deal with forecasting problems based on fuzzy time series .The proposed method belongs to the first order and time variant method Last 18 year total monsoon rainfall (in mm) of region Ambikapur, Chhattisgarh are used to illustrate the forecasting process of proposed method.

INTRODUCTION

Forecasting activity play an important role in our daily life. The classical time series method cannot deal with forecasting problem in which values of time series are linguistic terms represented by fuzzy sets .It has the advantage of reducing the calculation time and simplifying the calculation process. Chen et al. used the difference of the enrollment to present a method of forecast the enrollment of Alabama based on fuzzy time series. In [4], Huang extended chen's work presented and used simplify calculations with addition of heuristic rules to forecast the enrollment. In [3], chen presented a forecasting method based on high -order fuzzy time series for forecasting the enrollments of university of Alabama.In [2], chen and hwang presented a method based on fuzzy time series to forecast the temperature.[1], presented forecasting first order and time variant method for forecasting the enrollment of university of Alabama.

However the forecasting accuracy rates of the existing fuzzy time series method for forecasting enrollment are not good enough. In this paper , we present a method belonging to first order and time variant methods .It can get a higher forecasting accuracy rate for forecasting rainfall of a region Ambikapur in chhattisgarh.

FUZZY TIME SERIES

In this section we briefly review the concept of fuzzy time series .The main difference of fuzzy time series and traditional time series is that the value of fuzzy time series is represented by fuzzy sets rather than real values. Let U be the universe of discourse, where $U=\{u_1,u_2,u_3.....u_n\}$.A fuzzy sets defined in the universe of discourse U can be represented as follows:

$$A=f_A(u_1)/u_1 + f_A(u_2)/u_2 + \dots\dots\dots + f_A(u_n)/u_n, \qquad (1)$$

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where f_A denotes the membership function of the fuzzy set A, f_A:U→[0,1], and $f_A(u_i)$ denotes the degree of membership of u_i belonging to the fuzzy sets A and $f_A(u_i) \in [0,1]$,and $1 \leq i \leq n$

Definition :Let Y(t) (t =........0,1,2,......) be the universe of discourse and be a subset of R. Assume f_i (t) (i =1,2,......) are defined on Y(t), and assume that F(t) is collection of f_1(t) ,

f_2(t) , f_3(t) ,........, then F(t) is called fuzzy time series definition Y(t) (t=........0,1,2,......).

Definition: Assume that F(t) is caused by F(t-1)only ,denotes as F(t-1) → F(t) ,then this relationship can be expressed F(t) = F(t-1) → F(t) ◦ R(t,t-1) , where F(t) = F(t-1) → F(t) ◦ R(t,t-1) is called first order model of F(t) , R(t,t-1) is a fuzzy relation between F(t-1) and F(t) , and ◦ is the Max and Min composition operator.

Definition: Let R(t,t-1) be a first order model of F(t) . If for any t, R(t,t-1) = R(t-1,t-2) , F(t) is called time invariant fuzzy time series .Otherwise it is called variant time series.

Definition :Assume that fuzzified rainfall of i th year is A_j and that of (i+1) th A_k, where A_j and A_k are two fuzzy sets defined in the universe of discourse U , then fuzzy logical relationship can be represented by $A_j \rightarrow A_k$, where A_j is called the current state of the logical relationship.

Method for forecasting rainfall using fuzzy time series

The proposed method is as follows, it involves following step

Step 1: Define the universe of U and partition it into several even and equal length intervals u_1, u_2, u_3.... and u_n. For example, assume that the universe of discourse U=[500,1900] is partitioned into seven even and equal length intervals u_1,u_2,u_3 u4,u_5,u_6 and u_7 where

u_1 =[500,700] , u_2 =[700,900], u_3 =[900,1100] , u_4 =[1100,1300] , u_5 =[1300,1500] ,

u_6 =[1500,1700] , u_7 = [1700,1900].

Step 2 :Get a statistics of the distribution of the rainfall in each interval. Sort the interval based on the number of historical rainfall data in each interval from the highest to the lowest .Find the interval having the largest number of rainfall data and divide it into four sub -interval of equal length .Find the interval having second largest number of rainfall data and divide it into three sub interval of equal length .Find the interval of third largest rainfall data and divide it into two sub interval of equal length. Find the interval with fourth largest rainfall data and let the length of this interval remain unchanged .If no data is distributed in an interval , then discard this interval .For example , the distributions of rainfall data in different are summarized as shown in Table 2.

Table 1: Last 18 year total monsoon rain fall(in mm) Data time series region , Ambikapur (C.G)

Year	Rainfall(in mm)
1995	1146.7
1996	1619.7
1997	1139.4
1998	1049.3
1999	1229.5
2000	1236
2001	1820.5
2002	1086
2003	1240.6
2004	858.4
2005	952.7
2006	1066.8
2007	1046.8
2008	1358.4
2009	603.2
2010	649.7
2011	1445.5
2012	1181.8

Table 2: The distribution of rainfall data.

Intervals	[500,700]	[700,900]	[900,1100]	[1100,1300]	[1300,1500]	[1500,1700]	[1700,1900]
Number of Rainfall data	2	1	5	6	2	1	1

After executing this step ,universe of discourse [500,1900] is divided into following interval

$u_{1,1}$ = [500,600]	$u_{1,2}$ =[600,700]
u_2 = [700,900]	$u_{3,1}$ = [900, 966.66]
$u_{3,2}$=[966.66,1033.26]	$u_{3,3}$ = [1033.26,1100]
$u_{4,1}$ = [1100,1150]	$u_{4,2}$ =[1150 ,1200]
$u_{4,3}$ = [1200,1250]	$u_{4,4}$ =[1250 ,1300]
$u_{5,1}$ = [1300,1400]	$u_{5,2}$ =[1400,1500]
u_6 =[1500,1700]	u_7 = [1700,1900]

step3:Define each fuzzy set A_i based on the re-divided interval and fuzzify the rainfall data shown in Table 1, where fuzzy set A_i denote a linguistic value of rainfall data represented by a fuzzy set and $1 \leq i \leq 14$. For example , A_1=very very very very few, A_2 =very very very few ,A_3= very very few, A_4 = very few , A_6= moderate , A_7 = many , A_8 = many many , A_9 = very many, A_{10} = too many , A_{11} = too many many, A_{12} =too many many many , A_{13} =too many many many many , A_{14} =too many many many many many ,defined as follows.

$A_1 = 1/ u_{1,1} + 0.5/ u_{1,2} + 0/ u_2 + 0/ u_{3,1} + 0/ u_{3,2} + 0/ u_{3,3} + 0/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/ u_{5,1} + 0/ u_{5,2} + 0/ u_6 + 0/ u_7$

$A_2 = 0.5/ u_{1,1} + 1/ u_{1,2} + 0.5/ u_2 + 0/ u_{3,1} + 0/ u_{3,2} + 0/ u_{3,3} + 0/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/ u_{5,1} + 0/ u_{5,2} + 0/ u_6 + 0/ u_7$

$A_3 = 0/ u_{1,1} + 0.5/ u_{1,2} + 1/ u_2 + 0.5/ u_{3,1} + 0/ u_{3,2} + 0/ u_{3,3} + 0/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/ u_{5,1} + 0/ u_{5,2} + 0/ u_6 + 0/ u_7$

$A_4 = 0/ u_{1,1} + 0/ u_{1,2} + 0.5/ u_2 + 1/ u_{3,1} + 0/ u_{3,2} + 0/ u_{3,3} + 0/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/ u_{5,1} + 0/ u_{5,2} + 0/ u_6 + 0/ u_7$

$A_5 = 0/ u_{1,1} + 0/ u_{1,2} + 0/ u_2 + 0.5/ u_{3,1} + 1/ u_{3,2} + 0.5/ u_{3,3} + 0/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/ u_{5,1} + 0/ u_{5,2} + 0/ u_6 + 0/ u_7$

$A_6 = 0/ u_{1,1} + 0/ u_{1,2} + 0/ u_2 + 0/ u_{3,1} + 0.5/ u_{3,2} + 1/ u_{3,3} + 0.5/ u_{4,1} + 0/ u_{4,2} + 0/ u_{4,3} + 0/ u_{4,4}$

$+0/u_{5,1}+0/u_{5,2}+0/u_6+0/u_7$

$A_7=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0.5/u_{3,3}+1/u_{4,1}+0.5/u_{4,2}+0/u_{4,3}+0/u_{4,4}$

$+0/u_{5,1}+0/u_{5,2}+0/u_6+0/u_7$

$A_8=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0.5/u_{4,1}+1/u_{4,2}+0.5/u_{4,3}+0/u_{4,4}$

$+0/u_{5,1}+0/u_{5,2}+0/u_6+0/u_7$

$A_9=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0.5/u_{4,2}+1/u_{4,3}+0.5/u_{4,4}$

$+0/u_{5,1}+0/u_{5,2}+0/u_6+0/u_7$

$A_{10}=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0.5/u_{4,3}+1/u_{4,4}$

$+0.5/u_{5,1}+0/u_{5,2}+0/u_6+0/u_7$

$A_{11}=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0.5/u_{4,4}$

$+1/u_{5,1}+0.5/u_{5,2}+0/u_6+0/u_7$

$A_{12}=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}$

$+0.5/u_{5,1}+1/u_{5,2}+0.5/u_6+0/u_7$

$A_{13}=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}$

$+0/u_{5,1}+0.5/u_{5,2}+1/u_6+o.5/u_7$

$A_{14}=0/u_{1,1}+0/u_{1,2}+0/u_2+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}$

$$+0/u_{5,1}+0/u_{5,2}+0.5/u_6+1/u_7$$

For simplicity, the membership value of fuzzy set A_i either 0,0.5 or 1 , where $1 \leq i \leq 14$. then, fuzzify the rainfall data in Table 1 and linguistic value of data A_1 , A_2 , A_3 ,......... A_{14}. The reason for fuzzifying rainfall data into fuzzified data is to translate crisp value into fuzzy sets get a fuzzy time series.

Step 4: Establish fuzzy logical relationships based on the fuzzified rainfall data:

$A_j \rightarrow A_q,$

$A_j \rightarrow A_r$

Where the fuzzy logical relationship" $A_j \rightarrow A_q$ " denotes " if fuzzified rainfall of year n-1 is A_j ,then the fuzzified data of year n is A_q ".For example , based on the fuzzify rainfall data obtained in step 3, we can get the fuzzy logical relationship as shown in Table 3

Table3 Fuzzy logical relationship

$A_7 \rightarrow A_{13}$	$A_{13} \rightarrow A_7$	$A_7 \rightarrow A_6$
$A_6 \rightarrow A_9$	$A_9 \rightarrow A_9$	$A_9 \rightarrow A_{14}$
$A_{14} \rightarrow A_6$	$A_6 \rightarrow A_9$	$A_9 \rightarrow A_3$
$A_3 \rightarrow A_4$	$A_4 \rightarrow A_6$	$A_6 \rightarrow A_6$
$A_6 \rightarrow A_{11}$	$A_{11} \rightarrow A_2$	$A_2 \rightarrow A_{15}$
$A_{15} \rightarrow A_8$		

Step 5: Divide each interval derived in step 2 into four sub interval of equal length, where 0.25 points and 0.75 points of each interval is used as upward and down ward forecasting points of the forecasting. use the following rules to determine whether the trend of forecasting goes up and down and to forecast rainfall. Assume that the fuzzy logical relationship is $A_i \rightarrow A_j$ where A_i denotes fuzzified rainfall of year n-1 and A_j denotes fuzzified rainfall of the year n ,then (1) if $j > i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is positive , then the trend of forecasting will go up, and we use **Rule** 2 to fore cast rainfall ; (2) if $j > i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is negative , then the trend of forecasting will go down and we use **Rule 3** ; (3) if $j < i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is positive , then the trend of forecasting will go up, and we use **Rule** 2 to fore cast rainfall ; (4) if $j < i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is negative , then the trend of forecasting will go down and we use **Rule 3** to forecast the rainfall ; (5) if $j = i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is positive , then the trend of forecasting will go up, and we use **Rule** 2 to fore cast rainfall ; (6) if $j = i$ and the difference of difference of rainfall between years n-1 and n-2 and between n-2 and n-3 is negative , then the trend of forecasting will go down and we use **Rule 3** to forecast the rainfall

Rule 1, Rule 2, Rule 3 are shown as follows

Rule 1: When forecasting the data of year 1997 . there is no data before the rainfall of the year 1994, therefore we are not able to calculate difference of rainfall 1995 and 1994 and the difference of the difference year 1996 and 1995 and between 1995 and 1994. Therefore ,if |(the difference of rainfall between the year1996 and1995)|/2 >half of the length of interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will go upward , and the forecasting rainfall falls at the 0.75 of this interval ; if |(the difference of the rainfall between the year 1996 and 1995)|/2 = half of the length of interval corresponding to the fuzzified rainfall A_j with membership value equal to1, then the forecasting rainfall falls at the middle value of this interval ; if |(the difference of the rainfall between the year 1996 and 1995)|/2 < half of the length of interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will be downward, and the forecasting rainfall falls at 0.25 point of the interval.

Rule 2 : If (|difference of differences between years n-1 and n-2 and between n-2 and n-3|× 2 +the rainfall of year n-1) or (rainfall of the year n-1 -|the difference of differences between yearn-1 and n-2 and n-2 and n-3|× 2) falls in the interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will go upward , and the forecasting rainfall falls at the 0.75 of this interval corresponding to the fuzzified rainfall A_j with membership value equal to1, If (|difference of differences between years n-1 and n-2 and between n-2 and n-3|/ 2 +the rainfall of year n-1) or (rainfall of the year n-1 -|the difference of differences between year n-1 and n-2 and n-2 and n-3|/ 2) falls in the interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will go downward , and the forecasting rainfall falls at the 0.25 point of this interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ; if neigther is the case, than we let the forecasting rainfall be the middle value of the interval corresponding to the fuzzified rainfall A_j with membership value equal to 1.

Rule 3 : If (|difference of differences between years n-1 and n-2 and between n-2 and n-3|/ 2 +the rainfall of year n-1) or (rainfall of the year n-1 -|the difference of differences between year n-1 and n-2 and n-2 and n-3|/ 2) falls in the interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will go downward , and the forecasting rainfall falls at the 0.25 point of this interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ; If (|difference of differences between years n-1 and n-2 and between n-2 and n-3|× 2 +the rainfall of year n-1) or (rainfall of the year n-1 -|the difference of differences between yearn-1 and n-2 and n-2 and n-3|× 2) falls in the interval corresponding to the fuzzified rainfall A_j with membership value equal to1 ,then trend of forecasting of this interval will go upward , and the forecasting rainfall falls at the 0.75 of this interval corresponding to the fuzzified rainfall A_j with membership value equal to1 if neither is the case, than we let the forecasting rainfall be the middle value of the interval corresponding to the fuzzified rainfall A_j with membership value equal to 1. Summarizes the forecasting result of proposed method from 1995 to 2012

Table 4, Actual rainfall and forecasting rainfall of Ambikapur (C.G)

	Rainfall(in mm)	Trend of forecasting	forecasting rainfall
1995	1146.7		
1996	1619.7	middle value	1600
1997	1139.4	upward, 0.75 points	1137.5
1998	1049.3	middle value	1066.62
1999	1229.5	downward, 0.25 point	1212.5
2000	1236	middle value	1225
2001	1820.5	middle value	1800
2002	1086	middle value	1066.62
2003	1240.6	middle value	1225
2004	858.4	middle value	800
2005	952.7	middle value	933.32
2006	1066.8	middle value	1066.58
2007	1046.8	downward, 0.25 point	1049.9
2008	1358.4	upward, 0.75 points	1375
2009	603.2	middle value	650
2010	649.7	middle value	650
2011	1445.5	middle value	1450
2012	1181.8	middle value	1175

In the following, we use the mean square error (MSE) to compare the forecasting results with different forecasting method, where the mean square error is calculated as follows:

$$MSE = \frac{\left[\sum_{i=1}^{n} (Actual\_rainfall_i - forecasted\_rainfall_i)^2 \right]}{n}$$

Where Actual_ rainfall$_i$ denotes actual rainfall of year i and forecasting_ rainfall$_i$ of denotes forecasting rainfall of year i .In following Table 5 we have calculated MSE for the proposed method to compare with exiting methods given by [4] and [3]

Table 5: MSE calculation

Year	Actual rainfall	forecasted rainfall	$(A_i-F_i)^2$
1995	1146.7		
1996	1619.7	1600	388.09
1997	1139.4	1137.5	3.61
1998	1049.3	1066.62	299.98
1999	1229.5	1212.5	289
2000	1236	1225	121
2001	1820.5	1800	400
2002	1086	1066.62	375.58
2003	1240.6	1225	225
2004	858.4	800	3364
2005	952.7	933.32	375.58
2006	1066.8	1066.58	0.048
2007	1046.8	1049.9	9.61
2008	1358.4	1375	275.56
2009	603.2	650	2190.24
2010	649.7	650	0.09
2011	1445.5	1450	20.25
2012	1181.8	1175	46.24
			Σ 8094.638

$$MSE = \sum_{i=1}^{n} (Ai - Fi)^2 / n = 8094.638/14$$

$$= 578.188$$

Conclusion

In this paper we have investigated using a new method for forecasting the rainfall of the region using fuzzy time series. The proposed method belongs to the first order and time invariant method.We see that MSE of the forecasting result of proposed method is smaller than previous existing methods. That is proposed method got a higher forecasting accuracy rate for forecasting rainfall in comparison with the existing methods.

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